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On the spacetime of a galaxy

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Abstract
We present an exact solution of the averaged Einstein's field equations in the presence of two real scalar fields and a component of dust with spherical symmetry. We suggest that the spacetime found provides the characteristics required by a galactic model that could explain the supermassive central object and the dark matter halo at once, since one of the fields constitutes a central oscillaton surrounded by dust and the other scalar field distributes far from the coordinate centre and can be interpreted as a halo. We show the behaviour of the rotation curves all along the background. Thus the solution could be a first approximation of a 'long exposition photograph' of a galaxy.

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1. Introduction

Doubtless, one of the most interesting open questions in physics now concerns the nature of the dark matter in the universe. In a series of recent papers, we have proposed that the dark matter in the Cosmos is of scalar field nature [1–4]. Following an analogous procedure as in particle physics, we wrote a Lagrangian with all the terms needed to reproduce the observed universe. In particular, using the scalar field potential of the scalar field dark matter (SFDM)

$$V(\Phi) = V_0[\cosh(\lambda\sqrt{\kappa_0}\Phi) - 1] \qquad (1)$$

where $\kappa_0 = 8\pi/M_{\rm pl}^2$, we were able to reproduce very well all the successes of the standard lambda cold dark matter model (ΛCDM) at the cosmological level including galactic scales [1, 2]. The free parameters of the scalar potential V_0 and λ can be fitted by cosmological observations obtaining $\lambda \simeq 20.23$ and $V_0 \simeq (3\times10^{-27}M_{\rm Pl})^4$, where $M_{\rm Pl}$ is the Planck mass [2]. The mass of the scalar particle then is $m_\Phi \simeq 9.1\times10^{-52}M_{\rm Pl} = 1.1\times10^{-23}$ eV (compare this value of the scalar mass with that in [5]). Under galactic scales there are some differences between the ΛCDM and the SFDM. The self-interaction of the scalar field Φ explains the

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observed dearth of dwarf galaxies and the smoothness of the galaxy core halos as well [2]. The question whether this scalar field is of fundamental origin or is a combination of some other fundamental particles is open. Nevertheless, in order to reproduce the high resolution *N*-body numerical simulations of self-interacting dark matter [6], we found that the renormalization scale of this scalar field is of the same order of the Planck mass [7]. This suggests that this scalar field dark matter has a fundamental origin. We have also studied the scalar field hypothesis at the galactic level in [3, 4]. The idea follows the standard idea of galaxy formation, namely that scalar field (dark matter) fluctuations are responsible for the origin of the galaxies. In the case of the scalar field potential (1), we have $\cosh(\lambda\Phi) \rightarrow \cosh(\lambda(\Phi + \delta\Phi)) \sim \exp(\lambda\delta\Phi)$ for regions where the scalar field fluctuation dominates $\delta\Phi > \Phi$. Of course, as in the standard theory of galaxy formation, the dark matter fluctuations are of different sizes in different regions of the universe for different galaxies. Therefore, at the galactic level, we have a scalar field potential which depends on local variables. Thus, the exponential potential approximates the cosh potential in some regimes of the scalar field.

On the other hand, new observations show that the supermassive central objects lying in the active galactic nuclei seem to be correlated with the velocity dispersion of the dark matter composing the dark halo, suggesting that the central object was formed at the same time as the halo [8]. This is probably contrary to the standard idea about galactic nuclei that proposes the existence of a central black hole, since it is formed at a time after the disc, i.e., much later after the formation of the halo. Going further, it is also shown that the supermassive objects which are supposed to be at the centre of galactic nuclei could be boson stars, obtaining the same predictions for the rate of accretion of matter due to the presence of a completely regular spacetime background without surface or horizon [9].

The question we now face is whether there is a dynamical mechanism that can provide a realistic scenario of galaxy formation using the scalar field dark matter hypothesis. First of all, there is a complete evolution of the galactic and under-galactic fluctuations that belong to the non-linear regime of perturbations. The right answer is provided by the numerical evolution of Einstein's equations. Fortunately, a partial answer is given in the numerical research on Einstein's equations developed since 1990. In particular, the collapse of a scalar field is studied in depth in [10–12] and it was found that there are final equilibrium and stable configurations for collapsed scalar field particles: boson stars (when the scalar field is complex) and oscillatons (when the scalar field is real and time dependent), both of them being formed through a process called gravitational cooling [11]. Based on these ideas, we present in section 2 the motivations leading us to build a galactic model with scalar fields, followed by the solution to the model in section 3; the physical features of such a toy model are shown in section 4 and finally we draw some conclusions.

2. The Galactic model with scalar field dark matter

We first recall the main results obtained in [10, 11]. Through the gravitational cooling process, a cosmological fluctuation of the scalar field collapses to form a compact oscillaton by ejecting a part of the scalar field. This ejected part of the cooling process carries the excess of kinetic energy out and plays the role of the halo of the collapsed object. The final configuration then consists of a central compact object, an oscillaton, surrounded by a diffuse cloud of the scalar field, both formed at the same time due to the same collapse process. This provides a correlation between the central object and the scalar halo.

If the central object is an oscillaton, its formation is due to coherent scalar oscillations around the minimum of the scalar potential (1). The scalar field Φ and the metric coefficients (considering the spherically symmetric case) are time dependent and it was proved that this configuration is stable, non-singular and asymptotically flat [10]. However, this is not the final answer because, from the quantum-mechanical point of view, a Bose–Einstein condensate forms and we must take into account that the scalar field Φ also decays or self-annihilates [13, 14]. In such a case, we should consider the scattering cross section $\sigma_{\Phi\to\Phi}$ in order to calculate the relaxation time of such condensation. In the models treated in [13–15], the coefficient of Φ^4 is responsible both for the size of the compact object and for the scattering cross section $\sigma_{\Phi\to\Phi}$. With potential (1) it is necessary to take into account all the couplings [7]. Nevertheless, it is possible to find a relaxation time smaller than the age of the universe [13] if

$$g > 10^{-15}(m_\Phi/\mathrm{eV})^{7/2} \tag{2}$$

with g being the effective Φ^4-theory coupling. Then, considering the value $m_\Phi \simeq 10^{-23}$ eV (see [2]), $g > 10^{-96}$. This condition is well satisfied in our case since $g \sim 10^{-48}$ [7]. This ensures that the relaxation time is shorter than the age of the universe.

In the case studied in [11], a massive scalar field without self-interaction collapses ejecting out 13% of the initial configuration of the scalar field. The critical mass of the final configuration for the oscillaton depends on the mass of the boson [10]. For our model (1), the mass of the scalar field is $m = 1.1 \times 10^{-23}$ eV and it is found that

$$M_{\mathrm{crit}} \sim 0.6\frac{M_{\mathrm{pl}}^2}{m} \sim 10^{12} M_\odot \tag{3}$$

which is a surprising result: the critical mass needed for the scalar field to collapse is of the same order of magnitude as the dark matter contents of a standard galaxy. Then we expect that the fluctuations of this scalar field due to Jeans instabilities will in general collapse to form objects with a mass of the same order as the mass of the halo of a typical galaxy. This is the reason by which we expect that the SFDM model can also work at the galactic level. Summarizing, we consider two working hypotheses. First, we identify the formation of a central compact object and a halo with the gravitational collapse of a scalar field. The compact object could be an oscillaton (since we are dealing only with a real scalar field) or a Bose–Einstein condensate. Second, we identify the ejected scalar field with the halo of this galaxy.

In a realistic model the metric and fields depend on time and their complete study involves numerical calculations within and beyond general relativity. One alternative is to study the behaviour of the galaxy numerically with all the hypotheses stated above. This procedure has the advantage that we do not need to eliminate any *a priori* consideration, but we can lose some important physical information within the numbers we obtain. Another alternative is to perform some approximations on the metric and fields and find exact solutions. These approximations also lead us to lose crucial information about the system but we can keep the physics under control. In our opinion, both procedures must be developed in order to have a better understanding of the hypotheses.

In this paper we adopt the second alternative, i.e. we build a toy model for the case stated above in purely geometrical terms and consider only the final stage of the collapse. We leave the dynamical evolution for future study [16]. To start with, we base our toy model on the numerical results studied in [10–12]. First, since the time dependence terms of the metric in an oscillaton are quite small [10], we suppose that the centre of this toy galaxy is an oscillaton which oscillates coherently but is considered to have a *static* metric. This is an approximation because neither the galactic nuclei nor the oscillaton are expected to be static. However, for the purposes of this analytic study, we suppose that the dynamics of the oscillaton can be frozen in time in a way we explain below. Second, we do not expect the scalar halo to possess the same properties as the collapsed oscillaton; in some sense, they must be different. Thus, we consider the scalar halo as another scalar field. Third, baryonic matter is considered to lie in the galaxy.

We suppose that the baryonic matter resides only at the galaxy centre and therefore that the galactic bulge is the luminous contributor to the curvature of the spacetime of the galaxy. This baryonic component is assumed to be dust. As in previous studies [3, 4], we leave the rest of the luminous matter around the galaxy as test particles, i.e. they do not essentially contribute to the curvature of the spacetime.

3. The analytical solution

The Lagrangian density that we start with reads $\mathcal{L} = R + \mathcal{L}_{\rm SFO} + \mathcal{L}_{\rm SFH} + \mathcal{L}_{\rm dust}$ being R, the scalar curvature and the other terms correspond to the Lagrangian of the scalar field oscillaton (SFO), the scalar field halo (SFH) and dust, which plays the role of a galactic bulge. For the scalar objects the following expressions are available:

$$\mathcal{L}_{\rm SFO} = -\tfrac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi\right) - V(\Phi) \qquad \mathcal{L}_{\rm SFH} = -\tfrac{1}{2}\left(\partial_\mu\psi\partial^\mu\psi\right) - U(\psi). \tag{4}$$

Here Φ is the scalar field corresponding to the oscillaton and $V(\Phi)$ its potential of self-interaction. ψ is the scalar field describing the scalar field halo and $U(\psi)$ its respective scalar potential. As a consequence of the non-coupling of the fields in the total Lagrangian, Einstein's equations are written as $G_{\mu\nu} = \kappa_0\left[T^{\rm SFO}_{\mu\nu} + T^{\rm SFH}_{\mu\nu} + T^{\rm dust}_{\mu\nu}\right]$, with

$$\begin{aligned} T^{\rm SFO}_{\mu\nu} &= \partial_\mu\Phi\partial_\nu\Phi - \tfrac{1}{2}g_{\mu\nu}\left[\partial^\sigma\Phi\partial_\sigma\Phi + 2V(\Phi)\right] \\ T^{\rm SFH}_{\mu\nu} &= \partial_\mu\psi\partial_\nu\psi - \tfrac{1}{2}g_{\mu\nu}\left[\partial^\sigma\psi\partial_\sigma\psi + 2U(\psi)\right] \\ T^{\rm dust}_{\mu\nu} &= du_\mu u_\nu \end{aligned}$$

d being the density of the dust and u^α the 4-velocity of its particles. Since the scalar field Φ is time-dependent and oscillating coherently around the minimum of its scalar potential, we can write $\Phi = P(r)\cos(\omega t)$ (see [12]). In order to handle Einstein's equations we average them over the period of a scalar oscillation, that is, we take $\left\langle G_{\mu\nu} = \kappa_0\left[T^{\rm SFO}_{\mu\nu} + T^{\rm SFH}_{\mu\nu} + T^{\rm dust}_{\mu\nu}\right]\right\rangle$. This procedure gives the lowest order approximation for an oscillaton and we are left with time-independent differential equations [12]; in the notation of [12] where the metric functions are written as $g(r,t) = g_0(r) + g_1(r)\cos(\omega t) + \cdots$ we are looking only for g_0 terms of the metric and the fields, which are the dominant terms. Besides Einstein's equations, there are two Klein–Gordon (KG) equations for the scalar fields,

$$\Phi^{;\mu}{}_{;\mu} + \frac{\mathrm{d}V}{\mathrm{d}\Phi} = 0 \qquad \psi^{;\mu}{}_{;\mu} + \frac{\mathrm{d}U}{\mathrm{d}\psi} = 0. \tag{5}$$

The KG equation on the left depends on time, but it can be shown that one can factorize an overall cosine term and then the resulting differential equation is time independent [12].

Then, starting from a spherically symmetric spacetime, using the *harmonic maps ansatz* we were able to find a solution of the system. Briefly, the main idea behind the harmonic maps ansatz is the reparametrization of the metric functions with convenient auxiliary functions which will obey a generalization of the Laplace equation along with some consistency relationships; the latter are usually quite difficult to fulfil, and great care and intuition must be taken in order to get a system of equations that are both workable and interesting [17]. The exact solution of the averaged Einstein's field equations is

$$\psi = \sqrt{\frac{v_a^2}{2\kappa_0}}\ln(r^2+b^2) + \psi_0 \qquad U(\psi) = \frac{2v_a^2}{\left(1-v_a^2\right)\kappa_0}\exp\left(-\sqrt{\frac{2\kappa_0}{v_a^2}}(\psi-\psi_0)\right) \tag{6}$$

for the scalar field ψ. The scalar field Φ for the oscillaton we obtain is

$$V(\Phi(r)) = -\frac{1}{4}\frac{\omega^2(r^2+b^2)^{-v_a^2} r P^2}{(-r+2M)B_0} + \frac{1}{2}\frac{\left(1+v_a^2\right)r^2 + b^2\left(1-v_a^2\right)}{(r^2+b^2)\kappa_0 r^2\left(1-v_a^2\right)} + \frac{1}{2}\frac{-\left(v_a^2+1\right)^2 r^4 + M v_a^2\left(1+2v_a^2\right)r^3 - 2b^2\left(1+2v_a^2\right)r^2 + 5v_a^2 M b^2 r - b^4}{\kappa_0 r^2\left(1-v_a^4\right)(r^2+b^2)^2} \tag{7}$$

where the function P is given by quadratures

$$P = \int\sqrt{-2\frac{\left(1-v_a^4\right)}{(r-2M)\kappa_0 r} + 2\frac{\left(1-v_a^4\right)r^4 - v_a^2 M\left(3-2v_a^2\right)r^3 + 2r^2b^2 - 3v_a^2 M b^2 r + b^4}{(r^2+b^2)^2(r-2M)\kappa_0 r}}\,\mathrm{d}r \tag{8}$$

where v_a is an asymptotic value of the tangential velocity of the test particles in our toy galaxy. The density of the dust is given by

$$d = \frac{1}{\kappa_0 r^2} - \frac{1}{2}\frac{\omega^2\left(r^2+b^2\right)^{-v_a^2} r P^2}{B_0(r-2M)} - \frac{\left(1-v_a^4\right)r^4 - v_a^2 M\left(3-2v_a^2\right)r^3 + 2b^2\left(1-v_a^2\right)r^2 + v_a^2 M b^2 r + b^4}{\kappa_0 r^2\left(1-v_a^4\right)\left(r^2+b^2\right)^2}. \tag{9}$$

Finally, the corresponding line element reads

$$\mathrm{d}s^2 = -B_0(r^2+b^2)^{v_a^2}\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \frac{A_0}{\left(1-\frac{2M}{r}\right)}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2 \tag{10}$$

with $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$ and M is a constant with the interpretation discussed below. This metric is singular at $r = 0$, but it has an event horizon at $r = 2M$. This metric does not represent a black hole because it is not asymptotically flat. Nevertheless, for regions where $r \ll b$ but $r > 2M$ the metric behaves like a Schwarzschild black hole. Inside the horizon the pressure of the perfect fluid is not zero anymore; thus our toy model is valid only in regions outside the horizon where it is an approximation of the galaxy. Metric (10) is not asymptotically flat, but it has a natural cut-off when the dark matter density equals the intergalactic density as mentioned in [4].

4. Physical features of the model

The dust density and the oscillaton scalar field potential depend on the value of the function P. We find that P goes very rapidly to a positive constant value, which depends on the boundary conditions one imposes. Let its asymptotic value be P_0 when $r \gg 2M$ whose interpretation would be the asymptotic amplitude of ψ.

In order to understand the other parameters of the metric, let us proceed in the following way. It is believed that in a standard galaxy, the central object has a mass of $M \sim$ 2–3 $\times\, 10^6 M_\odot \sim$ some AU. Far away from the centre of the galaxy, say from 1 pc up, the term $2M/r \ll 1$. In this limit metric (10) becomes

$$\mathrm{d}s^2 = -B_0(r^2+b^2)^{v_a^2}\,\mathrm{d}t^2 + A_0\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2. \tag{11}$$

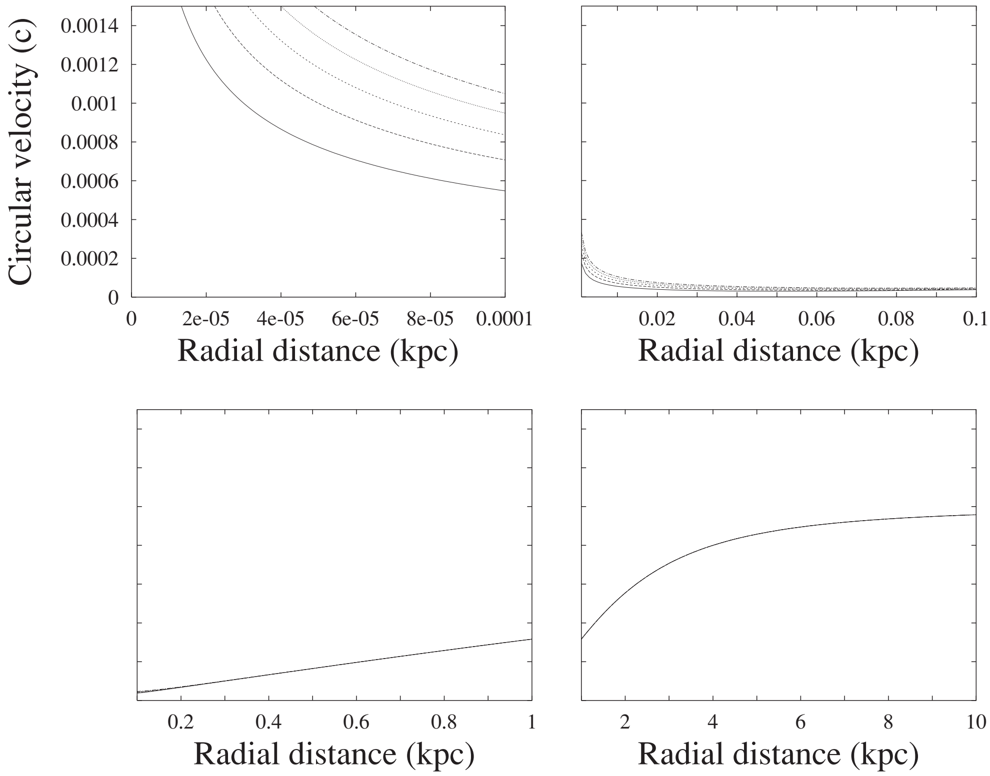

Figure 1. Rotation curves provided by the line element (10), for different values of the mass of the central object: $M = 3, 5, 7, 9, 11 \times 10^6 M_\odot$ and fixed $(b, v_a) = (3\ \text{kpc}, 10^{-3}c)$ are shown. At the top the rotation curve in the region near the galactic centre is shown, where a Keplerian fall-off can be observed. At the bottom is shown the region where it is evident the rotation curve is associated with the dark matter component.

This spacetime is very similar to metric (18) of [4], but now with the potential

$$U(\psi(r)) = \frac{2v_a^2}{\kappa_0\left(1 - v_a^2\right)}\frac{1}{(r^2 + b^2)}, \tag{12}$$

both solutions being the same in the limit $r \to \infty$. This implies that $A_0 = 1 - v_a^4$, recovering the asymptotic results shown in [4]. Parameter b is related to parameter b of metric (21) in [3] where it acts as a gauge parameter. Of course, this metric is only valid far away from the centre of the galaxy. With parameter b it is now possible to fit quite well the rotation curves of spiral galaxies. Therefore, metric (10) not only represents the exterior part of the galaxy, but it is a good approximation for the core part as well. Let us examine this point.

The rotation curves v^{rot} as seen by an observer at infinity for a spherically symmetric metric are given by $v^{\text{rot}} = \sqrt{r g_{tt,r}/(2g_{tt})}$ [4]. For metric (10) this reads

$$v^{\text{rot}}(r) = \sqrt{\frac{v_a^2(r - 2M)r^2 + M(r^2 + b^2)}{(r - 2M)(r^2 + b^2)}},$$

a formula that allows one to fit observational curves. In figure 1 the variation of the rotation curve when the value of M changes is shown, and it is evident that this affects only the kinematics in the central parts of the galaxy, exactly in the same way as the mass of the central object does [9].

Nevertheless, it calls the attention in figure 1 that there is a region where the velocity of the test particles is near zero and in fact the rotation curve decays in a Keplerian way. Let us see what happens in such a region near the galactic centre: the factor $(r^2 + b^2)^{v_a^2}$ in g_{tt} is almost 1 for $r \sim 10$ pc for reasonable values $b \sim$ kpc and $v_a \sim 10^{-3}$; therefore, the timelike geodesics are determined by the factor $1 - 2M/r$. Under such conditions the contribution of ψ is very close to zero for an observer far away from the galaxy. The interesting situation arises after calculating the density of all the components of the central density; it reads

$$\rho_{\rm dust} + \rho_{\rm SFO} + \rho_{\rm SFH} = -\frac{1}{2}\frac{M\,(r - 2\,M)\,(r^2 + b^2)^{v_a^2 - 2}B_0}{r^6\kappa_0\left(1 - v_a^4\right)} \times \left(-4\,r^6 v_a^2 + b^2\left(4 - 7\,v_a^2 + 2\,v_a^4\right)r^4 + 3\,b^4\left(3 - v_a^2\right)r^2 + 3\,b^6\right) \tag{13}$$

where the approximations $v_a \ll 1$ and $(r^2 + b^2)^{v_a^2} \sim 1$ are considered valid for reasonable values of the parameters. Moreover, we can infer that the total energy density as seen by an observer far away from the centre of the galaxy diverges when r approaches zero and goes rapidly to zero as $1/r^5$ when $r \to \infty$. But when an observer at infinity measures the mass of the central object, it is not possibile to distinguish between the density of each component.

Now let us explore an ADM-like concept of mass associated with the central region; we stress that metric (10) is not asymptotically flat and therefore the ADM concept of mass fails to be strictly valid, so we recall that there is a region in the interval $2M \ll r < b$ that is geometrically almost flat, where we will define a useful infinity ∞_f that should serve to calculate the mass of the central configuration through the standard metric

$$\mathrm{d}s^2 = -\mathrm{e}^{2\delta}\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{2m}{r}\right)} + (1 - \alpha)r^2\,\mathrm{d}\Omega^2 \tag{14}$$

where $m = m(r)$ is interpreted as the mass function and $\delta = \delta(r)$ as the gravitational potential. This form of the metric is convenient because in these coordinates $m_{,r} = 4\pi r^2\rho_{\rm T}$, where $\rho_{\rm T}$ is the total density of the object. This interpretation is correct in regions where the spacetime is almost flat, i.e. far away from the horizon $r = 2M$. Close to the horizon or inside it, function m is a quantity that should be similar to the mass of the object, but it is not since it contains the contribution of all the components together; in this region, where the curvature of the spacetime is huge, the volume element is different from $4\pi r^2\,\mathrm{d}r$. Furthermore, inside the horizon we are not able to know the real physics of the object. On the other side, far away from the centre of the toy galaxy, this function can be interpreted as the mass of an infinitesimal shell at radius r. Anyway, we will call function m the mass function everywhere. Thus, the ADM-like mass is obtained by $M_{\rm ADM} = \lim_{r\to\infty_f} m$. We perform the coordinate transformation $\sqrt{A_0}r \to r$, $\sqrt{A_0}b \to b$ in order to compare metrics (10) and (14). We obtain

$$\mathrm{d}s^2 = -\frac{B_0}{A_0^{v_a^2}}(r^2 + b^2)^{v_a^2}\left(1 - \frac{2M\sqrt{A_0}}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(1 - 2M\sqrt{A_0}/r)} + \frac{r^2}{A_0}\,\mathrm{d}\Omega^2 \tag{15}$$

with $A_0 = 1 - v_a^4$. Thus, $M_{\rm ADM} = \sqrt{1 - v_a^4}M$. Probably an observer at any large r would see the mass $M_{\rm ADM}$ at the centre of the galaxy.

There is a good reason to consider the analysis near the horizon with care: we consider a time-averaged photograph of the spacetime, which in the centre would be strongly time dependent due to the presence of the oscillaton, and stationarity fails to be a good approximation; thus it is not possible to know under the present approach which are the exact roles played by the binding energy of the oscillaton, the negative energy of the dust and the self-energy of the whole central system; such features can be found by performing

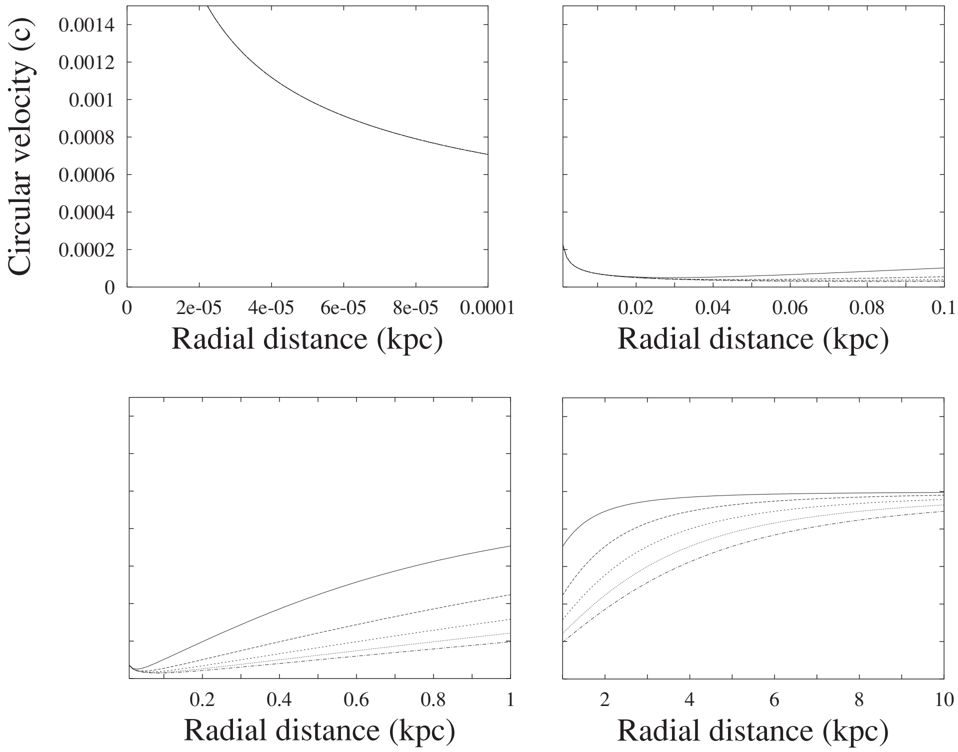

Figure 2. We show the rotational curves when parameter $b = 1, 2, 3, 4, 5$ kpc (and $(M, v_a) = 5 \times 10^6 M_\odot, 10^{-3}c)$ now fixed. It is evident that in this case the shape of the curve associated with the dark matter changes, but in the centre it remains unchanged.

the evolution of Einstein's equations and in fact represent a topic which is itself discussed elsewhere [16].

For this model, there is a contribution which does not fulfil the energy conditions; this is the reason to call it a 'toy model', although an observer at infinity sees the sum of all the energy densities of the components. In other words, the amount of the matter, with negative or positive energy density, is the quantity which determines the value of $m(r)$, the 'black hole mass' concentrated at radius r of the toy galaxy, not the contributors separately. Only if the observer measures the contributions of the energy density very close to the centre of the galaxy, could it distinguish them. At infinity the observer only measures M_{ADM}, i.e. will see a black-hole-like metric at the centre of the galaxy whose horizon lies at $r = 2M_{\mathrm{ADM}}$.

We present in figure 2 a situation where parameter b takes different values, thus playing the role of the core radius in the usual dark halo hypothesis. Here we show that for an observer at infinity, the test particles close to the galaxy centre behave as if there were a black hole of mass M_{ADM} in the centre. The event horizon prevents the observer seeing inside of the horizon surface, the velocity of test particles is higher for closer test particles and the rotation velocity decays as $1/\sqrt{r}$ in this region. For particles far away from the centre, the rotation curves behave just as the rotation curves given by the contribution of the dark matter halo in a typical galaxy.

In figure 3, the fit of the curves is done using the observed rotation curves of some dwarf galaxies whose dark matter contribution is extremely dominating and, therefore, is considered as the *test of fire* for a dark matter model in the galaxies. In general, for disc galaxies, the fit of

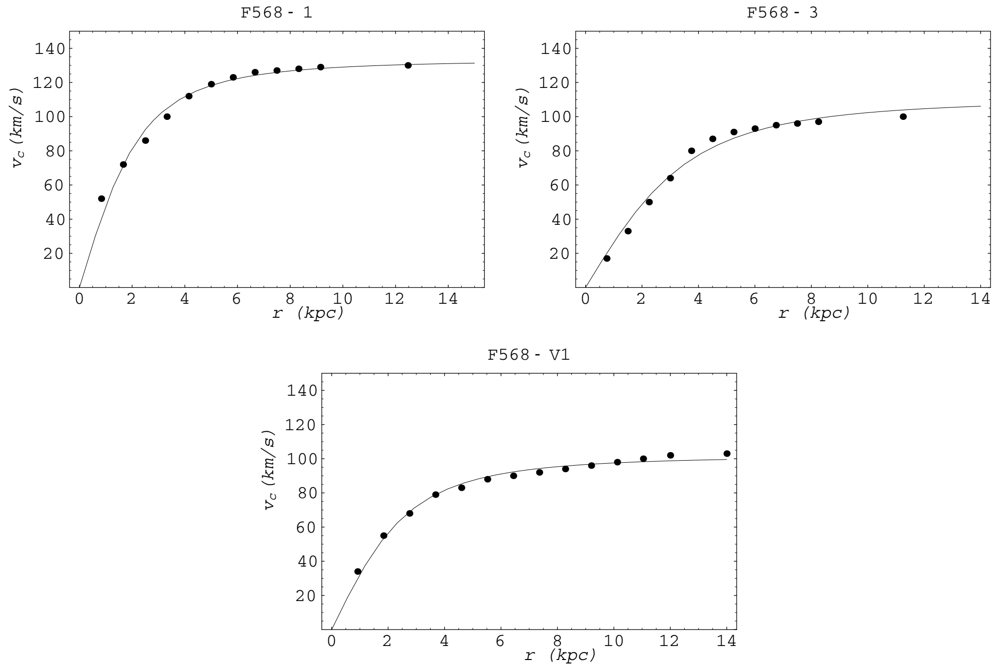

Figure 3. Rotation curves of three dwarf galaxies are shown. The units are as usual: km s^{-1} in the vertical axis and kpc in the horizontal one. The fitting parameters for F568-1, F568-3 and F568-V1 are: the asymptotic tangential velocity of last particles $v_a = 133.3$, 110.5, 101.9 km s^{-1} and the core radius of the halo $b = 2.6$, 4.06, 2.99 kpc respectively. Obviously an extremely wide range of values of M keep these fittings unaltered, since the value of M is important only in the central region (see figure 1).

the rotation curves using this metric is analogous to that in [3]. It seems then that metric (10) is a good approximation for some late state of the spacetime of a spiral galaxy; it is a good approximation of a 'long exposition photograph' of a galaxy.

5. Discussion

The configuration we found is as follows: in the innermost region of the toy galaxy, it lies on a time-averaged oscillaton and dust, looking like a picture of an object very similar to a black hole for an observer who is far away. Moreover, the resting scalar field should be accommodated in the outer parts. This picture is very similar to the one required for a current galaxy, whose centre is dominated by a supermassive object and it posseses a halo region dominated by the presence of the dark matter. The corresponding line element (10) is free of naked singularities and is not asymptotically flat. Instead, it behaves exactly as a black hole near the coordinate centre and is asymptotically constructed to provide the *flat curve condition* [4].

The main difference between these results and previous research supporting the scalar field dark matter hypothesis in galaxies is precisely that in this case we have been able to extend the solution towards the galactic core. We have shown two important statements: first, an averaged spherically symmetric source field exists that supports a real scalar field under the assumption of non-asymptotic flatness. Second, a fully relativistic treatment is able to explain the kinematics of the test particles in the whole background spacetime of a galaxy at once.

Therefore, we believe that the metric presented here is a first approximation of galactic spacetime providing the presence of a coherent field instead of the usual matter. Even when our solution accounts for negative energy densities of some components, observers at infinity are not able to know it. Furthermore, for such observers the test particles around the central object behave as if there would be a black hole of mass M_{ADM}.

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