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Vector field and rotational curves in dark galactic halos

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Abstract

We study equations of a non-gauge vector field in a spherically symmetric static metric. The constant vector field with a scale arrangement of components, the temporal component about the Planck mass $m_{\rm Pl}$ and the radial component about M suppressed with respect to the Planck mass, serves as a source of metric reproducing flat rotation curves in dark halos of spiral galaxies, so that the velocity of rotation v_0 is determined by the hierarchy of scales: $\sqrt{2}v_0^2 = M/m_{\rm Pl}$, and $M \sim 10^{12}$ GeV. A natural estimate of Milgrom's acceleration about the Hubble rate is obtained.

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1. Introduction

In contrast to the scalar field, appropriately elaborated in cosmology lately, the vector field has remained beyond intensive study in astro-particle problems.

Indeed, a slowly rolling scalar field drives an inflation [1], which produces the correct spectrum of inhomogeneous perturbations in matter density as was observed in the cosmic microwave background anisotropy [2]. Next, a scalar field naturally gives a state with negative pressure, a quintessence [3–5], so that it can cause an accelerated expansion of the universe [6] with a dynamical dark energy. Moreover, some authors try to ascribe dark matter in halos of galaxies [7–9] to a scalar field too, though the properties of such fields at the galaxy scale should be rather different from those of quintessence at the cosmic scale [10–12, 14]. So, the scalar field provides a preferable, more or less successful theoretical treatment of important astro-particle phenomena[1] though some tuning procedures might be

[1] An example of an alternative description due to a modification of gravity is given in [15].

required due to the arbitrariness of scalar potentials. So, the scalar particle of quintessence should, for instance, have a mass of Hubble constant scale. In addition, the dark matter distribution in the spiral galaxies falls as $1/r^2$, which could be a problem for a theoretical explanation in terms of both a scalar field or exotic weak interacting particles.

However, as was recently observed [16], a vector field dynamics possesses features, which make it attractive for cosmological studies. So, in the evolution of flat isotropic homogeneous universe a vector field gains a dynamical mass proportional to the Hubble constant at any, even trivial, potential[2]. Therefore, we do not need any synthetic assumptions to get such a characteristic property in the scalar field sector[3], but it is instead enough to introduce an isotropic vector field in the cosmology, and covariant derivatives automatically generate such dynamical mass terms.

We study a simple Lorentz-invariant form of Lagrangian for a vector field interacting gravitationally only[4],

$$\mathcal{L}_V = \xi \tfrac{1}{2} g^{\mu\nu} (\nabla_\mu \phi^m)(\nabla_\nu \phi^n) g_{mn} - V(\phi^2), \tag{1}$$

where ξ is a vector field signature[5] that can be normal ($\xi = -1$) or phantomic ($\xi = +1$), respectively[6].

Since a temporal component of the 4-vector has a negative kinetic term at the normal signature $\xi = -1$, we will not treat (1) as a fundamental Lagrangian in the sense of axiomatic field theory, but we address it as an effective phenomenological Lagrangian with a phantom: the field possessing the negative kinetic term. So, we accept the following *postulate of validity* for the phantom phenomenology:

- the phantom component is not observable in the flat spacetime, i.e., it does not interact with the matter field (one can use a standard technique in order to introduce a gauge-invariant Lagrangian for a vector field);
- in gravity, the phantom is valid in regions *beyond physical singularities*, if exist.

In this respect, a motivation for the Lagrangian of (1) could be manyfold.

First, a phenomenology of scalar quintessence shows that the state parameter of the quintessence, i.e., the ratio of pressure to density of energy $w = p/\rho$, can take values less than -1 [4],

$$w < -1,$$

that probably puts the quintessence into the phantom stage: the field with a negative kinetic energy [5, 21, 22]. The Lagrangian of the vector field in (1) *naturally* contains a phantom component, the temporal component at normal signature $\xi = -1$. An important physical difference from the case of the scalar phantom is the non-trivial covariant derivative of the vector field.

However, the negative kinetic energy of a fundamental field is not restricted from below, so the phantom cannot be considered as a fundamental field. Such treatment of the scalar phantom involves a cut-off for the kinetic term or, equivalently, higher derivative terms providing a low boundary of negative energy. For example, we can add a term of the form

$$\delta\mathcal{L} = \frac{1}{\Lambda^4}\dot{\phi}^4, \qquad \dot{\phi} = \frac{\mathrm{d}\phi}{\mathrm{d}t},$$

[2] Note that a reasonable form of potential for the vector field is more strictly constrained than that of the scalar field.
[3] The interacting scalar fields could give a dynamical mass of Hubble scale, as was shown in [17].
[4] A gauge vector field with a global symmetry was investigated in [18], while a nonlinear electrodynamics, which leads to an acceleration of universe, was studied in [19].
[5] The signature of metric is assigned to (+, −, −, −).
[6] Note that the 4-vector is generically composed by spin-1 and spin-0 components (see [20] for discussion on covariant object decomposition into components with definite spins).

which guarantees a 'stabilization' of negative kinetic term $\mathcal{L}_0 = -1/2\dot{\phi}^2$ for the scalar phantom. Nevertheless, a greater characteristic time scale of field changes with respect to the inverse stabilization scale $1/\Lambda$ is a more accurate leading approximation by the small negative kinetic term. So, we can expect that at cosmological and galactic scales the effective phantom theory with negative kinetic term could be sound if the stabilization scale is microscopic enough. Decay instabilities of the phantom in the presence of several phantoms were considered in [5]. Thus, standard arguments justifying the study of scalar phantoms are applicable for the introduction of Lagrangian (1) for the vector field.

Second, theories with extra dimensions generate cosmological equations including specific terms quadratic in energy–momentum tensors of matter propagating in four dimensions in addition to an ordinary energy–momentum term [23]. This fact can be considered as an effective redefinition of the energy–momentum tensor:

$$\rho \to \rho + \frac{1}{2\sigma}\rho^2,$$

where σ is a bare cosmological constant (a brane tension). So, the negative kinetic term of the phantom should generate a term quadratic in the kinetic energy, which effectively reproduces the cut-off mentioned above.

Third, let us show that, in contrast to naive expectation, a free phantom interacting with gravity only cannot propagate in a homogeneous isotropic spacetime at all. Indeed, in the Friedmann–Robertson–Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2], \tag{2}$$

we get the following independent field equations for the time-dependent homogeneous scalar phantom ϕ:

$$H^2 = \frac{8\pi G}{3}\left(-\frac{1}{2}\dot{\phi}^2 + V_0\right), \qquad \ddot{\phi} + 3H\dot{\phi} = 0, \qquad H \equiv \frac{\dot{a}}{a}, \tag{3}$$

where we have added a cosmological constant V_0 for reference. At the flat limit of $V_0 = 0$, we get the evident condition

$$H^2 \geqslant 0 \quad \Rightarrow \quad \dot{\phi} \equiv 0,$$

and the phantom is a constant field, which does not evolve (or propagate). Moreover, in the case of positive cosmological constant $V_0 > 0$, the 'pathological' kinetic energy is automatically restricted

$$\dot{\phi}^2 \leqslant 2V_0.$$

Such a restriction remains valid also in the presence of ordinary matter with a density ρ and pressure p satisfying the conservation law for the tensor of energy–momentum

$$\dot{\rho} + 3H(\rho + p) = 0,$$

in the isotropic homogeneous expanding universe, so that a finite density of energy falls down and the permitted region of phantom energy becomes narrower. The same arguments are valid for the temporal component of the vector field too.

In addition, we emphasize that the metric of (2) substituted into the Einstein–Hilbert action of gravity

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G}\int R\sqrt{-g}\,\mathrm{d}^4x$$

after integration by parts, takes the form

$$S_{\mathrm{FRW}} = -\frac{3}{8\pi G}\int \dot{a}^2 a\,\mathrm{d}^4x, \tag{4}$$

which is the action for a scalar field $a(t)$ up to a normalization factor with a 'phantom' kinetic term sign. That is why a cosmological singularity is possible. A reason for the consistency of such a theory of gravity is due to the fact that the FRW metric does not give 'free gravitational waves', but refers to the coupled case, i.e., we deal with a virtual gravitational field, while the harmonic oscillations are well behaved. Therefore, the same features can be ascribed to fields of a 'gravitational nature', as we mean for the vector field.

Next, we consider a vector field, which Lagrangian is not invariant under gauge transformations. For comparison, any gauge field $\mathcal{A}_\mu$, for example an Abelian field, contains an unphysical degree, i.e., a purely gauge component:

$$\mathcal{A}_\mu = \partial_\mu f.$$

If the energy–momentum tensor of the gauge field is gauge invariant, then the purely gauge component does not propagate. If we add a gauge-fixing term, the gauge invariance guarantees that the longitudinal component preserves its bare propagator, i.e., it is not renormalized. Hence, the longitudinal field undetectable by gauge interactions remains arbitrary and gauge dependent anyway. So, it would be reasonable to study a gauge-noninvariant vector field interacting with gravity only, in order to examine the main effects due to such a field.

Finally, a current–current interaction

$$\mathcal{L}_{\text{int}} = j^\mu \mathcal{A}_\mu$$

preserves its gauge independence if the matter current, j^μ, is transversal:

$$\nabla_\mu j^\mu \equiv 0.$$

This note could be important, since it provides a chance for transversal components of the vector field to interact with the matter if they do not kinematically mix with longitudinal ones.

Further, the observed quasi-isotropic cosmic microwave background (CMB) fixes a universe rest frame, i.e., it forms an ether. Indeed, experimentalists measure a velocity of Earth motion in the CMB in order to extract a small true anisotropy (so-called dipole subtraction). Thus, the expanding universe determines a specific rest frame, which allows one to introduce fields, corresponding to the symmetry of the frame. Particularly, the vector field preserving the homogeneous and isotropic metric should have a temporal component only: $\phi^m = (\phi_0, \mathbf{0})$. It is *a priori* clear that the curved metric of the expanding universe breaks the Lorentz invariance of course. That is why the vector field components have a fixed form required by the symmetry of the physical system. In addition, we expect that an effective potential of the vector field should be of gravitational origin, so that its dimensional parameters are posed in the Planck mass range. If the potential has a stable point, we expect that a corresponding value of the temporal component ϕ_0 is given by $\phi_* \sim m_{\text{Pl}}$. We will see that the expansion causes a small time dependence of $\phi_0(t)$ in the vicinity of ϕ_*. So, the variation of ϕ_0 at galactic scales of distance and time is negligible. This fact means that we get a factorization: the field equations give a true function of $\phi_0(t)$, which can be considered as a constant external field ϕ_* at galactic level.

Furthermore, given the external vector field source $\phi_0 = \phi_*$, we can allow small stochastic fluctuations of spatial components, $\boldsymbol{\phi}$, only. These fluctuations could be related to matter fields. For instance, a complex scalar matter field, χ, can develop vacuum fluctuations (condensates) at a characteristic scale $M \ll m_{\text{Pl}}$, so that the spatial derivatives, $\nabla\chi$, coupled with the spatial components of the vector field can stochastically get nonzero vacuum expectations $\langle\partial_i\chi^\dagger\partial_j\chi\rangle$ at the same scale M, and the effective potential of spatial vector field could acquire a form yielding a stochastically stable point of $\langle\boldsymbol{\phi}^2\rangle = \phi_\star^2 \sim M^2$ too. Thus, we arrive at the position with two scales of expectation values for the vector field: the temporal component ϕ_* of the order of m_{Pl} and the spatial component $\phi_\star$ of the order of $M \ll m_{\text{Pl}}$. This small parameter is a characteristic for the problem, and the arrangement of scales is valid, say, at M determined by a spontaneous symmetry breaking in the great unification theory, $M \sim M_{\mathrm{GUT}}$. The constant vector field with the scale arrangement serves as the external source in the gravity equations at the galactic scale.

The consideration is essentially transformed in the spherically symmetric case, since the symmetry of the system allows the spatial component directed along the radius vector: $\boldsymbol{\phi} = \phi_\star \boldsymbol{n}$ with the unit vector $\boldsymbol{n} = \boldsymbol{r}/r$. Therefore, the external vector field takes the form $\phi^m = (\phi_*, \phi_\star \boldsymbol{n})$, which is purely gauge field with a gauge function $f(t,r) = \phi_* t - \phi_\star r$ and $\phi_m = \partial_m f(t,r)$ in a flat spacetime limit, so that this field can only be detected gravitationally. In addition, this point gives an extra argument for the Lagrangian of (1), since one could completely substitute a scalar field derivative for the vector field: $\phi_m = \partial_m f(t,r)$, so that the potential near the extremal point, say, $V \approx V_0 - \bar{\mu}^2(\phi_0^2 - \phi^2)$, would be transformed to an ordinary kinetic term of scalar field f up to a normalization factor, while the other terms represent higher derivative contributions in an affective Lagrangian.

In this paper, we study vector field dynamics in a static spherically symmetric metric, which should be in halos of spiral galaxies. Rotational curves in such galaxies become flat in the regions of dark halos that correspond to $1/r^2$ dependence of dark matter density on the distance from the galaxy centre. We show that the covariant derivatives of vector fields naturally and uniquely generate such dependence. The physical reason for the conclusion is rather simple. First, the vector field gains a dynamical mass term determined by a spatial curvature. Second, the problem introduces a small parameter, a constant velocity of rotation in the dark halos v_0. Third, following the factorization of cosmological and galactic scales, at large distances we have to reach a cosmological limit: the vector field negligibly slowly evolving with time and distance, i.e., the constant field at the galaxy scale with the hierarchy of expectations $\kappa = \phi_\star/\phi_* \ll 1$. We find the relation between two small parameters, the velocity and scale ratio: $v_0^2 = \kappa/\sqrt{2}$. Then, the curvature should fall as $1/r^2$, only that reproduces the flat rotational curves.

The paper is organized as follows. In section 2, we study Einstein equations with the vector field in a static spherically symmetric metric

$$\mathrm{d}s^2 = \mathfrak{f}(r)\,\mathrm{d}t^2 - \frac{1}{\mathfrak{h}(r)}\,\mathrm{d}r^2 - r^2[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2], \tag{5}$$

by deriving the energy–momentum tensor. Then, we recall a condition following from the observation of flat rotational curves: the function $\mathfrak{f}(r)$ should take a specific form up to small neglected corrections, while $\mathfrak{h}(r)$ should be close to 1. Then, we expand in small v_0^2 and κ and find the solution[7]

$$v_0^2 = \kappa/\sqrt{2}, \qquad \mathfrak{h}(r) = 1 - 2v_0^2, \qquad \mathfrak{f}'(r) = \frac{2v_0^2}{r},$$

giving a $1/r^2$ profile of the curvature and a flat asymptotic behaviour for the velocity of rotation[8] so that the velocity squared is determined by the small ratio of spatial component to temporal one, i.e., the ratio of characteristic matter scale to that of gravity, the Planck mass, that gives $M \sim 10^{-7} m_{\mathrm{Pl}} \sim 10^{12}$ GeV, a scale in the range of GUT breaking[9]. In section 3, we analyse the factorization. Section 4 is devoted to the description of applicability regions

[7] We use ordinary notation for the derivative with respect to the distance, the prime symbol $\partial_r f(r) = f'(r)$.

[8] The same metric was first found exactly by Nucamendi, Salgado and Sudarsky (the last reference of [10]). They also studied light bending and gravitational lensing, which do not conflict with observations, so we will not concern ourselves with this question in the present paper.

[9] One can also make $M \sim \mu^2/m_{\mathrm{Pl}}$ yielding $\mu \sim 10^{16}$ GeV, which is closer to the GUT scale. However, this treatment suffers from 'tending to the desirable result', I think.

for the flat rotational curves in the framework of constant vector fields that gives a natural estimate of Milgrom's acceleration. Other approaches are briefly discussed for comparison. The results are summarized in the conclusion.

2. Generic equations

General expressions for the Christoffel symbols, Ricci tensor and scalar curvature for the spherically symmetric static metric of (5) are listed in the appendix.

Then, a tensor entering the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{6}$$

i.e.,

$$G_\mu^\nu = R_\mu^\nu - \tfrac{1}{2}\delta_\mu^\nu R,$$

takes the form

$$G_t^t = \frac{1-\mathfrak{h}}{r^2} - \frac{\mathfrak{h}'}{r}, \qquad G_r^r = \frac{1-\mathfrak{h}}{r^2} - \frac{\mathfrak{f}'}{r}\frac{\mathfrak{f}}{\mathfrak{h}}, \tag{7}$$

$$G_\theta^\theta = G_\varphi^\varphi = -\frac{1}{2}\frac{\mathfrak{h}}{\mathfrak{f}}\mathfrak{f}'' - \frac{\mathfrak{h}}{2r}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} + \frac{\mathfrak{h}'}{\mathfrak{h}}\right) + \frac{1}{4}\mathfrak{h}\frac{\mathfrak{f}'}{\mathfrak{f}}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} - \frac{\mathfrak{h}'}{\mathfrak{h}}\right). \tag{8}$$

In polar coordinates $x^m = (t, r, \theta, \varphi)$, covariant derivatives of vector field $\phi^m = (\phi_0, \phi_r, 0, 0)$ in the metric of (5) are equal to

$$\begin{aligned}
&\phi^t{}_{;t} = \frac{1}{2}\frac{\mathfrak{f}'}{\mathfrak{f}}\phi_r, && \phi^t{}_{;r} = \phi_0' + \frac{1}{2}\frac{\mathfrak{f}'}{\mathfrak{f}}\phi_0,\\
&\phi^r{}_{;t} = \frac{1}{2}\mathfrak{h}\mathfrak{f}'\phi_0, && \phi^r{}_{;r} = \phi_r' - \frac{1}{2}\frac{\mathfrak{h}'}{\mathfrak{h}}\phi_r,\\
&\phi^\theta{}_{;\theta} = \phi^\varphi{}_{;\varphi} = \frac{1}{r}\phi_r, && \sqrt{-g} = \sqrt{\mathfrak{f}/\mathfrak{h}}\,r^2\sin^2\theta,
\end{aligned} \tag{9}$$

where we have also shown the determinant of the metric.

Squaring $(\mathcal{D}\phi)^2 \equiv \phi^m{}_{;\mu}\phi^n{}_{;\nu} g_{mn} g^{\mu\nu}$ gives

$$(\mathcal{D}\phi)^2 = -\mathfrak{f}\mathfrak{h}\left(\phi_0' + \frac{1}{2}\frac{\mathfrak{f}'}{\mathfrak{f}}\phi_0\right)^2 - \frac{1}{4}\frac{\mathfrak{h}}{\mathfrak{f}}(\mathfrak{f}')^2\phi_0^2 + \frac{1}{4}\left(\frac{\mathfrak{f}'}{\mathfrak{f}}\right)^2\phi_r^2 + \left(\phi_r' - \frac{1}{2}\frac{\mathfrak{h}'}{\mathfrak{h}}\phi_r\right)^2 + \frac{2}{r^2}\phi_r^2. \tag{10}$$

The action of the vector field in the problem is written as

$$S_{\rm V} = -\frac{1}{2}\int \mathrm{d}t\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi(\mathcal{D}\phi)^2\sqrt{-g},$$

where we have not displayed a potential, whose properties have been described in the introduction, so that it makes a dominant contribution to the field equations for the vector field that results in the mentioned quasi-constant fields, as will be discussed in section 3.

For the moment we recall the restriction following from the flatness of the rotational curves. In this way, a particle motion in the metric of (5) is determined by the Hamilton–Jacobi equations

$$g^{\mu\nu}\partial_\mu S \partial_\nu S - m^2 = 0, \tag{11}$$

where m denotes the particle mass. Following the general framework, we write the solution in a form which incorporates two integrals of motion in the spherically symmetric static gravitational field,

$$S = -\mathcal{E}t + \mathfrak{M}\theta + \mathcal{S}(r), \tag{12}$$

where $\mathcal{E}$ and $\mathfrak{M}$ are the conserved energy and rotational momentum, respectively. Then, from (11) we deduce

$$\left(\frac{\partial \mathcal{S}}{\partial r}\right)^2 = \frac{1}{\mathfrak{f}\mathfrak{h}}\mathcal{E}^2 - \frac{1}{\mathfrak{h}}\left(\frac{\mathfrak{M}^2}{r^2} + m^2\right), \tag{13}$$

which results in

$$\mathcal{S}(r) = \int_{r_0}^{r(t)} \mathrm{d}r \frac{1}{\sqrt{\mathfrak{f}\mathfrak{h}}}\sqrt{\mathcal{E}^2 - V^2(r)}, \tag{14}$$

where V^2 is an analogue of the potential,

$$V^2(r) = \mathfrak{f}\left(\frac{\mathfrak{M}^2}{r^2} + m^2\right).$$

The trajectory is implicitly determined by the equations

$$\frac{\partial S}{\partial \mathcal{E}} = \text{const} = -t + \int_{r_0}^{r(t)} \mathrm{d}r \frac{1}{\sqrt{\mathfrak{f}\mathfrak{h}}} \frac{\mathcal{E}}{\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{15}$$

$$\frac{\partial S}{\partial \mathfrak{M}} = \text{const} = \theta - \int_{r_0}^{r(t)} \mathrm{d}r \frac{1}{\sqrt{\mathfrak{f}\mathfrak{h}}} \frac{\mathfrak{f}}{r^2} \frac{\mathfrak{M}}{\sqrt{\mathcal{E}^2 - V^2(r)}}. \tag{16}$$

Taking the derivative of (15) and (16) with respect to the time[10], we get

$$1 = \dot{r}\frac{\mathcal{E}}{\sqrt{\mathfrak{f}\mathfrak{h}}\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{17}$$

$$\dot{\theta} = \frac{\dot{r}}{r^2}\frac{\mathfrak{f}}{\sqrt{\mathfrak{f}\mathfrak{h}}}\frac{\mathfrak{M}}{\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{18}$$

and hence,

$$\mathcal{E} = \frac{\mathfrak{f}}{rv}\mathfrak{M}, \tag{19}$$

relating the energy and the rotational momentum, where we have introduced the velocity

$$v \stackrel{\text{def}}{=} r\dot{\theta}.$$

The points of return are determined by

$$\dot{r} = 0 \quad \Rightarrow \quad \mathcal{E}^2 - V^2 = 0 \quad \Rightarrow \quad \mathfrak{M}^2 = m^2 r^2 \frac{v^2}{\mathfrak{f} - v^2}.$$

The circular rotation takes place if two return points coincide with each other, i.e., we have a stability of zero $\dot{r}$ condition. Introducing a 'proper distance' λ by

$$\frac{\partial}{\partial \lambda} = \frac{\partial r}{\partial \lambda}\frac{\partial}{\partial r} = \sqrt{\mathfrak{f}\mathfrak{h}}\frac{\partial}{\partial r}$$

we deduce the wave equation with spectral parameter $\mathcal{E}^2$ and 'potential' V^2

$$\left(\frac{\partial \mathcal{S}}{\partial \lambda}\right)^2 = \mathcal{E}^2 - V^2, \tag{20}$$

[10] As usual, $\partial_t f(t) = \dot{f}$.

so that the stability of circular motion implies the stability of potential,

$$\frac{\partial V^2}{\partial r} = 0. \tag{21}$$

Then, we get

$$v^2 = \frac{1}{2}\frac{\mathrm{d}\mathfrak{f}(r)}{\mathrm{d}\ln r}. \tag{22}$$

Introducing a re-scaled velocity with respect to the proper time,

$$\mathfrak{v}^2 = \frac{1}{\mathfrak{f}} v^2,$$

we get the result of [24]

$$\mathfrak{v}^2 = \frac{1}{2}\frac{\mathrm{d}\ln\mathfrak{f}}{\mathrm{d}\ln r}.$$

An accuracy of observations do not allow us to distinguish v from $\mathfrak{v}$, since $\mathfrak{f}(r) \to 1$ at large distances in dark halos. So, in the halo we make

$$v = v_0 = \text{const}.$$

Therefore,

$$\mathfrak{f}' = \frac{2v_0^2}{r} \tag{23}$$

gives the profile of the flat rotational curves[11]. Non-relativistically, we get

$$\mathfrak{h} = 1 - q, \qquad q \ll 1, \tag{24}$$

while the form of dependence on the distance is not fixed by the flatness.

Let us show that for the vector field serving as an external source, there is a solution of Einstein equations with

$$q' \approx 0, \tag{25}$$

where the approximation means the leading order in a small parameter of v_0^2, which is, in practice, $v_0^2 \sim 10^{-7}$, since the characteristic velocity in the halos is about 100–150 km s^{-1}, i.e., $v_0 \sim (1/3\text{–}1/2) \times 10^{-3}$.

Remember that we consider the following limit for the external vector field in a spiral galaxy:

$$\phi_0 = \phi_*, \qquad \phi_r = \phi_\star, \qquad \kappa = \frac{\phi_\star}{\phi_*} \ll 1, \tag{26}$$

so that

$$\phi_0' = 0, \qquad \phi_r' = 0, \tag{27}$$

and we refer the system of (24)–(27) as a C-surface condition. Then,

$$(\mathcal{D}\phi)^2|_{\mathrm{C}} = -\frac{1}{2}\frac{\mathfrak{h}}{\mathfrak{f}}(\mathfrak{f}')^2\phi_*^2 + \frac{1}{4}\left(\frac{\mathfrak{f}'}{\mathfrak{f}}\right)^2\phi_\star^2 + \frac{2}{r^2}\phi_\star^2, \tag{28}$$

so that we get terms quadratic in the field that implies the appearance of ‘induced’ mass. This fact is characteristic for the vector field dynamics in curved spacetime. Combining the C-surface with (23) gives a $\bar{\mathrm{C}}$-surface.

[11] More specified consideration can be found in [25], for instance.

It is a straightforward task to calculate the energy–momentum tensor for the vector field

$$T^{\nu}_{\mu} = 2\frac{g^{\nu\alpha}}{\sqrt{-g}}\frac{\delta S_{\mathrm{V}}}{\delta g^{\mu\alpha}},$$

so that

$$T^{t}_{t} = -2\mathfrak{f}\frac{\sqrt{\mathfrak{h}/\mathfrak{f}}}{r^2\sin\theta}\frac{\delta S_{\mathrm{V}}}{\delta\mathfrak{f}}, \qquad T^{r}_{r} = 2\frac{\mathfrak{h}}{\mathfrak{f}}\frac{\sqrt{\mathfrak{h}\mathfrak{f}}}{r^2\sin\theta}\frac{\delta S_{\mathrm{V}}}{\delta\mathfrak{h}},$$

while the angle components are given by

$$T^{\theta}_{\theta} = T^{\varphi}_{\varphi} = -\frac{\sqrt{\mathfrak{h}/\mathfrak{f}}}{r^2\sin\theta}\frac{\delta S_{\mathrm{V}}}{\delta\lambda},$$

where $\delta\lambda$ is the dilatation of φ:

$$\delta_{\lambda}\varphi = \delta\lambda\cdot\varphi.$$

Direct calculations result in

$$T^{t}_{t}\big|_{\mathrm{C}} = -\frac{1}{2}\phi_*^2\left[-2\mathfrak{h}\mathfrak{f}'' + \frac{1}{2}(\mathfrak{f}')^2\frac{\mathfrak{h}}{\mathfrak{f}} - \frac{4}{r}\mathfrak{h}\mathfrak{f}'\right] - \frac{1}{8}\phi_\star^2\left[-\frac{8}{r^2} - 3\left(\frac{\mathfrak{f}'}{\mathfrak{f}}\right)^2 + \frac{8}{r}\frac{\mathfrak{f}'}{\mathfrak{f}} + 4\frac{\mathfrak{f}''}{\mathfrak{f}}\right]. \tag{29}$$

Applying a $\bar{\mathrm{C}}$-surface in the leading order over $v_0 \ll 1$ and $\kappa \ll 1$ we get

$$T^{t}_{t}\big|_{\bar{\mathrm{C}}} \approx \frac{\phi_*^2}{r^2}\left(2v_0^2 + \kappa^2\right), \tag{30}$$

and

$$G^{t}_{t}\big|_{\bar{\mathrm{C}}} \approx \frac{q}{r^2}. \tag{31}$$

The Einstein equation for the temporal components reads

$$G^{t}_{t}\big|_{\bar{\mathrm{C}}} = 8\pi G T^{t}_{t}\big|_{\bar{\mathrm{C}}} \quad\Rightarrow\quad q = 8\pi G\phi_*^2\left(2v_0^2 + \kappa^2\right). \tag{32}$$

The radial component of the energy–momentum tensor equals

$$T^{r}_{r}\big|_{\mathrm{C}} = \frac{1}{4}\phi_*^2\frac{\mathfrak{h}}{\mathfrak{f}}(\mathfrak{f}')^2 + \frac{1}{r^2}\phi_\star^2,$$

transformed to

$$T^{r}_{r}\big|_{\bar{\mathrm{C}}} \approx \frac{\phi_*^2}{r^2}\left(v_0^4 + \kappa^2\right), \tag{33}$$

while

$$G^{r}_{r}\big|_{\bar{\mathrm{C}}} = \frac{q - 2v_0^2}{r^2}, \tag{34}$$

and

$$G^{r}_{r}\big|_{\bar{\mathrm{C}}} = 8\pi G T^{r}_{r}\big|_{\bar{\mathrm{C}}} \quad\Rightarrow\quad q - 2v_0^2 = 8\pi G\phi_*^2\left(v_0^4 + \kappa^2\right). \tag{35}$$

Finally, the angle component is equal to

$$T^{\varphi}_{\varphi}\big|_{\mathrm{C}} = T^{\theta}_{\theta}\big|_{\mathrm{C}} = -\frac{1}{4}\frac{\mathfrak{h}}{\mathfrak{f}}(\mathfrak{f}')^2\phi_*^2 + \frac{1}{r^2}\phi_\star^2,$$

yielding

$$T^{\varphi}_{\varphi}\big|_{\bar{\mathrm{C}}} \approx \frac{\phi_*^2}{r^2}\left(\kappa^2 - v_0^4\right), \tag{36}$$

so that

$$G^{\varphi}_{\varphi}\big|_{\bar{\mathrm{C}}} = \frac{v_0^4}{r^2}, \tag{37}$$

and

$$G^{\varphi}_{\varphi}\big|_{\bar{\mathrm{C}}} = 8\pi G T^{\varphi}_{\varphi}\big|_{\bar{\mathrm{C}}} \quad \Rightarrow \quad v_0^4 = 8\pi G\phi_*^2\big(\kappa^2 - v_0^4\big). \tag{38}$$

Equations (32), (35) and (38) are satisfied at

$$\kappa = \sqrt{2}v_0^2 + \mathcal{O}\big(v_0^4\big), \tag{39}$$

$$q = 2v_0^2 + \mathcal{O}\big(v_0^4\big), \tag{40}$$

$$8\pi G\phi_*^2 = 1 + \mathcal{O}\big(v_0^2\big). \tag{41}$$

Therefore, this solution results in the fact that the temporal component of the energy–momentum tensor dominates and has the required profile with the distance:

$$T^r_r \sim T^{\varphi}_{\varphi} \sim T^{\theta}_{\theta} \sim \mathcal{O}\big(v_0^2\big)\cdot T^t_t \sim \mathcal{O}\left(\frac{1}{r^2}\right).$$

Numerically, we get

$$\kappa \sim 10^{-7} \quad \Rightarrow \quad M \sim 10^{12}\ \mathrm{GeV},$$

hence, the characteristic scale of matter influence on the vector field is in the range of the GUT and effective scale responsible for the small neutrino masses.

Thus, the small ratio of two natural energetic scales determines the rotation velocity in dark galactic halos. In addition, we have found condition (41), which is analogous to a definition. Let us test this problem in the next section.

3. Factorization

The previous section treats the vector field as an external source. That would be valid if the vector-field equations are satisfied. In that case we should take into account the field potential as we suggest that the characteristic scales in this potential are much greater than the scales induced by the size of the galactic halos as well as the Hubble rate or large-scale inhomogeneous structures.

So, first, suppose that we deal with a constant isotropic homogeneous vector field. This means that the radial component is equal to zero, while the temporal component is posed at the extremal point of its potential:

$$\frac{\partial V}{\partial\phi_0}\bigg|_{\mathrm{C}} = 0, \qquad \frac{\partial^2 V}{\partial\phi_0^2}\bigg|_{\mathrm{C}} = m_0^2, \qquad |m_0| \sim \phi_0 = \phi_* \sim m_{\mathrm{Pl}}.$$

If we consider an expanding isotropic homogeneous universe, then the potential gets an additional term determined by the Hubble rate [16],

$$\delta V = \tfrac{3}{2}H^2\phi_0^2,$$

so that the temporal component acquires a slow variation with time due to the displacement of the stable point, since the field equation takes the form

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 - 3H^2\phi_0 - \frac{\partial V}{\partial\phi_0} = 0,$$

and neglecting the time derivative, we get

$$3H^2\phi_0 + \frac{\partial V}{\partial \phi_0} = 0.$$

Expanding in ϕ_0 at $\phi_0 = \phi_* + \delta\phi_0$ gives

$$3H^2\phi_* + \left(3H^2 + m_0^2\right)\delta\phi_0 = 0,$$

so that at $H \ll m_{\mathrm{Pl}}$

$$\frac{\delta\phi_0}{\phi_*} \approx -\frac{3H^2}{m_0^2}, \tag{42}$$

which is a really small correction we justify. The induced time dependence is due to the Hubble rate

$$\delta\dot{\phi}_0 \approx -H\phi_* 6\frac{H^2}{m_0^2}\left(\frac{1}{H^2}\frac{\ddot{a}}{a} - 1\right) \quad \Rightarrow \quad |\delta\dot{\phi}_0/\phi_* H| \ll 1.$$

Analogous arguments are valid for the spatial component of the vector field: the corresponding effective mass of the radial component is $m_r \sim M$, introduced above, while the correction due to the covariant derivative

$$\delta_r V = \frac{2}{r^2}\phi_r^2$$

generates a small correction at macroscopic scales $1/r \ll M$, so, an induced dependence of ϕ_r on the distance is actually suppressed, and $\phi_r \approx \phi_\star$.

If both the expansion and the radial components are nonzero, then the covariant derivatives

$$\phi^r_{\ ;r} = H\phi_0, \qquad \phi^\theta_{\ ;\theta} = \phi^\varphi_{\ ;\varphi} = \frac{1}{r}\phi_r + H\phi_0, \tag{43}$$

induce the correction to the potential

$$\delta_H V = \frac{1}{2}H^2\phi_0^2 + \left(\frac{1}{r}\phi_r + H\phi_0\right)^2, \tag{44}$$

which gives similar restrictions on the suppressed variations of the vector field of course.

Therefore, both the evolution equations and Einstein equations at galactic scales can be considered with the constant vector field serving as the external source of gravity.

Note that as was shown in [16], the dependence of ϕ_0 on time generates a variation of effective gravitational constant with time. However, such dependence should be small as is forced by experimental observation. We have seen that in the scheme described above the time dependence of the vector field is negligible.

Finally, let us discuss the relation

$$8\pi G\phi_*^2 = 1, \tag{45}$$

which looks like a definition, but is the constraint. Introduce a bare gravitational constant G_0 and an extra interaction of vector field with the gravity, so that[12]

$$\frac{1}{16\pi G}R \quad \to \quad \frac{1}{16\pi G_0}R + \frac{c}{4}\phi_0^2 R.$$

Then, according to [16], the evolution equation with a constant vector field $\phi_0 = \phi_*$ looks like

$$3\left(\frac{1}{8\pi G_0} + \frac{c}{2}\phi_*^2\right)H^2 = \rho - \frac{3}{2}H^2\phi_*^2,$$

[12] We consider the FRW metric, while generically, instead of ϕ_0^2, one should introduce the invariant square of the vector field in the interaction of course.

which reproduces the ordinary form, if we make

$$\frac{1}{G} = \frac{1}{G_0} + (1+c)4\pi\phi_*^2.$$

Taking into account (45), we get

$$G_0 = \frac{2}{1-c}G.$$

For instance, at $c = 1$ we have $1/G_0 = 0$, and we find that ϕ_*^2 defines the gravitational constant, or the Planck mass.

Thus, we have shown that (45) preferably gives a defining relation for the gravitational constant, which has been used as the starting point for the scale arrangement. This statement seems to be justified. On the other hand, it points to a deep connection between the vector field dynamics and the gravity.

4. Indicating Milgrom's acceleration

As we have just shown in the previous section, the expansion of universe causes the time dependence of metric, which produces a negligible time dependence of the temporal component of the vector field. Therefore, the 'cosmological limit' of the vector field is consistently reached. However, the radial component of the vector field can induce the angle components of covariant derivatives (43) resulting in the anisotropic potential of (44). The anisotropy can be neglected at large distances when

$$\left|\frac{1}{r}\phi_r\right| \ll H\phi_0.$$

Then, the 'cosmological limit' of the constant vector field can be disturbed by the potential of (44), at distances less than r_0, defined by

$$\frac{1}{r_0}\phi_r = \varepsilon H\phi_0 \quad \Rightarrow \quad \frac{1}{r_0}\frac{\phi_\star}{\phi_*} = \varepsilon H_0, \tag{46}$$

where $H_0 = H(t_0)$ is the value of Hubble constant at the current moment of time t_0 and ε is a parameter of the order of 1–0.1. Thus, at distances[13] less than r_0 the flatness may be disturbed, since the vector field can acquire a distance dependence[14]. Substituting the ratio $\phi_\star/\phi_* = \sqrt{2}v_0^2$ into (46), we get

$$\frac{v_0^2}{r_0} = \frac{\varepsilon}{\sqrt{2}}H_0, \tag{47}$$

while the quantity

$$a_0 = \frac{v_0^2}{r_0} \tag{48}$$

is the centripetal acceleration at a 'critical radius' of r_0. Then, the critical acceleration is determined by the Hubble rate[15],

$$a_0 = \frac{\varepsilon}{\sqrt{2}}H_0, \tag{49}$$

[13] The actual parameter is the ratio of the radial component to the distance of course.
[14] The dependence of the radial component should dominate.
[15] In units with the speed of light equal to 1.

and it determines the acceleration below which the limit of the flat rotational curves becomes justified[16]. That is exactly a direct analogue of the critical acceleration introduced by Milgrom in the framework of the modified Newtonian dynamics (MOND) [26]. In MOND, Milgrom's acceleration, a_0, separates two regimes, the Newtonian and modified ones, so that at gravitational accelerations $a < a_0$, the dynamics reaches the limit reproducing the non-Newtonian flat rotational curves. With empirical data, Milgrom, surprisingly, found that $a_0 \sim H_0$, which is quite an amazing relation. Then, equations (46)–(49) show that the scale of Milgrom's acceleration naturally appears in the framework of the vector field embedded into the theory of gravitation.

Further, we could suppose that in the case where the gravitational acceleration produced by the visible matter in the galactic centres exceeds the critical value, we cannot reach the limit of the flat rotational curves. Indeed, in that case, the distance dependence cannot be excluded for the vector field. The Newtonian acceleration at the 'critical scale', r_0, is equal to

$$a_0^* = \frac{G\mathcal{M}}{r_0^2},$$

where $\mathcal{M}$ is a visible galactic mass. According to (49), the critical acceleration is a universal quantity slowly depending on the time, while (48) implies that the 'critical distance' can be adjusted by the variation of parameter v_0. Therefore, we should make

$$a_0^* = a_0, \tag{50}$$

which yields

$$v_0^4 = G\mathcal{M}a_0. \tag{51}$$

The galaxy mass is proportional to the H-band luminosity of the galaxy L_H, so that (51) reproduces the Tully–Fisher law

$$L_H \propto v_0^4.$$

Then, other successes of MOND can easily be incorporated in the framework under consideration too.

Nevertheless, one could look at (50) as a coincidence, which can be treated twofold. Indeed, it implies that

$$\frac{\phi_\star^2}{\phi_*^2} \sim G\mathcal{M}H \quad \Rightarrow \quad \phi_\star^2 \sim \mathcal{M}H.$$

So, first, the extremal point of potential for the radial component is fixed by the galaxy mass and the Hubble rate, or, second, if we arrange the scale of $\phi_\star \sim M \sim 10^{12}$ GeV, then the characteristic masses of galaxies should be given by $\mathcal{M} \sim M^2/H$. The preference for one of two viewpoints is subjective. Despite that, a possible challenge is the coincidence problem for the radial field adjusted to get a critical acceleration.

For completeness, let us discuss two modern approaches also incorporating Milgrom's acceleration. First, the empirical suggestion of MOND was reformulated as a nonrelativistic mechanics in the Lagrange form [27], where the relativistic theory as a scalar–tensor gravity was also given, though superluminal velocities of the scalar field were found as well as extragalactic gravitational lensing being too weak. A 'stratified' theory by Sanders [28] involves *a priori* constant timelike vector field as a disformal extension of a curved metric to a physical one. Recently, a tensor–vector–scalar theory for the MOND paradigm succeeded by Bekenstein [29] was presented as a consistent theory with dynamical fields in agreement with all observational data. It involves an unknown function, which guarantees the MOND effect

[16] In practice, $\varepsilon \approx 1/4$.

of critical acceleration in a relativistic theory. In this theory, Milgrom's acceleration is an ad hoc quantity, while in the framework we have presented, its scale is naturally obtained. Note that our consideration of Tully–Fisher law has repeated the arguments of Bekenstein taken from [29].

The same note concerns the second approach: a nonsymmetric gravitation theory (NGT) by Moffat [30], where the scale of Milgrom's acceleration has been fixed empirically in order to extract the model parameters, say, a characteristic scale. The advantage of NGT is the direct possibility of explicitly fitting the astronomical data on the rotation curves by theoretical formulae, though the nonsymmetric rank-2 tensor itself (instead of metric) is rather an exotic object. In addition, the NGT points to an exponential decrease of rotation velocity at infinitely large distances.

Thus, in this section we have obtained a natural estimate of critical acceleration, Milgrom's acceleration, determining the region of consistency for the flat rotational curves.

5. Conclusion

In this paper, we have found that the spherically symmetric static Einstein equations with the source given by the energy–momentum tensor of a constant vector field have a solution characterized by the metric corresponding to flat rotation curves in spiral galaxies at large distances. A feature of such a vector field is the scale hierarchy: the temporal component is of the order of the Planck mass $m_{\rm Pl}$, while the radial component about M should be suppressed so that the small parameter $\kappa = M/m_{\rm Pl} \ll 1$ determines the square of small rotation velocity $v_0^2 = \kappa/\sqrt{2}$, implying $M \sim 10^{12}$ GeV. The arrangement of scales should be caused by a specific potential of the vector field, so that its dynamics at cosmological scales is factorized from the dynamics at galactic scales. Thus, the vector field can provide an explanation of the dark matter in galactic halos. This statement is enforced by the natural estimate of Milgrom's acceleration, below which the flatness can be consistently justified.

We also have to point out the possibility that the vector field, whose spatial components in the asymptotic region are directed along the radius vector (in agreement with the symmetry of the problem), can be a manifestation of a monopole (see papers by Nucamendi, Salgado and Sudarsky in [10]). In that case, the magnitude of the radial component can fall to zero in the vicinity of the galactic centres, which can be exploited for an explanation of the decrease in the dark matter contribution to the rotation velocity near the galactic centres.

In the model offered, an analysis of cosmological evidence for the dark matter requires additional investigations, which deserve separate publications. Nevertheless, we can point out that our preliminary estimates of gravitational lensing by the vector field dark matter in a leading approximation (a paper in preparation for publication) are in a good agreement with observations (a ratio of dark matter mass to visible baryonic mass ~10–20 or greater[17]), which favour recent measurements of the homogeneous dark matter energy fraction in the universe budget ($\Omega_M \approx 0.27$). Moreover, we have seen that in the present model, the pressure is suppressed by a factor of $v^2 \sim 10^{-6}$ in comparison to the energy density, which should be for the dark matter, so we can reasonably hope that the model gives a meaningful explanation for the observed fraction of dark matter energy with no involvement of extra dark fluid.

The next question is the role of long-scale perturbations, which can be observed in angular anisotropy of the cosmic microwave background and in large-scale structure surveys

[17] The accuracy of the method is rather low.

of universe. It requires more accurate studies, the results of which are not yet clear to us. This is a challenge for further investigations.

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Appendix. Metric values

The spherically symmetric metric of (5) produces the following Christoffel symbols:

$$\begin{aligned}
&\Gamma^t_{tr} = \frac{1}{2}\frac{\mathfrak{f}'}{\mathfrak{f}}, &&\Gamma^r_{rr} = -\frac{1}{2}\frac{\mathfrak{h}'}{\mathfrak{h}}, &&\Gamma^r_{tt} = \frac{1}{2}\mathfrak{f}'\mathfrak{h},\\
&\Gamma^\theta_{\theta r} = \Gamma^\varphi_{\varphi r} = \frac{1}{r}, &&\Gamma^r_{\theta\theta} = -r\mathfrak{h}, &&\Gamma^r_{\varphi\varphi} = -r\sin^2\theta\,\mathfrak{h},\\
&\Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta, &&\Gamma^\varphi_{\theta\varphi} = \frac{\cos\theta}{\sin\theta},
\end{aligned} \tag{A.1}$$

where the symbols are symmetric over the contra-variant indices, and other symbols not listed above are equal to zero. In equations (A.1) we do not explicitly show the dependence of metric components on distance. Then, the nonzero elements of the Ricci tensor are given by

$$\begin{aligned}
R_{tt} &= \frac{1}{2}\mathfrak{h}\mathfrak{f}'' + \frac{1}{r}\mathfrak{h}\mathfrak{f}' - \frac{1}{4}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} - \frac{\mathfrak{h}'}{\mathfrak{h}}\right)\mathfrak{h}\mathfrak{f}',\\
R_{rr} &= -\frac{1}{2}\frac{\mathfrak{f}''}{\mathfrak{f}} - \frac{1}{r}\frac{\mathfrak{h}'}{\mathfrak{h}} + \frac{1}{4}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} - \frac{\mathfrak{h}'}{\mathfrak{h}}\right)\frac{\mathfrak{f}'}{\mathfrak{f}},\\
R_{\theta\theta} &= 1 - \mathfrak{h} - r\mathfrak{h}' - \frac{1}{2}r\mathfrak{h}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} - \frac{\mathfrak{h}'}{\mathfrak{h}}\right), \qquad R_{\varphi\varphi} = R_{\theta\theta}\sin^2\theta,
\end{aligned} \tag{A.2}$$

while the scalar curvature is equal to

$$R = \frac{\mathfrak{h}}{\mathfrak{f}}\mathfrak{f}'' - \frac{1}{2}\frac{\mathfrak{h}}{\mathfrak{f}}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} - \frac{\mathfrak{h}'}{\mathfrak{h}}\right)\frac{\mathfrak{f}'}{\mathfrak{f}} + 2\frac{\mathfrak{h}}{r}\left(\frac{\mathfrak{f}'}{\mathfrak{f}} + \frac{\mathfrak{h}'}{\mathfrak{h}}\right) - \frac{2}{r^2}(1 - \mathfrak{h}). \tag{A.3}$$

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