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HIGHER-DIMENSIONAL THEORIES, DILATON FIELDS AND SPONTANEOUS SYMMETRY BREAKING

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We consider an eight-dimensional gravitational theory, which possesses a principle fiber bundle structure, with Lorentz-scalar fields coupled to the metric. One of them plays the role of a Higgs field and the other one of a dilaton field. The effective cosmological constant is interpreted as a Higgs potential. As usual we introduce by hand the Yukawa couplings. The extra dimensions are a $SU(2)_L \times U(1)_Y \times SU(2)_R$ group manifold. A Dirac field is coupled to the metric. As a result, we obtain an effective four-dimensional theory which contains all couplings of a Weinberg–Salam–Glashow theory in a curved space–time. The true masses of the gauge bosons and of the first two fermions families are given by the theory.

1. Introduction

Interest in higher-dimensional theories has never disappeared, it has waned and waxed but has never ceased. Recently, one of the authors[1–3] has analyzed the problem of the explanation for the mass of some elementary particles in the context of higher-dimensional models. In particular,[2] a five-dimensional model was studied, where a scalar field coupled to the metric could play the role of a Higgs field. In a further work,[3] the fermionic sector of higher-dimensional theories was studied, this fermionic sector consisted of the first fundamental families, the space–time was endowed with the internal symmetry that corresponds to that of the $U(1) \times SU(2)$ group. It is noteworthy to mention that in this work there are no scalar fields at all. Unfortunately, the results yielded masses that showed some problems, namely, the neutrino is massive whereas the gauge fields associated to the weak interaction are massless and the ratio between the leptonic and hadronic masses is one third, a result clearly denied by the experiment. It must be mentioned that the fifth dimension, precisely, the radius of the S^1 circle, is the only parameter of the extra

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dimensions involved in the masses of the fermionic sector, that is, the S^3 sphere has no influence at all in these masses.

It seems that one of the most natural steps to be taken in the pursuit of this line of thought is the introduction of dilaton fields that could play the role of a Higgs field and thereby could yield the terms that are necessary to solve some of the aforementioned problems.

To consider non-Abelian symmetries[4,5] we could just take the structure of the principal fiber bundle for the whole space–time. In this case we do not need to assume any spatial dependence on the metric components. If we want to introduce the electroweak interactions the structure of the principal fiber bundle is $G - P \to^{\pi} B^4$, where B^4 is a four-dimensional Riemannian manifold and the fiber is assumed to be the group manifold of a compact non-Abelian group G, that for the particular case that comprises the electroweak interaction must be at least four-dimensional. When this is combined with the four-dimensional space–time part, we are led to a gravity theory which is at least eight-dimensional. The line element takes the form[3,6,7]

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu - I_1(x)^2 \left[dx^5 + \kappa L_1^{-1} B_\mu(x) dx^\mu\right]^2$$
$$- I(x)^\dagger I(x) \gamma_{ij}(y) \left[dy^i + \kappa L^{-1} \kappa^i_\alpha(y) A^\alpha_\mu(x) dx^\mu\right]^2 , \qquad (1.1)$$

where γ_{ij} is the metric tensor on the group manifold of G and the functions $\kappa^i_\alpha(y)$ are Killing vectors on G. The fields $A^\alpha_\mu(x)$ are gauge fields of an arbitrary non-Abelian gauge group as components of the gravitational field in $4+n$ dimensions.

In the effective four-dimensional theory, the gauge transformations are a remnant of the original coordinate invariance group in $4+n$ dimensions, which has been spontaneously broken down through dimensional reduction to the symmetries of the four-dimensional coordinate transformation group and a local gauge group; $I_1(x)$ and $I(x)$ are Lorentzian scalar fields, precising I_1 can be identified with a dilatonic field while I has isospin structure. These two fields depend only on the space–time coordinates. The most obvious step to be taken here is to consider both fields, I_1 and I, as isoscalar fields. Unfortunately, if this is done then the problems of the earlier work stand and we must dismiss this possibility.

In the present paper we construct an eight-dimensional space–time, where the new coordinates y^i have to be interpreted as a parametrization of the manifold of the non-Abelian group $\mathrm{SU}(2)_L \times \mathrm{U}(1)$. The group manifold of $\mathrm{SU}(2)_L$ is the sphere S^3 and that of U(1) is the circle S^1. Therefore the space $S^1 \times S^3$ has the desired $\mathrm{U}(1) \times \mathrm{SU}(2)_L \times \mathrm{SU}_R(2)$ symmetry[8] and it is the natural manifold for this group. In this framework we investigate an eight-dimensional gravity theory with two fields coupled to the metric [see (2.1)], which possesses a principal fiber bundle structure, where $I_1(x)$ is a singlet with respect to $\mathrm{U}(1) \times \mathrm{SU}(2)_L$; namely, it is a dilaton field with its usual linear vacuum behavior, in which it tends to a constant value, but $I(x)$ is now endowed with an isospin structure. A Dirac field is coupled to the metric one; this field contains the first two fermionic families, i.e.

$$\psi_{1a} = \begin{pmatrix} \nu_e \\ e \\ u \\ d \end{pmatrix}, \quad \psi_{2a} = \begin{pmatrix} \nu_\mu \\ \mu \\ c \\ s \end{pmatrix}, \quad a = 1, 2, 3, 4. \tag{1.2}$$

A potential term related to the field $I(x)$ that contains a mass and quartic self-interaction terms is introduced. It behaves as the effective four-dimensional cosmological constant; in other words the four-dimensional cosmological constant may be identified with this potential.[9,10] By hand, we introduce Yukawa terms, which consist of two contributions; namely, the preonic one, which compensates the preonic contribution of the fifth dimension that leads to nonphysical results and the usual Yukawa coupling which generates through GIM mechanism the true fermionic masses. This means that the group structure of the right-hand part is U(1). Through the spontaneous symmetry breaking of the $\mathrm{U}(1) \times \mathrm{SU}(2)_L$ symmetry of our Lagrangian and employing Weinberg decomposition we achieve mass terms for the gauge fields related to the weak interaction as well for the electron, muon, s, c, u, and d quarks, whereas the gauge field related to the electromagnetic interaction remains massless. The Z, W^+ and W^- bosons acquire mass and their masses are in accord with the usual relations in the four-dimensional Weinberg–Salam–Glashow theory; namely, the ratio between the mass of the Z boson and that of the W^+ or W^- is $\cos^{-2}\theta_W$, where θ_W stands for the Weinberg angle. Through the process of symmetry breaking the fermionic sector acquires mass, but there is a mass term related to both of our neutrinos that does not come from symmetry breaking. Once again[2,3] it is the influence of the fifth dimension, which produce the preonic masses of the electron, muon, u, s, c and d quarks.

We may conclude that for the electron, muon, u, s, c, and d quarks, the mass term contains two contributions — one comes from symmetry breaking and the other one emerges from the presence of the radius of the fifth dimension — as a consequence of the dimensional reduction and must be cancelled by the Yukawa couplings. This applies as well for the neutrinos.

As usual,[2,3] as a consequence of the dimensional reduction, in the four-dimensional effective theory Pauli terms emerge. They may be understood as an anomalous weak momentum and an anomalous electromagnetic momentum. In other words in this theory the neutrino has no electric charge, but it generates an electromagnetic field and this fact may be seen in the polarization currents that emerge in the Yang–Mills equations. This paper is easily extended to include the third fundamental family,

$$\psi_{3a} = \begin{pmatrix} \nu_\tau \\ \tau \\ t \\ b \end{pmatrix}. \tag{1.3}$$

This work is organized as follows. In Sec. 2 we construct the scalar curvature. In Sec. 3 we build the eight-dimensional Dirac–Lagrangian density. In Sec. 4 we calculate the Yukawa couplings and afterward carry out the breaking of symmetry.

In Sec. 5 we perform the dimensional reduction and obtain the field equations. Section 6 contains Weinberg decomposition of the field equations by means of the mixing angle and the ensuing results are then discussed.

2. Scalar Curvature

The local principal fiber bundle line element for the product space–time $M_4 \times G$, with $M_4 \subset B^4$ is

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu - I_1(x)^2 \left[dx^5 + \kappa L_1^{-1} B_\mu(x)dx^\mu\right]^2$$
$$- I(x)^\dagger I(x)\gamma_{ij}(y)\left[dy^i + \kappa L^{-1} K^i_\alpha(y) A^\alpha_\mu(x)dx^\mu\right]^2 , \tag{2.1}$$

where μ, $\nu, \ldots = 0, 1, 2, 3$; $i, j, \ldots = 5, 6, 7, 8$; $\alpha, \beta, \ldots =$ group indices. As usual, we identify $g_{\mu\nu}$ with the metric of the four-dimensional space–time, $\gamma_{ij}(y)$ is the Killing metric on $S^1 \times S^3$, $K^i_\alpha(y)$ are the Killing vectors and $A^\alpha_\mu(x)$ the corresponding gauge fields, $I_1(x)$, is a dilaton field while $I(x)$ is an isospin quadruplet. We adopt the higher dimensional analogue of the Einstein–Dirac–Higgs action

$$I_8 = \int d^4x \int d^4y \sqrt{\hat{g}} \left[\frac{1}{16\pi GV}\left(\hat{R} + V(I) + Y_u\right) + \mathcal{L}_D\right] \tag{2.2}$$

to be the basic action. Here $\hat{R}$ is the eight-dimensional scalar curvature, $V(I)$ is a Higgs potential term given as follows[9,10]:

$$V(I) = \frac{\mu^2}{2} I^\dagger I + \frac{\lambda}{4!}(I^\dagger I)^2 , \tag{2.3}$$

and $\mathcal{L}_D$ is a straightforward generalization in eight dimensions of the well-known four-dimensional Dirac–Lagrangian density, which will be seen in the next section, whereas Y_u denotes the Yukawa term

$$Y_u = h(\bar{L}IR + \bar{R}I^\dagger L) , \tag{2.4}$$

and V is the volume of the internal space.

We are going to employ the horizontal lift basis (HLB)[6,11]

$$\hat{\theta}^\nu = dx^\nu , \tag{2.5}$$

which contains no reference at all of the internal space, and

$$\hat{\theta}^i = dy^i + \frac{\kappa}{L} K^i_\alpha(y) A^\alpha_\nu(x) dx^\nu \tag{2.6}$$

as basis one-forms. The basis dual to (2.5) and (2.6) is

$$\hat{e}_\mu(x, y) = \partial_\mu - \frac{\kappa}{L} A^\alpha_\mu K^i_\alpha \partial_i , \tag{2.7}$$

$$\hat{e}_i(y) = \partial_i \,. \tag{2.8}$$

On dimensional grounds we introduce the length scales L^{-1} y L_1^{-1} of S^3 and S^1 respectively. The metric in this basis is simply

$$g_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & -g_{ij}(y) \end{pmatrix} . \tag{2.9}$$

The eight-dimensional curvature scalar is given by

$$\hat{R} = g^{\mu\rho}\hat{R}^{\lambda}_{\mu\lambda\rho} + g^{\mu\rho}\hat{R}^{k}{}_{\mu k\rho} - g^{ij}\hat{R}^{\lambda}{}_{i\lambda j} - g^{ij}\hat{R}^{k}{}_{ikj} \,, \tag{2.10}$$

or explicitly

$$\begin{aligned}\hat{R} = R &+ \frac{1}{I^\dagger I}R_{s^3} + 2(\partial_\nu \ln I_1)(\partial^\nu \ln I_1) + \frac{3}{I^\dagger I}(D_\nu I^\dagger)(D^\nu I) \\ &- \frac{1}{4}\kappa^2(I_1^2 F^{\mu\nu}F_{\mu\nu} + I^\dagger I F^{\alpha\mu\nu}F_{\alpha\mu\nu}) \,, \end{aligned} \tag{2.11}$$

where R is the scalar curvature of M_4 and R_{s^3} is the scalar curvature of $S^1 \times S^3$, which is defined by

$$R_{s^3} = -\gamma^{ij}R^{k}{}_{ikj} \,, \tag{2.12}$$

so that $R_{s^3} > 0$ for the sphere. Here we have $g_{55} = I_1^2 L_1^2$ and $g_{ij} = I^2\gamma_{ij}$.

3. Eight-Dimensional Dirac–Lagrangian Density

In order to calculate explicitly the Dirac–Lagrangian density and the scalar curvature we need the Killing vectors associated to the internal space $S^1 \times S^3$, which determine the metric γ_{ij}, through the expression $\gamma^{ij} = K^{i}{}_{\alpha}K^{j\,\alpha}$.

As basis for the tangent bundle of S^1,[12,13] we use the vector ∂_5 and for the tangent bundle of S^3 the three Killing vectors which can be written in terms of the Euler angles (θ, ρ, ψ).

$$\mathbf{K}_5 = L_1\partial_5 \,, \tag{3.1}$$

$$\mathbf{K}_6 = L[\cos\psi\partial_\theta - \sin\psi(\cot\theta\partial_\psi - \csc\theta\partial_\rho)] \,, \tag{3.2}$$

$$\mathbf{K}_7 = L[\sin\psi\partial_\theta + \cos\psi(\cot\theta\partial_\psi - \csc\theta\partial_\rho)] \,, \tag{3.3}$$

$$\mathbf{K}_8 = L\partial_\rho \,. \tag{3.4}$$

(Note that $y^6 = \theta$, $y^7 = \rho$, $y^8 = \psi$). Thus the Killing metric γ_{ij} for $S^1 \times S^3$ may be easily evaluated. It is found to be

$$\gamma_{ij} = \begin{pmatrix} L_1^2 & 0 & 0 & 0 \\ 0 & L^2 & 0 & 0 \\ 0 & 0 & L^2 & L^2\cos\theta \\ 0 & 0 & L^2\cos\theta & L^2 \end{pmatrix} . \tag{3.5}$$

We can now calculate the achtbein necessary to introduce spinors, in the HLB. The achtbein like the metric is simply the block diagonal

$$e^{\hat{A}}_{\hat{\mu}} = \begin{pmatrix} e^{A}{}_{\mu} & 0 \\ 0 & e^{(k)}_{j} \end{pmatrix} = \begin{pmatrix} e^{A}{}_{\mu} & 0 & 0 & 0 & 0 \\ 0 & I_1 L_1 & 0 & 0 & 0 \\ 0 & 0 & \tilde{I}L & 0 & 0 \\ 0 & 0 & 0 & \tilde{I}L & -\tilde{I}L\cos\theta \\ 0 & 0 & 0 & \tilde{I}L\cos\theta & \tilde{I}L\sin\theta \end{pmatrix} . \tag{3.6}$$

Here $e^{\hat{A}}_{\hat{\mu}}$ satisfy $(\hat{A}, \ldots,\ \hat{\mu}, \ldots = 0, 1, 2, \ldots, 8)$ the usual relation.

$$\hat{g}_{\hat{\mu}\hat{\nu}} = e^{\hat{A}}_{\hat{\mu}} e^{\hat{B}}_{\hat{\nu}} \eta_{\hat{A}\hat{B}} , \tag{3.7}$$

with $\eta_{\hat{A}\hat{B}} = \mathrm{diag}\,(+1, -1, \ldots, -1)$ and $e^{A}{}_{\mu}, e^{(k)}_{j}$ are the vierbeins which satisfy the usual algebra

$$g_{\mu\nu} = e^{A}_{\mu} e^{B}_{\nu} \eta_{AB} , \tag{3.8}$$

$$\gamma_{ij} = e^{(k)}_{i} e^{(l)}_{j} \delta_{kl} , \tag{3.9}$$

respectively. The spin covariant derivative is defined by

$$\hat{\nabla}_{\hat{\mu}} \psi_{jR,L} = (\hat{e}_{\hat{\mu}} + \hat{\Gamma}_{\hat{\mu}}) \psi_{jR,L} . \tag{3.10}$$

Here $\hat{\Gamma}_{\hat{\mu}}$ are the spin connections

$$\hat{\Gamma}_{\hat{\mu}} = \frac{1}{2} e^{\hat{\nu}}_{\hat{A}}\, e_{\hat{B}\hat{\nu};\hat{\mu}}\, \sigma^{\hat{A}\hat{B}} , \tag{3.11}$$

with

$$\sigma^{\hat{A}\hat{B}} = \frac{1}{4} [\Gamma^{\hat{A}}, \Gamma^{\hat{B}}] . \tag{3.12}$$

The size of the eight-dimensional spinors is sixteen. Let γ^A and γ^k denote the Dirac matrices on M_4 and $S^1 \times S^3$ respectively. Then we may take the Dirac matrices on $M_4 \times S^1 \times S^3$ to be given by the following tensor products:

$$\begin{aligned} \Gamma^A &= I \otimes \gamma^A , \quad A = 0, 1, 2, 3 , \\ \Gamma^k &= \gamma^k \otimes \hat{\gamma}^5 , \quad k = 5, 6, 7, 8 , \end{aligned} \tag{3.13}$$

where $\hat{\gamma}^5$ is the usual γ^5-matrix on M_4. The matrices in (3.13) satisfy

$$\{\Gamma^{\hat{A}}, \Gamma^{\hat{B}}\} = 2\eta^{\hat{A}\hat{B}} . \tag{3.14}$$

Here γ^A are the usual four-dimensional Dirac matrices and γ^k are given by

$$\gamma^5 = \begin{pmatrix} i1 & 0 \\ 0 & -i1 \end{pmatrix} , \quad \gamma^l = \begin{pmatrix} 0 & \sigma^l \\ -\sigma^l & 0 \end{pmatrix} , \quad l = 6, 7, 8 . \tag{3.15}$$

The eight-dimensional Dirac–Lagrangian density is defined by

$$\mathcal{L}_D = \sum_{j=1}^{2} \frac{i}{2}(\bar{\Psi}^{jL}\Gamma^{\hat{A}}\ e_{\hat{A}}^{\ \hat{\mu}}\ \hat{\nabla}_{\hat{\mu}}\Psi_{jL} - e_{\hat{A}}^{\ \hat{\mu}}\ \hat{\nabla}_{\hat{\mu}}\bar{\Psi}^{jL}\Gamma^{\hat{A}}\Psi_{jL})$$

$$+\sum_{j=1}^{2} \frac{i}{2}(\bar{\Psi}^{jR}\Gamma^{\hat{A}}\ e_{\hat{A}}^{\ \hat{\mu}}\ \hat{\nabla}_{\hat{\mu}}\Psi_{jR} - e_{\hat{A}}^{\ \hat{\mu}}\ \hat{\nabla}_{\hat{\mu}}\bar{\Psi}^{jR}\Gamma^{\hat{A}}\Psi_{jR}), \tag{3.16}$$

where Ψ_{jL} is the left-hand part and Ψ_{jR} is the right-hand part of our two fundamental families and the covariant derivatives for Ψ_{jR} and Ψ_{jL} are given by

$$\nabla_\nu \Psi_{jR} = \left[\partial_\nu - \frac{\kappa}{L_1}B_\nu\partial_5 + \Gamma_\nu\right]\Psi_{jR}, \tag{3.17}$$

$$\nabla_\nu \Psi_{jL} = \left[\partial_\nu - \frac{\kappa}{L_1}B_\nu\partial_5 - \frac{\kappa}{L}A_\nu^\alpha K_\alpha^i\partial_i + \Gamma_\nu\right]\Psi_{jL}. \tag{3.18}$$

We choose the following x^i dependence for the left part of our Dirac spinor in terms of the Euler angles in S^3:

$$\Psi_{jL}(x^\mu, y^i) = V^{-1/2}e^{iy^5Y_L/2}e^{i\rho\tau^3/2}e^{i\theta\tau^1/2}e^{i\psi\tau^3/2}\psi_{jL}(x^\mu), \quad j = 1,2, \tag{3.19}$$

where we have

$$\psi_{1a}(x^\mu) = \begin{pmatrix} \nu_e \\ e \\ u \\ d \end{pmatrix}, \quad \psi_{2a}(x^\mu) = \begin{pmatrix} \nu_\mu \\ \mu \\ c \\ s \end{pmatrix}. \tag{3.20}$$

The right-hand part of our Dirac fields is a singlet with respect to SU(2). As a consequence we may choose the following dependence for the right-hand part:

$$\Psi_{jR}(x^\mu, y^i) = V^{-1/2}e^{iy^5Y_R/2}\tilde{\psi}_{jR}(x^\mu), \quad j = 1,2, \tag{3.21}$$

where $\tilde{\psi}_{jR} = e_R, \mu_R, u_R, c_R, d_R, s_R$ are the right handed singlets.

Here Y_L is the left handed fermionic hypercharge matrix and Y_R are the corresponding right handed values, and τ^i are the SU(2) generators which are given by

$$Y_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & -1/3 \end{pmatrix}, \quad \tau^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{3.22}$$

with $Y_R = -2$ for right handed leptons e_R, μ_R, and $Y_R = \frac{4}{3}$ for right handed quarks u_R, c_R and $Y_R = -\frac{2}{3}$ for d_R, s_R.

$$\tau^2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{3.23}$$

where τ^i satisfy the usual SU(2) algebra

$$[\tau^i, \tau^j] = 2\delta^{ij}\mathbf{1}\,, \quad [\tau^i, \tau^j] = 2i\epsilon^{ijk}\tau_k\,. \tag{3.24}$$

The form for the hypercharge matrix, Y_ψ has been selected by convenience to obtain the correct values of the electric charge, using the Gellman–Nishima expression[14]

$$Q = T_3 + \frac{Y}{2}\,, \tag{3.25}$$

Q being the electric charge.

4. Yukawa Couplings, GIM Mechanism and Symmetry Breaking

The effective four-dimensional potential is given by

$$V(I) = \frac{\mu^2}{2}(I^\dagger I) + \frac{\lambda}{4!}(I^\dagger I)^2\,, \tag{4.1}$$

where we have the isospin quadruplet given by

$$I_a = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}\,. \tag{4.2}$$

The Yukawa couplings term for the leptons may be written as

$$\begin{aligned} \mathcal{L}_{Yl} = \tilde{G}_1[(\bar{\nu}_e, \bar{e}, 0, 0)_L I e_R + \bar{e}_R I^\dagger(\nu_e, e, 0, 0)^t_L] \\ + \tilde{G}_2[(\bar{\nu}_e, \bar{e}, 0, 0)_L I \mu_R + \bar{\mu}_R I^\dagger(\nu_e, e, 0, 0)^t_L] \\ + \tilde{G}_3[(\bar{\nu}_\mu, \bar{\mu}, 0, 0)_L I e_R + \bar{e}_R I^\dagger(\nu_\mu, \mu, 0, 0)^t_L] \\ + \tilde{G}_4[(\bar{\nu}_\mu, \bar{\mu}, 0, 0)_L I \mu_R + \bar{\mu}_R I^\dagger(\nu_\mu, \mu, 0, 0)^t_L]\,. \end{aligned} \tag{4.3}$$

Consider the matrix C defined by the expression

$$C = i\tau_2\,. \tag{4.4}$$

It is readily seen that if we define $\tilde{I}$ as

$$\tilde{I} = C\,{}^t I^\dagger\,, \tag{4.5}$$

then $\tilde{I}$ is an isospin quadruplet, but it has as hypercharge matrix the negative of the hypercharge associated to I.

For the hadronic part, using Cabbibo angle, θ_c, we may introduce the well-known Cabbibo mixture

$$d_c = d\cos(\theta_c) + s\sin(\theta_c)\,, \tag{4.6}$$

$$s_c = s\cos(\theta_c) - d\sin(\theta_c)\,. \tag{4.7}$$

Let us now proceed to carry out the GIM mixture[18,19] in the usual way

$$
\begin{aligned}
\tilde{\Psi}_{1L} &= \Psi_{1L}\cos(\alpha) - \Psi_{2L}\sin(\alpha)\,,\\
\tilde{\Psi}_{2L} &= \Psi_{2L}\cos(\alpha) + \Psi_{1L}\sin(\alpha)\,,\\
\tilde{u}_R &= u_R\cos(\beta) - c_R\sin(\beta)\,,\\
\tilde{c}_R &= c_R\cos(\beta) + u_R\sin(\beta)\,,\\
\tilde{d}_R &= d_R\cos(\gamma) - s_R\sin(\gamma)\,,\\
\tilde{s}_R &= s_R\cos(\gamma) + d_R\sin(\gamma)\,.
\end{aligned}
\tag{4.8}
$$

These last definitions enable us to write the Yukawa couplings for the hadronic part as

$$
\begin{aligned}
\mathcal{L}_{Yh} = {} & \tilde{G}_5[(0,0,\bar{u},\bar{d}_c)_L I d_R + \bar{d}_R I^\dagger(0,0,u,d_c)^t_L]\\
&+ \tilde{G}_6[(0,0,\bar{u},\bar{d}_c)_L I s_R + \bar{s}_R I^\dagger(0,0,u,d_c)^t_L)]\\
&+ \tilde{G}_7[(0,0,\bar{c},\bar{s}_c)_L I d_R + \bar{d}_R I^\dagger(0,0,c,s_c)^t_L]\\
&+ \tilde{G}_8[(0,0,\bar{c},\bar{s}_c)_L I s_R + \bar{s}_R I^\dagger(0,0,c,s_c)^t_L]\\
&+ \tilde{G}_9[(0,0,\bar{u},\bar{d}_c)_L \tilde{I} u_R + \bar{u}_R \tilde{I}^\dagger(0,0,u,d_c)^t_L]\\
&+ \tilde{G}_{10}[(0,0,\bar{u},\bar{d}_c)_L \tilde{I} c_R + \bar{c}_R \tilde{I}^\dagger(0,0,u,d_c)^t_L]\\
&+ \tilde{G}_{11}[(0,0,\bar{c},\bar{s}_c)_L \tilde{I} u_R + \bar{u}_R \tilde{I}^\dagger(0,0,c,s_c)^t_L]\\
&+ \tilde{G}_{12}[(0,0,\bar{c},\bar{s}_c)_L \tilde{I} c_R + \bar{c}_R \tilde{I}^\dagger(0,0,c,s_c)^t_L]\,.
\end{aligned}
\tag{4.9}
$$

We know that there is a set of points that render a minimum for our potential, these points meet the condition

$$
I^{0\dagger} I^0 = \sum_{j=1}^{4} |\phi_j^0| = a^2 = -\frac{6\mu^2}{\lambda}\,. \tag{4.10}
$$

We will now proceed to break the symmetry.[15-17] As usual, we have two Goldstone bosons, namely ϕ_1 and ϕ_3.

Thus, we have the following hypercharge matrix for I:

$$
Y_I = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{4.11}
$$

Consequently, the hypercharge matrix of $\tilde{I}$ is

$$
Y_{\tilde{I}} = \begin{pmatrix} -1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{4.12}
$$

The ground state for the Higgs field is chosen as

$$I^0 = \begin{pmatrix} 0 \\ am \\ 0 \\ an \end{pmatrix}, \quad \tilde{I}^0 = \begin{pmatrix} am \\ 0 \\ an \\ 0 \end{pmatrix}, \tag{4.13}$$

where $n^2 + m^2 = 1$; without any loss of generality we may assume $m > 0$ and $n > 0$.

The covariant derivative of the I field is

$$D_\nu I = \left[\partial_\nu - \frac{i}{2} g_1 B_\nu Y_I - \frac{i}{2} g_2 A_\nu^\alpha \tau_\alpha\right] I. \tag{4.14}$$

Employing (4.2) and (4.12) in Eq. (4.3), we find that the Yukawa term, after symmetry breaking, for the leptons is given as follows:

$$\mathcal{L}_{Yl} = G_1 am(\bar{e}_L e_R + \bar{e}_R e_L) + G_2 am(\bar{\mu}_R \mu_L + \bar{\mu}_L \mu_R), \tag{4.15}$$

where G_1 and G_2 are linear combinations of $\tilde{G}_j$, with $j = 1, 2, 3, 4$.

The GIM mechanism[18,19] allows us to choose the angles α, β and γ such that $G_5 = G_9$, $G_6 = G_{10}$ and $G_{11} = G_{12} = 0$, G_5 to G_{12} being linear combinations of $\tilde{G}_j$, $j = 5, \ldots, 12$. Therefore we are entitled to cast Eq. (4.9) as

$$\begin{aligned} \mathcal{L}_{Yh} = {} & G_5 an(\bar{d}_L d_R + \bar{d}_R d_L) + G_6 an(\bar{s}_L s_R + \bar{s}_R s_L) \\ & + G_7 an(\bar{u}_L u_R + \bar{u}_R u_L) + G_8 an(\bar{c}_L c_R + \bar{c}_R c_L). \end{aligned} \tag{4.16}$$

In order to achieve the Weinberg–Salam–Glashow model in the low energy limit we will add to the Yukawa terms a preonic contribution to compensate for the preonic influence of the fifth dimension on the fermionic masses, which leads to results excluded by the experimental data.[3]

5. Dimensional Reduction and Field Equations

We are going to carry out a conformal transformation in the internal metric g_{ij}. The idea is to isolate General Relativity from the fields I_1 and I. This implies that the determinant of the internal metric will be taken as the conformal factor. By this means the determinant of the transformed metric becomes one. This fact leads us to write

$$\hat{g}_{ij} = (I_1^2 I^6 \sin^2\theta)^{-\frac{1}{4}} g_{ij}. \tag{5.1}$$

The new internal metric is written down in the following way:

$$\hat{g}_{ij} = (I_1^2 I^6 \sin^2\theta)^{-\frac{1}{4}} \begin{pmatrix} I_1^2 L_{\ 1}^2 & 0 & 0 & 0 \\ 0 & I^2 L^2 & 0 & 0 \\ 0 & 0 & I^2 L^2 & I^2 L^2 \cos\theta \\ 0 & 0 & I^2 L^2 \cos\theta & I^2 L^2 \end{pmatrix}. \tag{5.2}$$

This could be done in a different manner,[20,21] performing two conformal transformations, one in the external and the other in the internal space. The first one isolates General Relativity from the scalar fields and the second one isolates the Yang–Mills field strength from these fields. In our case it is enough to perform only the internal conformal transformation, because we interpret the effective four-dimensional cosmological constant as the Higgs potential.[9,10]

The ensuing achtbein is

$$e^{\hat{A}}_{\hat{\mu}} = \begin{pmatrix} e^{A}{}_{\mu} & 0 \\ 0 & e^{(k)}_{j} \end{pmatrix} = \begin{pmatrix} e^{A}_{\mu} & 0 & 0 & 0 & 0 \\ 0 & I_1 L_1 & 0 & 0 & 0 \\ 0 & 0 & \tilde{I}L & 0 & 0 \\ 0 & 0 & 0 & \tilde{I}L\cos\frac{\theta}{2} & -\tilde{I}L\sin\frac{\theta}{2} \\ 0 & 0 & 0 & \tilde{I}L\cos\frac{\theta}{2} & \tilde{I}L\sin\frac{\theta}{2} \end{pmatrix}. \tag{5.3}$$

The volume element dv_y in terms of the Euler angles for S^3 is given as follows:

$$dv_y = dy^5 L^3 \sin\theta \, d\theta \, d\rho \, d\psi \,, \tag{5.4}$$

where $0 \leq y^5 \leq 2\pi L_1$, $0 \leq \theta \leq \pi$, $0 \leq \rho \leq 2\pi$, $0 \leq \psi \leq 4\pi$ y $\sqrt{\tilde{g}} = \sqrt{-g} L_1 L^3 \sin\theta$.

Performing the integration over S^1 and S^3 the action (2.2) reduces to

$$\begin{aligned}
I_4 = \int d^4x\sqrt{-g}\Bigg\{ & \frac{1}{16\pi G}\bigg[R + 2(\partial_\nu \ln I_1)(\partial^\nu \ln I_1) + \frac{3}{I^\dagger I}[(D_\nu I^\dagger)(D^\nu I) \\
& + I^\dagger I V(I)] - \frac{1}{4}\kappa^2(I_1^2 F^{\mu\nu}F_{\mu\nu} + I^\dagger I F^{\alpha\mu\nu}F_{\alpha\mu\nu}) + \mathcal{L}_{Yl} + \mathcal{L}_{Yh}\bigg] \\
& + \sum_{j=1}^{2} \frac{i}{2}\bar{\Psi}_{jL} e^\nu_A \Gamma^A \left[\partial_\nu - \frac{i}{2}g_1 B_\nu Y_\psi - \frac{i}{2}g_2 A^\alpha_\nu \tau_\alpha + \Gamma_\nu\right]\Psi_{jL} \\
& + \sum_{j=1}^{2} \frac{i}{2}\bar{\Psi}_{jR} e^\nu_A \Gamma^A \left[\partial_\nu - \frac{i}{2}g_1 B_\nu Y_\psi + \Gamma_\nu\right]\Psi_{jR} \\
& - \sum_{j=1}^{2}\bigg[\frac{i}{2}e^\nu_A\left[\partial_\nu\bar{\Psi}_{jL} - \frac{i}{2}g_1 B_\nu \bar{\Psi}_{jL} Y_\psi - \frac{i}{2}g_2 A^\alpha_\nu \bar{\Psi}_{jL}\tau_\alpha + \Gamma_\nu \bar{\Psi}_{jL}\right]\Gamma^A\Psi_{jL} \\
& + \frac{1}{2I_1L_1}\bar{\Psi}_{jL}\Gamma^5\Psi_{jL} - \frac{i}{2}\kappa I_1 e^\mu_A e^\nu_B F_{\nu\mu}\bar{\Psi}_{jL}\sigma^{AB}\Gamma^5\Psi_{jL}\bigg] \\
& - \sum_{j=1}^{2}\bigg[\frac{i}{2}e^\nu_A\left[\partial_\nu\bar{\Psi}_{jR} - \frac{i}{2}g_1 B_\nu \bar{\Psi}_{jR} Y_\psi + \bar{\Psi}_{jR}\Gamma_\nu\right]\Gamma^A\Psi_{jR} \\
& + \frac{1}{2I_1L_1}\bar{\Psi}_{jR}\Gamma^5\Psi_{jR} - \frac{i}{2}\kappa I_1 e^\mu_A e^\nu_B F_{\nu\mu}\bar{\Psi}_{jR}\sigma^{AB}\Gamma^5\Psi_{jR}\bigg]\Bigg\}\,,
\end{aligned} \tag{5.5}$$

where $\mathcal{L}_{Yl}$ and $\mathcal{L}_{Yh}$ are given by Eqs. (4.3) and (4.9), respectively. Note that we have made the substitution

$$g_1 = \frac{\kappa}{L_1}, \quad g_2 = \frac{\kappa}{L}, \tag{5.6}$$

where g_1, g_2 are the U(1) and SU(2) coupling constants respectively.

Now we make the usual identification, namely

$$\kappa^2 = 16\pi G. \tag{5.7}$$

Notice that it is possible to elicit, from this four-dimensional action, (i) the Einstein–Yang–Mills action, (ii) there is also the complete fermionic action of the Weinberg–Salam–Glashow model in a curved space–time, (iii) we have Pauli terms and mass terms, and (iv) our Higgs field shows up.

Performing the variation with respect to the gauge fields we find that the Yang–Mills equations turn out to be

$$\begin{aligned}(I_1^2 F^{\mu\nu})_{;\nu} &+ \frac{3ig_1}{16\pi G}[I^\dagger Y_I D^\mu I - (D^\mu I)^\dagger Y_I I] \\ &= \frac{g_1}{2} e^\mu_A \sum_{j=1}^{2} [\bar{\Psi}_{jL} \Gamma^A Y_\psi \Psi_{jL}] + \frac{g_1}{2} e^\mu_A \sum_{j=1}^{2} [\bar{\Psi}_{jR} \Gamma^A Y_\psi \Psi_{jR}] \\ &\quad + \sum_{j=1}^{2} (\kappa I_1 e^\mu_A e^\nu_B [\bar{\Psi}_{jL} \sigma^{AB} \Gamma^5 \Psi_{jL} + \bar{\Psi}_{jR} \sigma^{AB} \Gamma^5 \Psi_{jR}])_{;\nu}\,,\end{aligned} \tag{5.8}$$

$$\begin{aligned}(I^\dagger I F^{\alpha\mu\nu})_{;\nu} &+ g_2 I^\dagger I F^{\nu\mu}_\gamma A_{\beta\nu} f^{\alpha\gamma\beta} + \frac{3ig_2}{16\pi G} I^\dagger I [I^\dagger \tau^\alpha D^\mu I - (D^\mu I)^\dagger \tau^\alpha I] \\ &= \frac{g_2}{2} e^\mu_A \sum_{j=1}^{2} [\bar{\Psi}_{jL} \Gamma^A \sigma^\alpha \Psi_{jL}].\end{aligned} \tag{5.9}$$

Performing now the variation with respect to the left-hand and right-hand parts of Ψ_1 and Ψ_2 we obtain the Dirac equations:

$$\begin{aligned}&ie^\nu_A \Gamma^A \left[\partial_\nu - \frac{i}{2} g_1 B_\nu Y_\psi - \frac{i}{2} g_2 A^\alpha_\nu \tau_\alpha + \Gamma_\nu\right] \Psi_{1L} - \frac{1}{2I_1 L_1} \Gamma^5 Y_\psi \Psi_{1L} \\ &\quad - \frac{i}{2} I_1 \kappa e^\mu_A e^\nu_B F_{\nu\mu} \sigma^{AB} \Gamma^5 \Psi_{1L} + Y_{1L} = 0\,,\end{aligned} \tag{5.10}$$

$$ie^{\nu}_{A}\Gamma^{A}\left[\partial_{\nu}-\frac{i}{2}g_{1}B_{\nu}Y_{\psi}-\frac{i}{2}g_{2}A^{\alpha}_{\nu}\tau_{\alpha}+\Gamma_{\nu}\right]\Psi_{2L}-\frac{1}{2I_{1}L_{1}}\Gamma^{5}Y_{\psi}\Psi_{2L}$$

$$-\frac{i}{2}I_{1}\kappa e^{\mu}_{A}e^{\nu}_{B}F_{\nu\mu}\sigma^{AB}\Gamma^{5}\Psi_{2L}+Y_{2L}=0\,, \tag{5.11}$$

$$ie^{\nu}_{A}\Gamma^{A}\left[\partial_{\nu}-\frac{i}{2}g_{1}B_{\nu}Y_{\psi}+\Gamma_{\nu}\right]\Psi_{1R}-\frac{1}{2I_{1}L_{1}}\Gamma^{5}Y_{\psi}\Psi_{1R}$$

$$-\frac{i}{2}I_{1}\kappa e^{\mu}_{A}e^{\nu}_{B}F_{\nu\mu}\sigma^{AB}\Gamma^{5}\Psi_{1R}+Y_{1R}=0\,, \tag{5.12}$$

$$ie^{\nu}_{A}\Gamma^{A}\left[\partial_{\nu}-\frac{i}{2}g_{1}B_{\nu}Y_{\psi}+\Gamma_{\nu}\right]\Psi_{2R}-\frac{1}{2I_{1}L_{1}}\Gamma^{5}Y_{\psi}\Psi_{2R}$$

$$-\frac{i}{2}I_{1}\kappa e^{\mu}_{A}e^{\nu}_{B}F_{\nu\mu}\sigma^{AB}\Gamma^{5}\Psi_{2R}+Y_{2R}=0\,, \tag{5.13}$$

where the Yukawa couplings are given after symmetry breaking by

$$\begin{gathered}
Y_{1L}=\begin{pmatrix} -\frac{1}{2L_1I_1}\nu_e \\ -\frac{1}{2L_1I_1}e_L+G_1ame_R \\ -\frac{1}{6L_1I_1}u_L+G_2amu_R \\ -\frac{1}{6L_1I_1}d_L+G_3amd_R \end{pmatrix}, \\
Y_{2L}=\begin{pmatrix} -\frac{1}{2L_1I_1}\nu_\mu \\ -\frac{1}{2L_1I_1}\mu_L+G_4am\mu_R \\ -\frac{1}{6L_1I_1}c_L+G_5amc_R \\ -\frac{1}{6L_1I_1}s_L+G_6ams_R \end{pmatrix},
\end{gathered} \tag{5.14}$$

$$\begin{gathered}
Y_{1R}=\begin{pmatrix} Y_{e_R}=-\frac{1}{2L_1I_1}e_R+G_1ame_L \\ Y_{u_R}=-\frac{1}{6L_1I_1}u_R+G_2amu_L \\ Y_{d_R}=-\frac{1}{6L_1I_1}d_R+G_3amd_L \end{pmatrix}, \\
Y_{2R}=\begin{pmatrix} Y_{\mu_R}=-\frac{1}{2L_1I_1}\mu_R+G_4am\mu_L \\ Y_{c_R}=-\frac{1}{6L_1I_1}c_R+G_5amc_L \\ Y_{s_R}=-\frac{1}{6L_1I_1}s_R+G_6ams_L \end{pmatrix}.
\end{gathered} \tag{5.15}$$

The Einstein equations are

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}V(I) = 8\pi \Bigg\{ & -\frac{i}{2}\sum_{j=1}^{2}[\bar{\Psi}_{jL}\Gamma_\mu\nabla_\nu\Psi_{jL} - \nabla_\nu\bar{\Psi}_{jL}\Gamma_\mu\Psi_{jL}] \\
& -\frac{i}{2}\sum_{j=1}^{2}[\bar{\Psi}_{jR}\Gamma_\mu\nabla_\nu\Psi_{jR} - \nabla_\nu\bar{\Psi}_{jR}\Gamma_\mu\Psi_{jR}] \\
& - i\kappa I_1\sum_{j=1}^{2} e_{A\nu}e_B^\rho F_{\mu\rho}[\bar{\Psi}jL\Gamma^5\Psi_{jL} + \bar{\Psi}jR\Gamma^5\Psi_{jR}] \\
& + I_1^2\left[F_\mu^\rho F_{\nu\rho} - \frac{1}{4}g_{\nu\mu}F^{\lambda\rho}F_{\lambda\rho}\right] \\
& + I^\dagger I\left[F_\mu^{\alpha\rho}F_{\alpha\nu\rho} - \frac{1}{4}g_{\nu\mu}F^{\alpha\lambda\rho}F_{\alpha\lambda\rho}\right] \\
& \times\frac{2}{I_1^2}\left[(\partial_\mu I_1)(\partial_\nu I_1) - \frac{1}{2}g_{\mu\nu}(\partial^\lambda I_1)(\partial_\lambda I_1)\right] \\
& + \frac{6}{(I^\dagger I)^2}\left[(\nabla_\mu I^\dagger)(\nabla_\nu I) - \frac{1}{4}g_{\mu\nu}(\nabla^\lambda I^\dagger)(\nabla_\lambda I)\right]\Bigg\}\,.
\end{aligned}
\tag{5.16}
$$

The field equation for the dilaton field is

$$
\begin{aligned}
& \frac{4}{I_1^2}D^\lambda D_\lambda I_1 + \kappa^2 I_1 F^{\mu\nu}F_{\mu\nu} + \frac{1}{I_1^2 L_1}\sum_{j=1}^{2}[\bar{\Psi}_{jL}\Gamma^5\Psi_{jL} + \bar{\Psi}_{jR}\Gamma^5\Psi_{jR}] \\
& \quad - \frac{i}{2}\kappa e_A^\mu e_B^\nu F_{\nu\mu}\bar{\Psi}_{jL}\sigma^{AB}\Gamma^5\Psi_{jL} - \frac{i}{2}\kappa e_A^\mu e_B^\nu F_{\nu\mu}\bar{\Psi}_{jR}\sigma^{AB}\Gamma^5\Psi_{jR} = 0\,.
\end{aligned}
\tag{5.17}
$$

The field equation for the Higgs field is

$$
\begin{aligned}
& \left[D^\nu D_\nu I + \mu^2 I + \frac{\lambda}{6}(I^\dagger I)I)\right] - \frac{1}{4}IF^{\alpha\nu\mu}F_{\alpha\nu\mu} \\
& \quad - 2[\tilde{G}_1\bar{e}_R(\nu_e, e, 0, 0)_L^t + \tilde{G}_2\bar{\mu}_R(\nu_e, e, 0, 0)_L^t + \tilde{G}_3\bar{e}_R(\nu_\mu, \mu, 0, 0)_L^t \\
& \quad + \tilde{G}_4\bar{\mu}_R(\nu_\mu, \mu, 0, 0)_L^t + \tilde{G}_5\bar{d}_R(0, 0, u, d_c)_L^t + \tilde{G}_6\bar{s}_R(0, 0, u, d_c)_L^t \\
& \quad + \tilde{G}_7\bar{d}_R(0, 0, c, s_c)_L^t + \tilde{G}_8\bar{s}_R(0, 0, c, s_c)_L^t + \tilde{G}_9\bar{u}_R C^t(0, 0, u, d_c)_L^t \\
& \quad + \tilde{G}_{10}\bar{c}_R C^t(0, 0, u, d_c)_L^t + \tilde{G}_{11}\bar{u}_R C^t(0, 0, u, s_c)_L^t + \tilde{G}_{12}\bar{c}_R C^t(0, 0, u, s_c)_L^t] = 0\,.
\end{aligned}
\tag{5.18}
$$

We have now all the field equations and using the symmetry breaking of Sec. 4, we proceed to make the Weinberg decomposition in order to obtain the final equations and to interpret the results. This will be done in the next section.

6. Weinberg Decompositions

We proceed to carry out the decomposition of the field equations, to do so we employ the Weinberg decomposition[22]

$$Z_\mu = A^3_\mu c_W + B_\mu s_W\,, \tag{6.1}$$

$$A_\mu = -A^3_\mu s_W + B_\mu c_W\,, \tag{6.2}$$

$$W^\pm_\mu = A^1_\mu \mp i A^2_\mu\,, \tag{6.3}$$

where $C_W = \cos(\theta_W)$, $s_W = \sin(\theta_W)$ and θ_W is the Weinberg angle. A_μ is now the electromagnetic gauge field and Z_μ the neutral boson. Proceeding as usual in the Weinberg–Salam theory,

$$Z_{\mu\nu} = Z_{\nu;\mu} - Z_{\mu;\nu}\,, \tag{6.4}$$

$$A_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}\,, \tag{6.5}$$

$$W^\pm_{\mu\nu} = W^\pm_{\nu;\mu} - W^\pm_{\mu;\nu}\,, \tag{6.6}$$

the electron charge is given by

$$\hat{e} = g_1 c_w = g_2 s_w\,. \tag{6.7}$$

The decomposed field equations are the following:

Dirac equations:

for ν_e

$$\begin{aligned} &i\gamma^\mu(\partial_\mu + \Gamma_\mu)\nu_e + \frac{1}{2}g_2\gamma^\mu W^+_\mu e_L + \frac{1}{2}(g_2 c_w + g_1 s_w)\gamma^\mu Z_\mu \nu_e \\ &\quad + \frac{i}{2} I_1 e_A{}^\mu e_B{}^\nu \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB}\nu_e = 0\,, \end{aligned} \tag{6.8}$$

for e_L

$$\begin{aligned} &i\gamma^\mu(\partial_\mu - i\hat{e}A_\mu + \Gamma_\mu)e_L + \frac{1}{2}g_2\gamma^\mu W^-_\mu \nu_e - \frac{1}{2}(g_2 c_w - g_1 s_w)\gamma^\mu Z_\mu e_L \\ &\quad + \frac{i}{2} I_1 e^\mu_A e^\nu_B \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} e_L + G_1 ame_R = 0\,, \end{aligned} \tag{6.9}$$

for u_L

$$\begin{aligned} &i\gamma^\mu\left(\partial_\mu + \frac{2}{3}i\hat{e}A_\mu + \Gamma_\mu\right)u_L + \frac{1}{2}g_2\gamma^\mu W^+_\mu d_L + \frac{1}{2}\left(g_2 c_w - \frac{1}{3}g_1 s_w\right)\gamma^\mu Z_\mu u_L \\ &\quad - \frac{i}{2} I_1 e^\mu_A e^\nu_B \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} u_L + G_2 anu_R = 0\,, \end{aligned} \tag{6.10}$$

for d_L

$$i\gamma^\mu\left(\partial_\mu - \frac{1}{3}i\hat{e}A_\mu + \Gamma_\mu\right)d_L + \frac{1}{2}g_2\gamma^\mu W_\mu^- u_L - \frac{1}{2}\left(g_2 c_w + \frac{1}{3}g_1 s_w\right)\gamma^\mu Z_\mu d_L$$
$$- \frac{i}{2}I_1 e_A^{\ \mu} e_B^{\ \nu}\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} d_L + G_3 an d_R = 0\,, \tag{6.11}$$

for ν_μ

$$i\gamma^\mu(\partial_\mu + \Gamma_\mu)\nu_\mu + \frac{1}{2}g_2\gamma^\mu W_\mu^+ \mu_L + \frac{1}{2}(g_2 c_w + g_1 s_w)\gamma^\mu Z_\mu \nu_\mu$$
$$+ \frac{i}{2}I_1 e_A{}^\mu e_B{}^\nu\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB}\nu_\mu = 0\,, \tag{6.12}$$

for μ_L

$$i\gamma^\mu(\partial_\mu - i\hat{e}A_\mu + \Gamma_\mu)\mu_L + \frac{1}{2}g_2\gamma^\mu W_\mu^- \nu_\mu - \frac{1}{2}(g_2 c_w - g_1 s_w)\gamma^\mu Z_\mu \mu_L$$
$$+ \frac{i}{2}I_1 e_A^\mu e_B^\nu\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB}\mu_L + G_4 am\mu_R = 0\,, \tag{6.13}$$

for c_L

$$i\gamma^\mu\left(\partial_\mu + \frac{2}{3}i\hat{e}A_\mu + \Gamma_\mu\right)c_L + \frac{1}{2}g_2\gamma^\mu W_\mu^+ s_L + \frac{1}{2}\left(g_2 c_w - \frac{1}{3}g_1 s_w\right)\gamma^\mu Z_\mu c_L$$
$$- \frac{i}{2}I_1 e_A^{\ \mu} e_B^{\ \nu}\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} c_L + G_5 anc_R = 0\,, \tag{6.14}$$

for s_L

$$i\gamma^\mu\left(\partial_\mu - \frac{1}{3}i\hat{e}A_\mu\Gamma_\mu\right)s_L + \frac{1}{2}g_2\gamma^\mu W_\mu^- c_L - \frac{1}{2}\left(g_2 c_w + \frac{1}{3}g_1 s_w\right)\gamma^\mu Z_\mu s_L$$
$$- \frac{i}{2}I_1 e_A^{\ \mu} e_B^{\ \nu}\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} s_L + G_6 ans_R = 0\,, \tag{6.15}$$

for e_R

$$i\gamma^\mu(\partial_\mu - i\hat{e}A_\mu + \Gamma_\mu)e_R + g_1 s_w\gamma^\mu Z_\mu e_R$$
$$- \frac{i}{2}I_1 e_A^\mu e_B^\nu\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} e_R + G_1 ame_L = 0\,, \tag{6.16}$$

for u_R

$$i\gamma^\mu\left(\partial_\mu + \frac{2}{3}i\hat{e}A_\mu + \Gamma_\mu\right)u_R - \frac{1}{6}g_1 s_w\gamma^\mu Z_\mu u_R$$
$$- \frac{i}{2}I_1 e_A^{\ \mu} e_B^{\ \nu}\sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} u_R + G_2 anu_L = 0\,, \tag{6.17}$$

for d_R

$$i\gamma^\mu\left(\partial_\mu - \frac{1}{3}i\hat{e}A_\mu\Gamma_\mu\right)d_R - \frac{1}{6}g_1 s_w \gamma^\mu Z_\mu d_R$$

$$-\frac{i}{2}I_1 e_A^\mu e_B^\nu \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} d_R + G_3 and_L = 0\,, \quad (6.18)$$

for c_R

$$i\gamma^\mu\left(\partial_\mu + \frac{2}{3}i\hat{e}A_\mu + \Gamma_\mu\right)c_R - \frac{1}{6}g_1 s_w \gamma^\mu Z_\mu c_R$$

$$-\frac{i}{2}I_1 e_A^\mu e_B^\nu \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} c_R + G_5 anc_L = 0\,, \quad (6.19)$$

for s_R

$$i\gamma^\mu\left(\partial_\mu - \frac{1}{3}i\hat{e}A_\mu\Gamma_\mu\right)s_R - \frac{1}{6}g_1 s_w \gamma^\mu Z_\mu s_R$$

$$-\frac{i}{2}I_1 e_A^\mu e_B^\nu \sqrt{16\pi G}(s_w Z_{\mu\nu} + c_w A_{\mu\nu})\sigma^{AB} s_R + G_6 ans_L = 0\,. \quad (6.20)$$

The Yang–Mills equations in the lower energy limit [where $I_1 = I_0$, see Eq. (6.27) below] are the following:

for photon:

$$A^{\mu\lambda}_{;\mu} + 2\hat{e}\bigg\{i(W^{[+(\mu)}_{;\mu}W^{-]\lambda)} + \frac{i}{2}[W^{+\mu\lambda}W^-_\mu + W^{-\mu\lambda}W^+_\mu] - \frac{\hat{e}}{2}\bigg[\left(\frac{c_W}{s_W}Z^\mu - A^\mu\right)$$

$$\times[(W^{+\lambda} + W^{-\lambda})(W^+_\mu + W^-_\mu) + i(W^{+\lambda} - W^{-\lambda})(W^+_\mu - W^-_\mu)]$$

$$-\left(\frac{c_W}{s_W}Z^\lambda - A^\lambda\right)[(W^{+\mu} + W^{-\mu})^2 - i(W^{+\mu} - W^{-\mu})^2]\bigg]\bigg\}$$

$$= \frac{\hat{e}}{I_1^2}\bigg[\bar{e}_L\gamma^\lambda e_L - \frac{2}{3}\bar{u}_L\gamma^\lambda u_L + \frac{1}{3}\bar{d}_L\gamma^\lambda d_L + \bar{\mu}_L\gamma^\lambda\mu_L - \frac{2}{3}\bar{c}_L\gamma^\lambda c_L$$

$$+\frac{1}{3}\bar{s}_L\gamma^\lambda s_L + \bar{e}_R\gamma^\lambda e_R - \frac{2}{3}\bar{u}_R\gamma^\lambda u_R + \frac{1}{3}\bar{d}_R\gamma^\lambda d_R$$

$$+\bar{\mu}_R\gamma^\lambda\mu_R - \frac{2}{3}\bar{c}_R\gamma^\lambda c_R + \frac{1}{3}\bar{s}_R\gamma^\lambda s_R\bigg] + \sqrt{16\pi G}\frac{c_W}{I_1}[(\bar{\nu}_e\sigma^{\mu\lambda}\nu_e + \bar{e}_L\sigma^{\mu\lambda}e_L$$

$$-\bar{u}_L\sigma^{\mu\lambda}u_L - \bar{d}_L\sigma^{\mu\lambda}d_L + \bar{\nu}_\mu\sigma^{\mu\lambda}\nu_\mu + \bar{\mu}_L\sigma^{\mu\lambda}\mu_L - \bar{c}_L\sigma^{\mu\lambda}c_L - \bar{s}_L\sigma^{\mu\lambda}s_L)]_{;\mu}\,, \quad (6.21)$$

for Z_μ boson:

$$Z^{\mu\lambda}_{;\mu} - 2g_2c_W\Bigg\{i(W^{[+(\mu})_{;\mu}W^{-]\lambda)} + \frac{i}{2}[W^{+\mu\lambda}W^-_\mu + W^{-\mu\lambda}W^+_\mu]$$

$$- \frac{g_2c_W}{2}\Bigg[\left(Z^\mu - \frac{s_W}{c_W}A^\mu\right)[(W^{+\lambda} + W^{-\lambda})(W^+_\mu + W^-_\mu)$$

$$+ i(W^{+\lambda} - W^{-\lambda})(W^+_\mu - W^-_\mu)]$$

$$- \left(Z^\lambda - \frac{s_W}{c_W}A^\lambda\right)[(W^{+\mu} + W^{-\mu})^2 - i(W^{+\mu} - W^{-\mu})^2]\Bigg]\Bigg\}$$

$$+ \frac{3a^2}{16\pi G}\left[\frac{g_1^2}{I_1^2} + g_2^2\right]Z^\mu$$

$$= \frac{1}{2I_1^2}\Bigg[(g_1s_W + g_2c_W)(\bar{\nu}_e\gamma^\lambda\nu_e + \bar{\nu}_\mu\gamma^\lambda\nu_\mu)$$

$$+ (g_1s_W - g_2c_W)(\bar{e}_L\gamma^\lambda e_L + \bar{\mu}_L\gamma^\lambda\mu_L)$$

$$+ \left(g_2c_W - \frac{1}{3}g_1s_W\right)(\bar{u}_L\gamma^\lambda u_L + \bar{c}_L\gamma^\lambda c_L)$$

$$- \left(g_2c_W + \frac{1}{3}g_1s_W\right)(\bar{d}_L\gamma^\lambda d_L + \bar{s}_L\gamma^\lambda s_L)\Bigg] + g_1s_W(\bar{e}_R\gamma^\lambda e_R + \bar{\mu}_R\gamma^\lambda\mu_R)$$

$$- \frac{1}{3}g_1s_W(\bar{u}_R\gamma^\lambda u_R + \bar{c}_R\gamma^\lambda c_R) - \frac{1}{3}g_1s_W(\bar{d}_R\gamma^\lambda d_R + \bar{s}_R\gamma^\lambda s_R)$$

$$+ \frac{\sqrt{16\pi G}}{I_1}s_W[(\bar{\nu}_e\sigma^{\mu\lambda}\nu_e + \bar{e}_L\sigma^{\mu\lambda}e_L - \bar{u}_L\sigma^{\mu\lambda}u_L - \bar{d}_L\sigma^{\mu\lambda}d_L$$

$$+ \bar{\nu}_\mu\sigma^{\mu\lambda}\nu_\mu + \bar{\mu}_L\sigma^{\mu\lambda}\mu_L - \bar{c}_L\sigma^{\mu\lambda}c_L - \bar{s}_L\sigma^{\mu\lambda}s)]_{;\mu}\,, \tag{6.22}$$

for W^+ boson:

$$W^{+\mu\lambda}_{;\mu} - ig_2\{[2W^{-[\lambda}(c_W\ Z^{\mu]} - s_W\ A^{\mu]})]_{;\mu} - W^{+\mu\lambda}(c_WZ_\mu - s_WA^{\mu\lambda})$$

$$+ W^-_\mu(c_WZ^{\mu\lambda} - s_WA^{\mu\lambda}) + ig_2[W^{+\mu}(W_{\mu i}W^i_\lambda + W_{\mu 3}W^{\lambda 3})$$

$$+ (W^\mu_iW^i_\mu + W^\mu_3W^3_\mu)W^+_\lambda]\} + \frac{3a^2g_2^2}{16\pi G}W^{+\mu}$$

$$= g_2[\bar{e}_L\gamma^\mu\nu_e + \bar{d}_L\gamma^\mu u_L + \bar{\mu_L}\gamma^\mu\nu_\mu + \bar{s}_L\gamma^\mu c_L]\,, \tag{6.23}$$

for W^- boson:

$$
\begin{aligned}
&W^{-\mu\lambda}_{;\mu} - ig_2\{[2W^{+[\lambda}(c_W\ Z^{\mu]} - s_W\ A^{\mu]})]_{;\mu} + W^{-\mu\lambda}(c_W Z_\mu - s_W A^{\mu\lambda}) \\
&\quad + W^+_\mu(c_W Z^{\mu\lambda} - s_W A^{\mu\lambda}) + ig_2[W^{-\mu}(W_{\mu i}W^{\lambda i} + W_{\mu 3}W^{\lambda 3}) \\
&\quad + (W^\mu_i W^i_\mu + W^\mu_3 W^3_\mu)W^-_\lambda]\} + \frac{3a^2g_2^2}{16\pi G}W^{-\mu} \\
&= g_2[\bar{\nu}_e\gamma^\lambda e_L + \bar{u}_L\gamma^\lambda d_L + \bar{\nu}_\mu\gamma^\lambda \mu_L + \bar{c}_L\gamma^\lambda s_L]\,. \qquad (6.24)
\end{aligned}
$$

Equations (6.9)–(6.20) give the mass terms for the electron, muon, u, d, c, and s quarks. They show two contributions, a preonic term[3] proportional to $\frac{1}{I_1 L_1}$, which is a consequence of the dimensional reduction and is cancelled by the Yukawa preonic contribution and another one that emerges from symmetry breaking i.e., $G_j am$ or $G_j an$.

From Eqs. (6.21)–(6.24) we find that the photon continues being massless, and that the squared mass for the W and Z bosons are, respectively,

$$M_W^2 = \frac{3a^2g_2^2}{16\pi G}\,, \qquad (6.25)$$

$$M_Z^2 = \frac{3a^2}{16\pi G}\left[\frac{g_1^2}{I_1^2} + g_2^2\right]. \qquad (6.26)$$

These results enable us to evaluate a and the Yukawa constants $\bar{G}_j$.

In order to match these theoretical predictions with the experimental results,[19,23] we must fulfill the requirement that $\frac{M_W^2}{M_Z^2} = \cos^2(\theta_W)$. This leads us to conclude that our dilatonic field, I_1 in its linear vacuum, must have a ground state, in which it is a constant, $I_0 \neq 0$, and $g_{\mu\nu} = \eta_{\mu\nu}, F_{\mu\nu} = 0, F^\alpha_{\mu\nu} = 0$ and $T_{00} = 0$ is a minimum. This is in fact the case according to our field equations; the ground state is degenerate and exists for any arbitrary constant value of $I = I_0$, as can be seen in (5.8) up to (5.17).[24–26]

From this point of view, we can choose an appropriate value of I_0 in order to obtain the true boson masses; one finds

$$I_1 = 1\,. \qquad (6.27)$$

Beside the usual terms of a Weinberg–Salam–Glashow theory in a curved space-time we find that as consequence of the dimensional reduction, there are two anomalous momenta, one related to the electromagnetic gauge field A_ν and the other one associated with the weak gauge field Z_ν.[3] In Gaussian units these momenta have the value

$$\frac{\hbar}{2c}\sqrt{16\pi G}\ c_w \approx 5.9 \times 10^{-32}\ \text{e cm} \qquad (6.28)$$

for the anomalous electromagnetic momentum and

$$\frac{\hbar}{2c}\sqrt{16\pi G}\, s_w \approx 3.2 \times 10^{-32} \text{ e cm} \tag{6.29}$$

for the weak anomalous momentum. The interaction of these momenta with their corresponding gauge fields appear in the Yang–Mills equations (6.21)–(6.24) and they produce additional polarization currents.

This model is readily extended to include the third fundamental family

$$\Psi_3 = \begin{pmatrix} \nu_\tau \\ \tau \\ t \\ b \end{pmatrix} \tag{6.30}$$

and all our previous conclusions stand.

In conclusion, we have shown that the standard model can be obtained from an eight-dimensional gravity theory taking the principal fiber bundle structure with an enlarged Yukawa coupling and interpreting the effective cosmological constant as the Higgs potential. The correct gauge boson masses as well as the fermionic ones are given by the theory. It seems to be that in order to introduce the gravitational interaction in a unified mathematically consistent theory, the extra dimensions are needed.

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References

1. A. Macías and H. Dehnen, *Class. Quantum Grav.* **8**, 203 (1991).
2. A. Macías and H. Dehnen, *Int. J. Mod. Phys.* **A7**, 5105–5113 (1992).
3. A. Macías and H. Dehnen, *Mod. Phys. Lett.* **A7**, 103 (1992).
4. A. Salam and J. Strathdee, *Ann. Phys. (N.Y.)* **141**, 316 (1982).
5. E. Witten, *Nucl. Phys.* **B186**, 412 (1981).
6. Y. M. Cho, *J. Math. Phys.* **16**, 2029 (1975).
7. T. Matos and J. A. Nieto, *Rev. Mex. Fis.* **39**, s81 (1993).
8. Y. Choquet-Bruhat, C. DeWitt-Morette and M. Dillard-Bleick, *Analysis, Manifolds and Physics* (North-Holland, Amsterdam, 1977).
9. P. Peebles and B. Ratra, *Astrophys. J.* **L17**, 325 (1988).
10. B. Ratra and P. Peebles, *Phys. Rev.* **D37**, 3406 (1988).
11. L. O. Pimentel and J. Socorro, *Gen. Relativ. Grav.* **25**, 1159 (1993).
12. C. Misner, K. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).
13. M. Rosenbaum, M. Ryan Jr., L. F. Urrutia and R. Matzner, *Phys. Rev.* **D36**, 1032 (1987).
14. M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
15. P. Higgs, *Phys. Lett.* **12**, 132 (1964a).

16. P. Higgs, *Phys. Lett.* **13**, 508 (1964b).
17. P. Higgs, *Phys. Rev.* **154**, 1156 (1966).
18. S. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev.* **D2**, 1285 (1970).
19. Chen, *et al.*, *Phys. Rev. Lett.* **51**, 634 (1983).
20. Y. Cho, *Phys. Rev. Lett.* **68**, 3133 (1992).
21. Y. Cho, *Phys. Rev.* **D41**, 2462 (1990).
22. S. Weinberg, *Phys. Lett.* **12**, 132 (1967).
23. E. Elliot and E. Predazzi, *Gauge Theories and the New Physics* (Cambridge University Press, N. Y., 1985).
24. R. Becerril and T. Matos, *Gen. Relativ. Grav.* **24**, 465 (1992).
25. R. Becerril and T. Matos, *Phys. Rev.* **D46**, 1540 (1992).
26. S. Chatterjee, *Astron. Astrophys.* **230**, 1 (1990).