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Scalar field dark matter, cross section and Planck-scale physics

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Abstract

In recent papers we have proposed that the dark matter of the Universe could be from scalar field origin. In this Letter, we find that if the scale of renormalization of the model is of order of the Planck mass, then a scalar field Φ endowed with the scalar potential $V = V_0[\cosh(\lambda\sqrt{\kappa_0}\Phi) - 1]$ can be a reliable model for dark matter in galaxies. The predicted scattering cross section fits the value required for self-interacting dark matter. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Recent cosmological observations suggest a flat Universe in a current accelerated expansion but full with 95% of unknown matter. 25% of the matter in the Universe, widely known as dark matter, is responsible of the formation of the large scale structure we observe in the sky: galaxies, clusters of galaxies, voids, walls, etc. This model of dark matter has been very successful [1]. However, at galactic scale, numerical simulations show some discrepancies between dark matter predictions and observations [2]. Dark matter simulations show cuspy halos of galaxies with an excess of small scale structure, while observations

suggest a constant halo core density [3] and a small number of subgalactic objects [4].

In order to solve these discrepancies, there are some proposals of scalar fields as dark matter in the literature nowadays [5–10]. All of them have assumed a polynomial scalar potential up to fourth order. At the cosmological scale, the main results are cold dark matter-like behavior, growing linear perturbations, the existence of a Jeans scale and suppression of the mass power at small scales, but these results depend upon initial conditions imposed on the scalar field (fine-tuning).² At the galactic scale, where the self-interaction becomes important, the scattering cross section is proportional to the quartic coupling square g^2 , which is a free parameter of the models. Even if there is a model-independent result in which a particle

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² Polynomial potentials [10,11] do not have a tracker nor a self-adjusting behavior as inverse power-law or exponential ones [12].

with a value for the scattering cross section given by (10) has, straightforwardly, a mean-free path of order of 1 Mpc (see Eq. (4) in [8]), the free parameters of the potential (the mass of the boson particle, the quartic coupling) are very related, as we can see in Eq. (6) of Ref. [8]. Depending of the coupling g the mass of the scalar field could be of order of MeV, inclusive. But, nobody has yet determined the value of this coupling.

In this Letter, we continue investigating the hypothesis of a scalar field as dark matter in the Universe with a cosh scalar potential [13–15]

$$V(\Phi) = V_0 [\cosh(\lambda\sqrt{\kappa_0}\Phi) - 1] \quad (1)$$

with

$$\kappa_0^{-1/2} = (8\pi G)^{-1/2} = M_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV}$$

being the inverse square of the reduced Planck mass. Using results given in renormalization theory, we calculate the $2 \rightarrow 2$ cross section at the lowest order for the potential (1). It is found that the result corresponds to an effective ϕ^4 -theory in which the effective coupling g is exponentially enhanced by the scale of renormalization Λ , which remains as a free parameter of the model. This result is compared with the value given for the cross section in self-interacting dark matter models. We found that if the scale of renormalization of potential (1) is of order of the Planck mass, $\Lambda = \mathcal{O}(M_{\text{Pl}})$, the predicted scattering cross section by mass of the scalar particles $\sigma_{2 \rightarrow 2}/m_\phi$ can fit the value predicted in numerical simulations of self-interacting dark matter in order to avoid high-density dark matter halos.

2. Scalar field dark matter with a cosh potential

We first mention the main results of the model, more details can be found in [13,14]. The model neither depends on initial conditions (those imposed on the scalar field and its perturbations at the end of inflation) nor has problems with nucleosynthesis (provided that $\lambda > 5$), and scales as radiation at early times, because of its exponential behavior [12]. It scales as cold dark matter once the scalar field Φ oscillates around the minimum of the potential. Its fluctuations are also the same than those of standard cold dark matter but it predicts a cut-off in the

Mass Power Spectrum, now due to its quadratic behavior. This results are enclosed in the following relations [13,14]

$$V_0 \simeq \frac{1.7}{9} \left(\frac{\Omega_{0\text{CDM}}}{\Omega_{0\gamma}} \right)^3 (\lambda^2 - 4)^3 \rho_{0\text{CDM}},$$

$$m_\phi^2 = \kappa_0 V_0 \lambda^2,$$

$$k_J \simeq 1.3\lambda \sqrt{\lambda^2 - 4} \frac{\Omega_{0\text{CDM}}}{\sqrt{\Omega_{0\gamma}}} H_0. \quad (2)$$

$\Omega_{0\text{CDM}}$ and $\Omega_{0\gamma}$ are the current contributions of dark matter and radiation to the critical energy density $\rho_{\text{crit}} = 3H_0^2/\kappa_0$ with H_0 the current Hubble parameter and then $\rho_{0\text{CDM}} = \Omega_{0\text{CDM}}\rho_{\text{crit}}$. m_ϕ is the mass of the scalar field and k_J is the Jeans wave number at which the Mass Power Spectrum has a cut-off, a very important difference with respect to the standard cold dark matter model.

We recall that the model, at this level, has only one free parameter: λ . Nevertheless, if we expect a cut-off in the Mass Power Spectrum about the wave number $k \simeq 4.5h \text{ Mpc}^{-1}$ [16], we can use the last of Eq. (2) to determine λ . It turns out that $\lambda \simeq 20.28$, then, we also obtain that $V_0 \simeq (3.0 \times 10^{-27} M_{\text{Pl}} \simeq 36.5 \text{ eV})^4$ and the corresponding ultra-light mass of the scalar field is $m_\phi \simeq 9.1 \times 10^{-52} M_{\text{Pl}} \simeq 1.1 \times 10^{-23} \text{ eV}$ [14]. In other words, the expected cut-off of the Mass Power Spectrum in the Universe fixes the last free parameter of the model. Up to this point, all parameters in potential (1) are completely determined, we recover the success of standard cold dark matter model at large scales and we have in addition a cut-off in the Mass Power Spectrum, alleviating problems of the standard model at subgalactic level.

Some differences with respect to other scalar field models appear because of the nonpolynomial behavior of (1) in certain regimes. For instance, at cosmological scale, the exponential-like behavior provokes no problem with initial conditions either for ρ_ϕ or the fluctuations (no fine-tuning). Also, the Jeans scale is related to the time at which the scalar field changes its behavior as radiation-like to cold dark matter-like, i.e., related to the intrinsic parameters of the potential [14], not to the time of cosmological radiation-matter equality as occurs for polynomial potentials [7].

Moreover, because of the presence of a scalar field potential (1), there must be an important self-interaction among the scalar particles. For example,

even if scalar particles are lighter than neutrinos, the former can cluster due to the presence of the scalar potential. That means that this scalar field dark matter model belongs to the so-called group of self-interacting dark matter models which are characterized by a $2 \rightarrow 2$ scattering cross section [2,17].

3. Renormalization and $2 \rightarrow 2$ scattering cross section

The scalar potential (1) can be written as a series of even powers in Φ ,

$$V = \sum_{n=1}^{\infty} V_0 \lambda^{2n} \kappa_0^n \Phi^{2n}.$$

Working on 4 dimensions, it is commonly believed that only Φ^4 and lower order theories are renormalizable. But, following the important work [18], if we consider that there is only one intrinsic scale Λ in the theory, we conclude that there is a momentum cut-off and that we can have an effective Φ^4 theory which depends upon all couplings in the theory. In addition, it was recently demonstrated [19] that scalar exponential-like potentials of the form (we have used the notation of potential (1) and for example, parameter μ in Eqs. (7), (8) in Ref. [19] is $\mu^{-1} = \lambda\sqrt{\kappa_0}$)

$$U_{\Lambda}(\Phi) = M^4 \exp\left(-\frac{\lambda^2 \kappa_0 \Lambda^2}{32\pi^2}\right) \exp(\pm \lambda\sqrt{\kappa_0} \Phi) \quad (3)$$

are nonperturbative solutions of the exact renormalization group equation in the Local Potential Approximation (LPA) (we have taken a massless Gaussian fixed point potential, i.e., parameter $m^2 = 0$ in Eq. (5) of [19]). The LPA is an approximation for the exact renormalization group equations, in fact a nonperturbative truncation, by considering only a potential term $U_{\Lambda}(\Phi)$. This allows us to consider all powers of Φ . Much more details can be found in [20].

Here M and λ are free parameters of the potential and Λ is the scale of renormalization. Being our potential a cosh-like potential (non-polynomial), it is then a solution to the renormalization group equations in the (LPA), too. Comparing Eqs. (1), (3), we can identify $V_0 = M^4 \tau$ with

$$\tau = \exp\left(-\frac{\lambda^2 \kappa_0 \Lambda^2}{32\pi^2}\right) \quad (4)$$

and we see that an additional free parameter appears, the scale of renormalization Λ . From this, we can assume that potential (1) is renormalizable with only one intrinsic scale Λ . Even though the parameters in the scalar potential (1) were fixed by cosmological observations, the scale Λ remains free.

We now proceed to calculate the $2 \rightarrow 2$ scattering cross section for the scalar particles following the procedure shown in [21]. For a potential with even powers of the dimensionless scalar field ϕ ,

$$U(\phi) = \sum_{n=1}^{\infty} u_{2n} \phi^{2n},$$

the scattering amplitudes are given by

$$A_{2n} = \sum_{m=n}^{\infty} u_{2m} \left(\frac{I}{2}\right)^{m-n} \frac{(2m)!}{(m-n)!}, \quad (5)$$

where, in our case, the dimensionless u_{2m} are

$$u_{2m} = \left(\frac{M}{\Lambda}\right)^4 \frac{\tau \lambda^{2m} (\Lambda^2 \kappa_0)^m}{(2m)!}, \quad (6)$$

and the internal contraction I reads

$$I \equiv \int_0^1 \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + 2u_2} = \frac{1}{16\pi^2} \left(1 + 2u_2 \ln \frac{2u_2}{1 + 2u_2}\right). \quad (7)$$

Summing the series (5) we find

$$A_{2n} = \left(\frac{M}{\Lambda}\right)^4 \lambda^{2n} (\Lambda^2 \kappa_0)^n \tau \exp\left(\frac{I \lambda^2 \kappa_0 \Lambda^2}{2}\right). \quad (8)$$

Since the only scale that appears naturally in gravitation is the Planck mass in κ_0 , we can expect that $\Lambda \sim \mathcal{O}(M_{\text{Pl}})$ and then $2u_2 = (m_{\phi}^2/\Lambda^2) \ll 1$. In consequence, $I \simeq 1/(16\pi^2)$. Then, the cross section for $2 \rightarrow 2$ scattering in the center-of-mass frame is

$$\sigma_{2 \rightarrow 2} = \frac{g^2}{16\pi E^2} \exp\left(\frac{\lambda^2 \kappa_0 \Lambda^2}{16\pi^2}\right), \quad (9)$$

where E is the total energy. At this point, we can notice the nonperturbative nature of potential (1). Had we taken perturbatively the scalar potential (1) with a coupling $g = 4!u_4 \simeq 2 \times 10^{-97}$, the self-interaction would have appeared as extremely weak. But, surprisingly, we can observe from Eqs. (4), (8) that the contribution of higher-order couplings points to an effective coupling $g_{\text{eff}} = g\tau^{-1}$.

4. Planck-scale physics and galactic consequences

An interesting dark matter model is that of a self-interacting dark matter [2,17]. This proposal considers that dark matter particles have an interaction characterized by a scattering cross section by mass of the particles given by [2,3]

$$\frac{\sigma_{2 \rightarrow 2}}{m} = 10^{-25} - 10^{-23} \text{ cm}^2 \text{ GeV}^{-1}. \quad (10)$$

This self-interaction provides shallow cores of galaxies and a minimum scale of structure formation, that it must also be noticed as a cut-off in the Mass Power Spectrum [4,16]. The cosh-potential (1) directly provides this cut-off, but the self-interaction among scalar particles would be important for other aspects of galaxy evolution.

We can guess the value of the scale Λ by considering the expected result (10). Near the threshold $E \simeq 4m^2$ and taking the values of λ , m_ϕ , we find that

$$\Lambda \simeq (1.93 \pm 0.01) M_{\text{Pl}} \simeq 2.3 \times 10^{19} \text{ GeV}. \quad (11)$$

The range of Λ is very narrow because of the exponential behavior. Observe that the scale of renormalization is of order of the Planck mass. Recalling that $V_0 \tau^{-1} = M^4$, we also find that

$$M = (6.7 \pm 1.9) \times 10^2 \text{ TeV}.$$

All parameters are completely fixed now.

An intriguing point is the appearance of two different energy scales, one in Λ and another one in M in (3), and it could be not only a coincidence that the former is of $\mathcal{O}(M_{\text{Pl}})$ and the latter is of $\mathcal{O}(100 \text{ TeV})$. It should be noted that it is the combination of these scales which determines the observable value of parameters V_0 and m_ϕ , at the cosmological scale, and the value of σ at the galactic one. This could be the first case in which the relevant scales are hidden into the effective theory at low-energies, as it was claimed that the solution (3) is a solution of the renormalization group equations “irrespectively of the high energy theory” [19]. This suggests that the cosh potential could be of some importance also in the early Universe. For instance, we have explored the idea of inflationary scalar field dark matter within the so-called braneworld cosmology [22].

5. Concluding remarks

The potential (1) is not motivated by particle physics. In principle, the model should be seen as completely phenomenological, but we have assumed the simplest hypothesis: a single minimally coupled real scalar field with a cosh potential. In fact, the results of the LPA presented above give theoretical support to the model. Furthermore, potential (1) has been widely studied in nonperturbative field theory in $1 + 1$ dimensions where it is known as the sinh-Gordon model (see [23] and references therein), a close relative of the sine-Gordon model studied in theory of solitons.

We would like to mention that a realistic model for dark matter should provide the right picture at both cosmological and galactic scales. Most of the models presented in the literature partially face the whole problem, focusing only on one aspect of dark matter [5,6,8,9,15,24]. This leads us to a wide range of models, but our opinion is that this degeneracy can only be broken by imposing on the models to fit observations at all scales.

Our first motivation was to investigate the hypothesis of scalar dark matter in galaxies, from a pure general relativistic point of view [25]. At the same time, we proved that a scalar field dark matter model with a cosh potential indeed provides a very simple model for dark matter at cosmological scales. In addition, the results presented in this Letter show, for the first time, that we can relate the cosmological model to galaxies, through the idea of self-interacting dark matter. Moreover, it is known that the formation of galaxies from dark matter involves nonlinear phenomena. We have begun to analyze that regime by evolving numerically the full Einstein–Klein–Gordon equations finding good agreement with observations using the *same* values (as suggested by the *cosmological* observations) of the free parameters in potential (1) [26].

But, even if the evolution of a galaxy could be described by self-interacting dark matter models [17], the scalar nature of dark matter can provide us with a more interesting picture: Bose condensation [6,9]. For instance, the relaxation time for the condensate would be smaller than the age of the Universe if [9]

$$g_{\text{eff}} > 6 \times 10^{-15} (m_\phi / \text{eV})^{7/2}. \quad (12)$$

We find that $\Lambda > 1.72 M_{\text{Pl}}$. Again, Λ should be of order of the Planck mass. From this, it is clear that,

at galactic scales, the scalar field dark matter model with the scalar potential (1) must be studied both in general relativity and quantum mechanics. More to the point, this would imply the study of objects in which quantum and gravitational interactions are at the same level, a first approximation to quantum gravity [27]. This is beyond the purpose of this paper and is left for future work.

Summarizing, a scalar field endowed with the scalar potential (1) is a reliable model as dark matter in the Universe, not only at cosmological level, but also at galactic level because of its self-interaction. Even if we are dealing with a nonperturbative case, the scalar potential is renormalizable at a energy scale of order of the Planck mass. Also, all relevant scales are hidden into the parameters of the effective theory, but they might be very important when performing quantum calculations.

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References

- [1] N.A. Bahcall, J.P. Ostriker, S. Perlmutter, P.J. Steinhardt, *Science* 284 (2000) 1481.
- [2] B.D. Wandelt, R. Dave, G.R. Farrar, P.C. McGuire, D.N. Spergel, P.J. Steinhardt, astro-ph/0006344.
- [3] C. Firmani, E. D’Onghia, G. Chincarini, X. Hernández, V. Avila-Reese, *Mon. Not. R. Astron. Soc.* 321 (2001) 713; V. Avila-Reese, C. Firmani, E. D’Onghia, X. Hernández, astro-ph/0007120.
- [4] P. Colín, V. Avila-Reese, O. Valenzuela, *Astrophys. J.* 542 (2000) 622.
- [5] P.J.E. Peebles, *Astrophys. J.* 534 (2000) L127.
- [6] J. Goodman, astro-ph/0003018.
- [7] W. Hu, R. Barkana, A. Gruzinov, *Phys. Rev. Lett.* 85 (2000) 1158.
- [8] M.C. Bento, O. Bertolami, R. Rosenfeld, L. Teodoro, *Phys. Rev. D* 62 (2000) 041302.
- [9] A. Riotto, I. Tkachev, *Phys. Lett. B* 484 (2000) 177.
- [10] A. Arbey, J. Lesgourgues, P. Salati, *Phys. Rev. D* 64 (2001) 123528, astro-ph/0105564; A. Arbey, J. Lesgourgues, P. Salati, *Phys. Rev. D* 65 (2002) 083514, astro-ph/0112324.
- [11] M.S. Turner, *Phys. Rev. D* 28 (1983) 1243; L.H. Ford, *Phys. Rev. D* 35 (1987) 2955.
- [12] B. Ratra, P.J.E. Peebles, *Phys. Rev. D* 37 (1988) 3406; C. Wetterich, *Nucl. Phys. B* 302 (1988) 302; E.J. Copeland, A.R. Liddle, D. Wands, *Phys. Rev. D* 57 (1998) 4686.
- [13] T. Matos, L.A. Ureña-López, *Class. Quantum Grav.* 17 (2000) L71.
- [14] T. Matos, L.A. Ureña-López, *Phys. Rev. D* 63 (2001) 63506.
- [15] V. Sahni, L. Wang, *Phys. Rev. D* 62 (2000) 103517.
- [16] M. Kamionkowski, A.R. Liddle, *Phys. Rev. Lett.* 84 (2000) 4525.
- [17] D.N. Spergel, P.J. Steinhardt, *Phys. Rev. Lett.* 84 (2000) 3760.
- [18] K. Halpern, K. Huang, *Phys. Rev. Lett.* 74 (1995) 3526; K. Halpern, K. Huang, *Phys. Rev. D* 53 (1996) 3252.
- [19] V. Branchina, *Phys. Rev. D* 64 (2001) 043513.
- [20] C. Bagnuls, C. Bervillier, *Phys. Rep.* 348 (2001) 91; C. Bagnuls, C. Bervillier, *Int. J. Mod. Phys. A* 16 (2001) 1825.
- [21] K. Halpern, *Phys. Rev. D* 57 (1998) 6337.
- [22] J.E. Lidsey, T. Matos, L.A. Ureña-López, astro-ph/0111292, *Phys. Rev. D* in press.
- [23] H. Saleur, cond-mat/0007309.
- [24] L. Hui, *Phys. Rev. Lett.* 86 (2001) 3467; A. Kusenko, P.J. Steinhardt, *Phys. Rev. Lett.* 87 (2001) 141301; D.E. Holz, A. Zee, *Phys. Lett. B* 517 (2001) 239.
- [25] F.S. Guzmán, T. Matos, *Class. Quantum Grav.* 17 (2000) L9; T. Matos, F.S. Guzmán, D. Núñez, *Phys. Rev. D* 62 (2000) 061301.
- [26] L.A. Ureña-López, gr-qc/0104093; T. Matos, F.S. Guzmán, *Class. Quantum Grav.* 19 (2002) 2617; M. Alcubierre et al., gr-qc/0110102.
- [27] I.M. Moroz, R. Penrose, P. Tod, *Class. Quantum Grav.* 15 (1998) 2733; K.R.W. Jones, D. Bernstein, *Class. Quantum Grav.* 18 (2001) 1513.