New Astronomy Reviews 46 (2002) 791–799

New Astronomy Reviews

www.elsevier.com/locate/newar

A light scalar field at the origin of galaxy rotation curves

Julien Lesgourgues[a,b,*], Alexandre Arbey[a,c], Pierre Salati[a,c]

[a] LAPTH, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France

[b] Theoretical Physics Division, CERN, CH-1211 Genève 23, Switzerland

[c] Université de Savoie, B.P. 1104, F-73011 Chambéry Cedex, France

Abstract

The nature of the dark matter that binds galaxies remains an open question. It is usually assumed to consist in a gas of massive particles with evanescent interactions; however, such particles—which have never been observed directly—should have a clumpy distribution on scales $\leq 10^{-2}$ kpc, which may be in contradiction with observations. We focus here on an exotic dark matter candidate: a light non-interacting (or only self-interacting) complex scalar field. We investigate the distribution of the field in gravitational interaction with matter, assuming no singularities (like black holes) at the galaxy center. This simplistic model accounts quite well for the rotation curve of low-luminosity spirals. A chi-squared analysis points towards a preferred mass $m \sim 0.4$ to 1.6×10^{-23} eV in absence of self-interaction. A rough calculation shows that allowing for a quartic self-coupling may shift the upper bound to around 1 eV. We conclude that a scalar field is a promising candidate for galactic dark matter. Our comparison should be extended to other rotation curves in order to derive better constraints on the scalar potential. We finally give a hint of the issues that appear when one tries to implement this scenario on cosmological time scales.

Keywords: Dark matter

1. Introduction

The observations of the Cosmic Microwave Background (CMB) anisotropies (Netterfield et al., 2002), combined either with the determination of the relation between the distance of luminosity and the redshift of supernovae SNeIa (Perlmutter et al., 1999), or with the large scale structure (LSS) information from galaxy and cluster surveys (Percival et al., 2001), give independent evidence for a dark matter density in the range $\Omega_{cdm}h^2 = 0.13 \pm 0.05$ (Netterfield et al., 2002), to be compared to a baryon density of $\Omega_b h^2 = 0.019 \pm 0.002$ as indicated by nucleosynthesis (Burles and Tytler, 1998) and the relative heights of the first acoustic peaks in the CMB data. The nature of that component is still unresolved insofar. The favorite candidate for the non-baryonic dark matter is a weakly-interacting massive particle (WIMP). The so-called neutralino naturally arises in the framework of supersymmetric theories. Depending on the numerous parameters of the model, its relic abundance Ω_{cdm} falls in the ballpark of the measured value. New experimental techniques have been developed in the past decade to detect these evading species. However, detailed numerical simulations have recently pointed to a few problems related to the extreme weakness with which that form of matter interacts. Neutralinos tend

*Corresponding author.

E-mail addresses: lesgourg@lapp.in2p3.fr (J. Lesgourgues), arbey@lapp.in2p3.fr (A. Arbey), salati@lapp.in2p3.fr (P. Salati).

1387-6473/02/$ – see front matter
PII: S1387-6473(02)00247-6

naturally to collapse in numerous and highly packed clumps (Moore et al., 1999) that are not seen. The halo of the Milky Way should contain ~ half a thousand satellites with mass in excess of 10^8 $M_\odot$ while a dozen only of dwarf-spheroidals are seen. The clumps would also heat and eventually shred the galactic ridge. More generally, this process would lead to the destruction of the disks of spirals. A neutralino cusp would form at their centers. This is not supported by the rotation curves of low-surface-brightness galaxies that indicate on the contrary the presence of a core with constant density. Finally, two-body interactions with halo neutralinos and its associated dynamical friction would rapidly slow down the otherwise observed spinning bars at the center of spirals like M31.

Solutions to these problems could be entirely of numerical or astrophysical nature. Indeed, the most recent *N*-body simulations tend to predict smoother halos (Weinberg and Katz, 2001), and less friction between halos and spinning bars (Valenzuela and Klypin, 2002). These recent results were obtained by increasing the resolution of the simulations, and by taking into account dissipative processes in the baryonic component. Some other works show that a clumpy halo is still compatible with the observed velocity distribution of stars in the galactic disk (Font et al., 2001), and also with microlensing data in the case of the Milky Way (Dalal and Kochanek, 2002): so, it is not impossible that the clumps are there, but remain invisible because they do not trace luminous matter.

However, these features may point towards the need for an alternative to standard ΛCDM in order to describe properly the galactic dynamics. A minimal change would be to alter slightly the properties of the dark matter particles. New candidates for the astronomical dark matter are under scrutiny such as warm dark matter (Colín et al., 2000; Narayanan et al., 2000; Barkana et al., 2001; Abazajian et al., 2001; Hansen et al., 2002), particles with self interactions (Spergel and Steinhardt, 2000), or non-thermally produced WIMPs (Lin et al., 2001). A more drastic solution is to assume deviations from Newtonian gravity on large scales, in order to account for galaxy rotation curves without the need for any dark matter particle. This is the main goal of the MOND theory (Milgrom, 1983). In the present contribution, we will study an approach which could—to some extent—unify the two points of view. We will introduce a scalar field that can be first considered as a new dark matter candidate, which plays almost the same role as usual WIMPs, but with slightly different clustering properties. Equivalently, this scalar field can be thought to be a modification of Newtonian gravity, as in tensor-scalar gravity theories. Although this model has not been elaborated as much as MOND theories, it appears as a possible competitor, and we believe that it provides a useful and complementary insight on the issue of galaxy rotation curves. Such as MOND, scalar field models would change drastically the usual understanding of galaxy rotation curves: the later would not result from complicated *N*-body dynamics, but would depend on the scalar field potential in a simple and direct way, namely, through the Klein–Gordon equation.

The scalar field scenario may also provide an exciting possibility to have a common explanation for for both the dark energy Ω_Λ and the dark matter $\Omega_{\rm cdm}$ components of the Universe. Before trying to reach such an ambitious goal[1] one could explore the relevance of scalar fields to the cosmological and galactic dark matter puzzles, as was done for dark energy with the so-called ''quintessence'' models (Bludman and Roos, 2001; Steinhardt and Caldwell, 1998). The archetypal example of quintessence is a neutral scalar field φ with Lagrangian density

$$L = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi). \tag{1}$$

Should the field be homogeneous, its cosmological energy density would be expressed as

$$\rho_\varphi \equiv T^0_{\ 0} = \frac{\dot{\varphi}^2}{2} + V(\varphi), \tag{2}$$

whereas the pressure would obtain from $T_{ij} \equiv -g_{ij}P$ so that

$$P_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi). \tag{3}$$

[1] Two possible directions for using a quintessence field as dark matter were proposed in Sahni and Wang (2000) and Wetterich (2001, 2002).

If the kinetic term $\dot{\varphi}^2/2$ is small with respect to the contribution from the potential $V(\varphi)$, the equation of state can match the condition for driving accelerated expansion in the Universe, $\omega_Q \equiv P_\varphi/\rho_\varphi < -1/3$. Instead, in order to behave as dark matter today, the field should be pressureless: $|P_\varphi| \ll \rho_\varphi$. So, the kinetic and potential energies should cancel out in Eq. (3), a condition automatically fulfilled by a quickly oscillating scalar field averaged over one period of oscillation. This well-known setup is that of the cosmological axion. It requires a quadratic scalar potential, so that the kinetic and potential energies both redshift as $\varphi^2 \propto a^{-3}$ with the Universe expansion and cancel out at any time during the field dominated stage, which is then equivalent to the usual matter dominated one.

Axions—or more generally, bosonic dark matter—were revived recently due to the undergoing CDM crisis. For instance, it was noticed in (Hu et al., 2000) that structure formation on small scales can be forbidden by quantum mechanics, for wavelengths smaller than the Compton wavelength—i.e., the minimal spreading of an individual boson wave function. The latter matches the scale of galactic substructures for an ultra-light mass of order $m \sim 10^{-22}$ eV. Alternatively, one may introduce a self-coupling term (Peebles, 2000; Goodman, 2000). As we have seen, the existence of a matter-like dominated stage requires that the contribution of non-quadratic terms to the potential energy remains subdominant. Nevertheless, a self-coupling would modify the field behavior in the early Universe, as well as its clustering properties today in regions where the field is overdense—exactly like for boson stars, which are crucially affected by the presence of a self-coupling (Colpi et al., 1986). The self-coupling is also relevant for the issue of field clumps stability, and can explain why dwarf and low-surface-brightness galaxies have cores with finite density (Riotto and Tkachev, 2000).

A remarkable feature with bosonic dark matter is the possibility to form Bose condensates, i.e., large domains where the field is coherent in phase and is in equilibrium inside its own gravitational potential—like boson stars—or in that of an external baryonic matter distribution. This opens the possibility to have a very simple and elegant model for galactic halos, in which rotation curves would follow from simple equations—essentially the Klein–Gordon wave equation, which governs classical scalar fields as well as Bose condensates. This situation strongly differs from the more conventional picture of a gas of individual particles—fermions or heavy bosons—for which gravitational clustering does not lead to universal density profiles and where the shape of galactic halos can only be studied through technically difficult N-body simulations.

The formation and stability of such condensates is a complicated issue (see e.g. Tkachev, 1986; Seidel and Suen, 1994; Khlebnikov, 2000, 2002) even when the field is complex and has a global charge—not to be understood as an electric charge, but as a conserved number of quanta like the baryon or lepton number. For instance, a large condensate can be unstable under fragmentation into smaller clumps. For a real scalar field with no conserved charge, the issue of stability is even more subtle since the field can self-annihilate, especially when the condensate core density exceeds a critical value (Tkachev, 1986). This property can improve the agreement with observations (Riotto and Tkachev, 2000), since the coupling constant will tune the upper limit on the density of dark matter cusps at the center of galaxies. However, such a positive feature is far from excluding models with a conserved charge. In fact, the issue of Bose condensation on galactic scales—in an expanding Universe—has never been studied in details. The result would depend very much on the scalar potential, and it is difficult to guess what would be the maximal core density today.

In our recent works (Arbey et al., 2001, 2002), we focused on a scenario with a conserved charge, and assumed that dark matter consists of a complex scalar field with a quasi-homogeneous density in the early Universe, producing later galactic halos through Bose-condensation. The Lagrangian density reads

$$L = g^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi - V(\phi). \tag{4}$$

Throughout this analysis, we will focus on the potential

$$V = m^2\phi^\dagger\phi + \lambda\{\phi^\dagger\phi\}^2. \tag{5}$$

Beside the issue of charge conservation, the case for a complex scalar field is somewhat richer than that of a real (neutral) scalar field, especially when this scenario is implemented on cosmological scales. In one limit, the complex field can behave as an effective real one, similar to the usual axion. On the other hand, it can be spinning in the complex plane with slowly-varying modulus, like in the so-called spintessence (Boyle et al., 2002) scenario. This depends on the dominant term in the kinetic energy, which can be either radial and oscillating, or orthoradial and slowly varying. As a result, during the field dominated era, the spintessence would have a continuously vanishing pressure, while the axion pressure would oscillate between $+\rho_\phi$ and $-\rho_\phi$.

2. Galactic halos

We are interested in galactic halos consisting in self-gravitating scalar field configurations—which can be seen as Bose–Einstein condensates spanning over very large scales. Since the typical velocities observed in galaxies do not exceed a few hundreds of km s^{-1}, it is enough to study the quasi-Newtonian limit where the deviations from the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ are accounted for by the vanishingly small perturbation $h_{\mu\nu}$. Inside galaxies, the latter is of order the gravitational potential

$$h_{\mu\nu} \sim \Phi \sim V_C^2, \tag{6}$$

where V_C is the rotation velocity—in the case of spirals—and where $\sqrt{2}V_C$ is the escape velocity from the system. In the harmonic coordinate gauge where it satisfies the condition

$$\partial_\alpha h^\alpha_\mu - \frac{1}{2}\partial_\mu h^\alpha_\alpha = 0, \tag{7}$$

the perturbation $h_{\mu\nu}$ is related to the source tensor

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\alpha_\alpha \tag{8}$$

through

$$\Box h_{\mu\nu} = -16\pi G\, S_{\mu\nu}. \tag{9}$$

The Poisson equation reads like

$$\Delta\Phi = 8\pi G\, S_{00}, \tag{10}$$

where $\Phi = h_{00}/2$ is the Newtonian potential. For pressureless matter, $2\,S_{00} = T_{00} = \rho$. On the other hand, for the complex scalar field, the gravitational potential is sourced by the effective mass density

$$\frac{\rho_\phi^{\mathrm{eff}}}{2} \equiv S_{00} = \left\{2\dot{\phi}^\dagger\dot{\phi} - V(\phi)\right\} \tag{11}$$

which is a priori different from the energy density T_{00}. So, inside a galactic halo, the gravitational potential is given by

$$\Delta\Phi = 4\pi G\left(\rho_\phi^{\mathrm{eff}} + \rho_{\mathrm{b}}\right), \tag{12}$$

where ρ_{b} is the distribution of baryonic matter forming the various galactic components—stellar disk, bar, bulge . . . In first approximation, the galaxy can be seen as spherically symmetric. In that case, one shows (Friedberg et al., 1987) that all stable field configurations must be in the form

$$\phi(r,t) = \frac{\sigma(r)}{\sqrt{2}}\,\mathrm{e}^{i\omega t} \tag{13}$$

where the amplitude σ depends only on the radius r. Then, the effective field density reads like

$$\rho_\phi^{\mathrm{eff}} = \left\{2\omega^2\sigma^2 - m^2\sigma^2 - \frac{1}{2}\lambda\sigma^4\right\}. \tag{14}$$

The radial distribution of the field $\sigma(r)$ and the gravitational potential $\Phi(r)$ are given by a system of two coupled equations: the Poisson equation (12) and the Klein–Gordon equation. The latter may be expressed as

$$\mathrm{e}^{-2v}\left\{\sigma'' + \left(u' + v' + \frac{2}{r}\right)\sigma'\right\} + \omega^2\,\mathrm{e}^{-2u}\sigma - m^2\sigma - \lambda\sigma^3 = 0 \tag{15}$$

in the isotropic metric where

$$\mathrm{d}\tau^2 = \mathrm{e}^{2u}\,\mathrm{d}t^2 - \mathrm{e}^{2v}\{\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2\}. \tag{16}$$

The prime denotes the derivative with respect to the radius r. The Newtonian approximation corresponds to

$$u \simeq -v \simeq \Phi. \tag{17}$$

Relation (15) simplifies into

$$\Delta\sigma + (1-4\Phi)\omega^2\sigma - (1-2\Phi)(m^2\sigma + \lambda\sigma^3) = 0. \quad (18)$$

For each value of the parameters (m, λ, ω) and a given baryon distribution, these equations form an eigenvalue problem with discrete solutions, labelled either by the central value $\sigma_0 > 0$ or by the number of nodes n in which $\sigma(r) = 0$. The lowest energy state—which is not identically null due to charge conservation—has $n = 0$. The self-consistency of the Newtonian limit requires $|\Phi| \ll 1$. Such solutions exist only for

$$0 < (m^2 - \omega^2) \ll m^2. \quad (19)$$

In Fig. 1, we give the rotation curves that would be predicited in a halo consisting in a pure free scalar field, for the fundamental state and for various excited states. The excited states are more flat at large radius, and may look like better candidates for explaining the flatness of rotation curves. However, for the data that we use later, only the raise of the dark matter velocity curve is important, and it is not possible to distinguish between the $n = 0$ and $n \neq 0$ solutions.

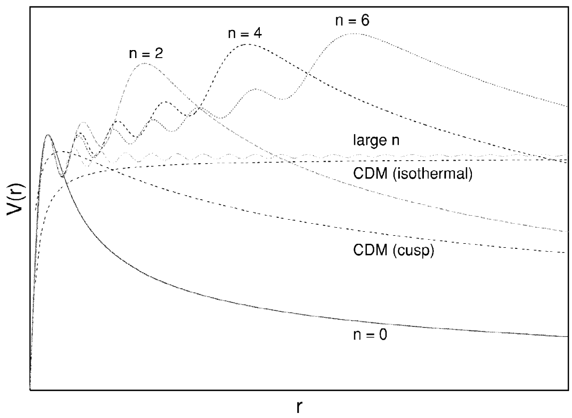

Fig. 1. Rotation curves inside a galactic halo that consists of a pure self-gravitating scalar field. The corresponding boson is massive but has no interactions. The fundamental and $n = 2, 4, 6$ states are featured together with an extremely excited field configuration for which $n \to \infty$. Conventional CDM halos are also presented for comparison [isothermal distribution, cuspy profile (Navarro et al., 1996)]. Each curve has an arbitrary core radius and normalization. We choose to normalize the five scalar field solutions to a common amplitude at the first maximum. These solutions possess $n + 1$ maxima, followed by a decay in $r^{-1/2}$—as for any bounded object. The amplitudes of the first inner maxima are approximately the same, while the outer ones are bigger. For $n \to \infty$, the last maximum and the $r^{-1/2}$ behavior are rejected far outside the figure, at infinity: we only see a quasi-flat region with small oscillations.

Note that all these configurations are assumed to be regular at the origin. Allowing for a black hole at the galactic center would lead to different solutions of the Klein–Gordon and Einstein equations (Wetterich, 2001, 2002).

2.1. Free field

In Arbey et al. (2001), we solved these equations for $\lambda = 0$. We found that halos consisting in the fundamental configuration of a free scalar field fit perfectly well the universal rotation curves of low-luminosity spiral galaxies (Persic et al., 1986). These data has three advantages for our purpose: the robustness of the points and error bars (obtained by averaging over many galaxies), the good determination of the baryon distribution—solely a stellar disk with exponential luminosity profile—and the low baryon contribution which justifies the approximation of spherical symmetry.

With a quadratic potential, the size of the halo is given by

$$l \sim \sqrt{\frac{M_P}{\sigma_0}\frac{\hbar}{mc}}, \quad (20)$$

where we neglected the dependence on the baryon density. If the central field value σ_0 is significantly smaller than the Planck mass, the coherence length of the condensate exceeds the Compton wavelength of an individual particle—$l_{\mathrm{compton}} = \hbar/(mc)$—but it is clear that only an ultra-light scalar field can condensate on distances of order 10 kpc. The typical orbiting velocity in such a halo is given by $v/c \sim \sqrt{\sigma_0/M_P}$. Therefore, requiring $v \sim 100$ km s^{-1} and $l \sim 10$ kpc fixes σ_0 around $10^{-6} M_P$ and m around 10^{-23} eV, as confirmed by a detailed fitting to the data (see Fig. 2).

Since the distribution of such halos only depends on the free parameters σ_0 and m—where we impose a unique value of m for all galaxies—we believe that their success in reproducing universal rotation curves is a significant argument in favor of this model. On

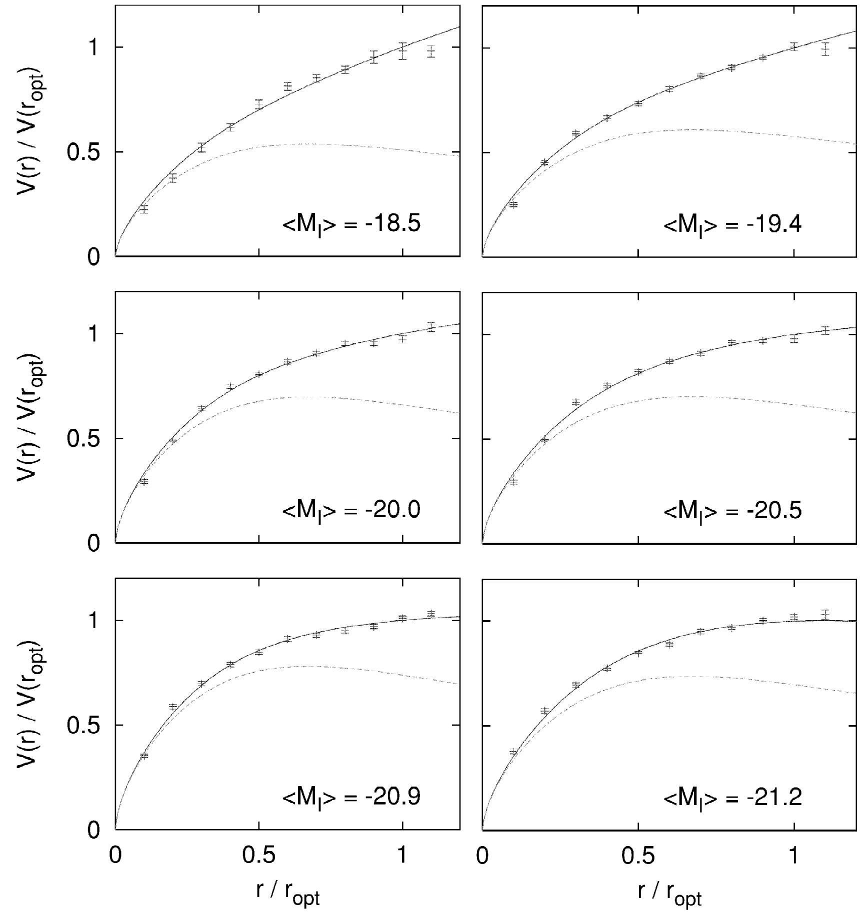

Fig. 2. A chi-square comparison with the universal rotation curves of (Persic et al., 1986) reveals that the best fit to different classes of magnitudes with a common value of the mass is given by $m = 6 \times 10^{-24}$ eV. The corresponding rotation curves are shown in solid lines. The dashed line shows the contribution from the disk only.

the other hand, the existence of such a low mass, even if not strictly forbidden by fundamental principles, is very unlikely due to unavoidable radiative corrections. This could motivate a systematic investigation of other potentials for the scalar field. The next level of complexity would consist in adding a quartic self-coupling.

2.2. Quartic self-coupling λ

As is well known for boson stars—which are exactly identical to our halos in the absence of a baryon component—the inclusion of a quartic term drastically modifies the mass m and the extension of the condensate, even when it contributes to a negligible fraction of the central energy density (Colpi et al., 1986; Liddle and Madsen, 1992). This is so because $\lambda\sigma^2$ can be very small with respect to $m^2 \simeq \omega^2$ and yet comparable to the difference $(m^2 - \omega^2)$ that appears in the Klein–Gordon equation. In the limit where $\Lambda \equiv \lambda/(4\pi G m^2) \gg 1$ and in the absence of a baryon population, we can even give an exact analytic solution for the field and for the orbiting velocity of test particles:

$$\sigma(r) = \sigma_0 \left\{ \frac{\sin\left(m\sqrt{2/\Lambda}\, r\right)}{m\sqrt{2/\Lambda}\, r} \right\}^{1/2}, \tag{21}$$

$$\frac{v(r)}{c} = r\Phi'(r) = 2\pi\,\Lambda\,\frac{\sigma_0^2}{M_P^2}\left\{\frac{\sin\left(m\sqrt{2/\Lambda}r\right)}{m\sqrt{2/\Lambda}r} - \cos\left(m\sqrt{2/\Lambda}r\right)\right\}, \tag{22}$$

with the requirement that

$$\Lambda^{-1} \ll \frac{\sigma_0}{M_P} \ll \Lambda^{-1/2}. \tag{23}$$

Because $\sigma_0/M_P \sim (v/c)/\sqrt{\Lambda}$, the second inequality follows from the Newtonian self-consistency condition $|\Phi| \sim v^2 \ll 1$. The first inequality translates into $\Lambda \gg (c/v)^2$. It implies that all the field spatial derivatives can be neglected in Eq. (18) and sets the maximal radius up to which the analytic solution is valid. This maximal radius is at most equal to half a period so that $r \leq \sqrt{\Lambda/2}(\pi/m)$. That bound is almost saturated for $\Lambda \rightarrow \infty$. Note that ω does not appear in the analytic solutions because it is only relevant at larger radii. However, the Newtonian self-consistency condition imposes that $1 - \omega^2/m^2 \ll 1$.

The field behavior (21) may be readily recovered by neglecting the spatial derivatives of σ in the Klein–Gordon equation (18) so that

$$\sigma^2(r) \simeq \frac{m^2}{\lambda}\left\{\frac{\Omega^2}{B} - 1\right\}, \tag{24}$$

where $B(r) = 1 + 2\Phi(r)$ and $\Omega = \omega/m$. In the Newtonian limit, the pressure reads like

$$P_\phi \equiv L \simeq \frac{m^4}{4\lambda}\left\{\frac{\Omega^2}{B} - 1\right\}^2, \tag{25}$$

while the effective mass density (14) of the Bose-condensate is

$$\rho_\phi^{\mathrm{eff}} \simeq m^2\sigma^2 \simeq \frac{m^4}{\lambda}\left\{\frac{\Omega^2}{B} - 1\right\}. \tag{26}$$

Both are related through the Lane–Emden polytropic equation of state

$$P_\phi = K\rho_\phi^{\mathrm{eff}\,1+1/n}, \tag{27}$$

with $K = \lambda/(4m^4)$ while the polytropic index is $n = 1$. For such a value, the gravitating system—in hydrostatic equilibrium—is shown to have a constant core radius $r_c = \pi a$ where

$$a^2 = \frac{1}{8\pi G}\frac{\lambda}{m^4}. \tag{28}$$

The field and density profiles are functions of the reduced radius $z = r/a$

$$\frac{\rho'_\phi(r)}{\rho'_\phi(0)} = \frac{\sigma^2(r)}{\sigma_0^2} = \frac{\sin(z)}{z}. \tag{29}$$

The most striking feature in the large Λ limit is as follows: although the quartic term remains subdominant in the energy density—Eq. (23) implies that $\lambda\sigma^4 \ll m^2\sigma^2$—the typical size of the system is very different from the free field case since now it reads like

$$l \sim \lambda^{1/2} M_P \hbar/(m^2 c). \tag{30}$$

As the central field value does not appear in this expression, different halo sizes would just result from different baryon contributions to the density, which bring corrections to (30). The central field value σ_0 still determines the rotation curve amplitude. In the large Λ limit and in the absence of baryons, the maximal rotation speed is given exactly by

$$\frac{v_{\max}^2}{c^2} = 2.13\,\pi\,\Lambda\frac{\sigma_0^2}{M_P^2} \quad \text{at } r = 1.94\Lambda^{1/2}/m. \tag{31}$$

We can use relations (30) and (31) in order to find the order of magnitude for λ/m^4 which has the best chance to provide a good fit to the data. By requiring that the rotation velocity peaks around 200 km s^{-1} at a typical radius of 10 kpc, we find

$$m \sim \lambda^{1/4}\,\mathrm{eV} \quad \text{and} \quad \sigma_0 \sim \Lambda^{-1/2} 10^{-3} M_P. \tag{32}$$

Taking for instance λ in the range [1, 10^{-4}], we obtain a mass of order 0.1 to 1 eV, i.e., a few orders of magnitude larger than the expected neutrino masses. So, the inclusion of a quartic self-coupling leads to a much more realistic value of the mass from a particle physics point of view.

3. Conclusion

We conclude that a free or a self-coupled scalar field is a promising candidate for the galactic dark matter component. The scalar mass is bound to be in the range 10^{-23} eV $< m < 1$ eV, depending on the value of the self-coupling constant. Clearly, our analysis can be improved in several ways: one should implement a better modelisation of the disk/halo geometry, use more galaxies, include some other baryonic components (gas, bulge . . .), compare with rotation curves at larger radius . . .

Note that some issues of a different type arise when the limits from galactic dynamics and some cosmological constraints are taken simultaneously into account. In Arbey et al. (2002), we investigated a scenario in which the field is quasi-homogeneous in the early Universe, and evolves under the Klein–Gordon and Einstein equation. For some sets of initial conditions, the density of the scalar field background takes over that of radiation at the usual redshift of equality, $z_{eq} \sim 10^3$. Then, during the field-dominated epoch (which replaces the usual matter-dominated epoch), the non-linear evolution produces galactic halos through Bose-condensation.

A detailed investigation of the background evolution reveals different problems for the two scalar potential discussed previously. A free complex field with a very small mass $m \sim 10^{-23}$ eV has a correct cosmological behavior in the early Universe, but behaves today mostly as a real axion, with a problematic value of its conserved quantum number: the number density of field quanta would exceed that of photons by 10^{24} orders of magnitude. On the other hand, an interacting field with a quartic coupling $\lambda \sim 0.1$ and a more realistic mass $m \sim 1$ eV carries a quantum number close to the photon number density. Unlike a free field, it would be spinning today in the complex plane—like the so-called ''spintessence''. Unfortunately, the cosmological evolution of such field in the early Universe is not compatible with nucleosynthesis. Indeed, during radiation domination, the scalar field decays as dark radiation, and can be described by an effective number of extra neutrinos $\Delta N \simeq 5$ (while the nucleosynthesis bound reads $\Delta N \leq 1$). A similar conclusion was reached in Peebles (2000) in the real scalar field case, and Peebles suggested to alter the expression of the self-interaction term: then, the field behavior in the early universe can be slightly different form that of radiation, leading to a smaller field density during nucleosynthesis. A more radical change would be to assume, in the spirit of Wetterich (2001, 2002), that the field fluctuations are already non-linear on some scales at the time of equality: then, back-reaction effects can modify the effective homogeneous density during the field-dominated stage. This alternative also deserves further studies—in the most optimistic prospect, it may offer an opportunity to unify the description of dark matter and dark energy.

Acknowledgements

We would like to thank C. Alard, S. Colombi and B. Fort for giving us the opportunity to present our work in this stimulating workshop, during which we got many useful comments from various participants.

References

Abazajian, K., Fuller, G.M., Tucker, W.H., 2001. ApJ 562, 593.
Arbey, A., Lesgourgues, J., Salati, P., 2001. Phys. Rev. D64, 123528.
Arbey, A., Lesgourgues, J., Salati, P., 2002. PhRvD 65, 083514.
Barkana, R., Haiman, Z., Ostriker, J.P., 2001. ApJ 558, 482.
Bludman, S., Roos, M., 2001. ApJ 547, 77.
Boyle, L.A., Caldwell, R.R., Kamionkowski, M., 2002. Phys. Lett. B 545, 17.
Burles, S., Tytler, D., 1998. ApJ 499, 699;
Burles, S., Tytler, D., 1998. ApJ 507, 732.
Colín, P., Avila-Reese, V., Valenzuela, O., 2000. ApJ 542, 622.
Colpi, M., Shapiro, S.L., Wasserman, I., 1986. PhRvD 57, 1485.
Dalal, N., Kochanek, C.S., 2002. ApJ 572, 25.
Font, A.S., Navarro, J.F., Stadel, J., Quinn, T., 2001. ApJ 563, L1.
Friedberg, R., Lee, T.D., Pang, Y., 1987. PhRvD D35, 3640.
Goodman, J., 2000. NewA 5, 103.
Hansen, S.H., Lesgourgues, J., Pastor, S., Silk, J., 2002. MNRAS 333, 544.
Hu, W., Barkana, R., Gruznikov, A., 2000. PhRvL 85, 1158.
Khlebnikov, S., 2000. PhRvD 62, 043519.
Khlebnikov, S., 2002. Preprint hep-ph/0201163.
Liddle, A.R., Madsen, M.S., 1992. Int. J. Mod. Phys. D1, 101.
Lin, W.B., Huang, D.H., Zhang, X., Brandenberger, R., 2001. PhRvL 86, 954.
Milgrom, M., 1983. ApJ 270, 365;
Milgrom, M., 1983. ApJ 270, 371;
Milgrom, M., 1983. ApJ 270, 384.
Moore, B. et al., 1999. ApJ 524, L19.
Narayanan, V.K., Spergel, D.N., Davé, R., Ma, C.-P., 2000. ApJ 543, L103.
Navarro, J.F., Frenk, C.S., White, S.D.M., 1996. ApJ 462, 563.
Netterfield, C.B. et al. (The Boomerang collaboration), 2002. ApJ 571, 604.
Peebles, P.J.E., 2000. ApJ 543, L127.

Percival, W.J. et al., 2001. MNRAS 327, 1297.
Perlmutter, S. et al., 1999. The Supernova Cosmology Project. ApJ 517, 565.
Persic, M., Salucci, P., Stel, F., 1986. MNRAS 281, 27.
Riotto, A., Tkachev, I., 2000. Phys. Lett. B484, 177.
Sahni, V., Wang, L., 2000. PhRvD 62, 103517.
Seidel, E., Suen, W.M., 1994. PhRvL 72, 2516.
Spergel, D.N., Steinhardt, P.J., 2000. PhRvL 84, 3760.
Steinhardt, P.J., Caldwell, R.R., 1998. In: Byun, Y.I., Ng, K.W. (Eds.), Cosmic Microwave Background and Large Scale Structure of the Universe, ASP Conference Series, Vol. 151, p. 13.
Tkachev, I.I., 1986. SvAL 12, 305.
Valenzuela, O., Klypin, A., 2002. Preprint astro-ph/0204028.
Weinberg, M.D., Katz, N., 2001. Preprint astro-ph/0110632.
Wetterich, C., 2001. Phys. Lett. B522, 5.
Wetterich, C., 2002. PhRvD 65, 123512.