Remarks on the spherical scalar field halo in galaxies

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Matos, Guzmán, and Nuñez proposed a model for the galactic halo within the framework of scalar field theory. We argue that an analysis involving the full metric can reveal the true physical nature of the halo only when a certain condition is maintained. We fix that condition and also calculate its impact on observable parameters of the model.

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One of the outstanding problems in modern astrophysics is the problem of dark matter which is invoked as an explanation for the observed flat rotation curves in the galactic halo. Doppler emissions from stable circular orbits of neutral hydrogen clouds in the halo allow the measurement of tangential velocity $v_{tg}(r)$ of the clouds treated as probe particles. According to Newton's laws, centrifugal acceleration v_{tg}^2/r should balance the gravitational attraction $GM(r)/r^2$, which immediately gives $v_{tg}^2 = GM(r)/r$. That is, one would expect a falloff of $v_{tg}^2(r)$ with r. However, observations indicate that this is not the case: v_{tg} approximately levels off with r in the halo region. The only way to interpret this result of observation is to accept that the mass $M(r)$ increases linearly with distance r. Luminous mass distribution in the galaxy does not follow this behavior; hence, the hypothesis that there must be huge amounts of nonluminous matter hidden in the halo. This unseen matter is given a technical name dark matter.

Despite the fact that the exact nature of dark matter is as yet unknown, several analytic halo models exist in the literature including those provided by scalar-tensor theories (see for instance [1]). In particular, the scalar field model first proposed by Matos, Guzmán, and Nuñez [2] has received considerable attention. It is important to note that the authors primarily constructed an exact solution of Einstein's field equations sourced by a scalar field that provides a density profile of $1/r^2$ together with other appealing features of the metric functions. As a particular application, they sketched a plausible interpretation of the halo dark matter problem. The problem being important in itself, we think that the interesting relativistic central feature of the solution, namely, a non-Newtonian halo, must be well grounded. The purpose of the present Brief Report is to fix the condition under which it is possible. In addition, we work out its impact on observable parameters.

It is to be mentioned that the solution in [2] has been criticized because of its singular behavior at the origin [3], but this singularity is not peculiar to that solution alone; there are other viable halo models in the literature that also possess such singularity (see for instance [4]). Subsequent to the work in Ref. [2], the authors and the associated research group have obtained several new results under the scalar field dark matter model in galaxies: solution with axial symmetry including the inner region [5], time-dependent spacetimes [6], the full nonlinear Newtonian evolution after the turn-around point [7], time evolution of density fluctuation [8], collision properties of two structures [9], and so on. While they obtained constraints coming from cosmological considerations, we believe that it is also useful to ascertain the constraint appearing from the local (halo) scale, which would clarify the relativistic nature of the spherically symmetric model under consideration.

Using the flat rotation curve condition [10], Matos, Guzmán, and Nuñez obtain the spherically symmetric static solution for the galactic halo as follows ($G = c = 1$, unless specifically restored):

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

$$B(r) = B_0 r^l \qquad A(r) = \frac{4 - l^2}{4 + D(4 - l^2)r^{-(l+2)}} \quad (2)$$

$$\phi(r) = \sqrt{\frac{l}{8\pi}}\ln r + \phi_0 \quad (3)$$

$$V(r) = -\frac{1}{8\pi(2 - l)r^2}, \quad (4)$$

where D is an arbitrary constant of integration, ϕ and V are the scalar field and potential, respectively. The parameter $l = 2(v_{tg}/c)^2$, $B_0 > 0$ is another constant. Observations of the frequency shifts in the HI radiation show that, in the halo region, v_{tg}/c is nearly constant at a value 7×10^{-4} [11]. Thus, in what follows, we take $l \sim 10^{-6}$.

Note that we can rewrite $A(r)$ in the standard Schwarzschild form,

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$$A(r) = \left[1 - \frac{2m(r)}{r}\right]^{-1}, \quad (5)$$

which is often convenient and will be useful later while discussing the observational parameters. Such a form has the advantage that it immediately reveals not only the mass parameter $m(r)$ but also shows that the proper radial length is larger than the Euclidean length because $r > 2m(r)$. This inequality, which is essential for signature protection, dictates that $A(r) > 1$. This is a crucial condition to be satisfied by any valid metric.

Now, for the sake of simplicity, Matos, Guzmán, and Nuñez choose $D = 0$, but this is not the best choice because it makes the metric component $A < 1$. As a consequence, whatever results follow from the reduced metric should be taken with caution. For instance, the stresses exhibit a density profile $\rho < 0$, meaning violation of weak energy condition and furthermore lead to $\omega < -1$ (see below), meaning repulsive gravity in the halo, contradicting observational facts. But these are actually not the true features of their model. To see the true picture, it is necessary to calculate the relevant quantities with $D \neq 0$.

We find the density and pressure profiles in the rest frame of the fluid as

$$\rho = \frac{1}{8\pi} \frac{r^{-(4+l)}[D(l^3 + l^2 - 4l - 4) + l^2 r^{2+l}]}{l^2 - 4} \quad (6)$$

$$p_r = \frac{1}{8\pi} \frac{r^{-(4+l)}[D(l^3 + l^2 - 4l - 4) - l(4 + l) r^{2+l}]}{l^2 - 4} \quad (7)$$

$$p_t = \frac{1}{8\pi} \frac{r^{-(6+l)}[D(l^3 + l^2 - 4l - 4) + l^2 r^{2+l}][(r^2 - 1)l - 2(r^2 + 1)]}{4(l^2 - 4)}, \quad (8)$$

where ρ is the energy density, p_r is the radial pressure, and p_t are the transverse pressures.

Matos, Guzmán, and Nuñez conclude that their model has huge pressure over density and thus it is non-Newtonian. We wish to emphasize that the role of the nonzero value of D is crucial not only for avoiding repulsive gravity (as alluded to above) but also for arriving at a correct conclusion about the relative strengths between pressure and density. For instance, let us take $D = 1$. In the distant halo region, we can take, typically, $r \sim$ 100–300 Kpc and with $l \sim 10^{-6}$, we find the numerical values to be $\rho \sim 10^{-9}$ and $p_r \sim 10^{-9}$, which means that they are of the same order. But on the other hand, $p_r + 2p_t \sim 10^{-11} \Rightarrow p_r + 2p_t \sim 10^{-2}\rho$, which indicates that total pressure is roughly 100 times *less* than the density. However, if we take $D = 10^{-5}$, we find that $p_r + 2p_t \sim 10^3\rho$. If we keep on decreasing the value of D further (but never exactly to zero for reasons stated above), we see that the total pressure dominates more and more over density reinforcing the non-Newtonian nature.

The next question is: How far can we go on decreasing D? We notice the following interesting scenario. When $D = 10^{-7}$, we find $p_r + 2p_t = 9 \times 10^5\rho$, which leads to $\omega = \frac{p_r + 2p_t}{3\rho} = 3 \times 10^5$ (attractive gravity) as shown in Fig. 1. This is the extreme possible non-Newtonian halo in the scalar field model under consideration. The reason is this. If $D = 10^{-8}$, we find that $\omega > 0$ up to $r = r_0 =$ 200 Kpc (attractive gravity) and becomes $\omega < -1$ after $r = r_0$ (repulsive gravity). At $r = r_0$, there is a singularity in ω. This value of D represents a transition from attraction to repulsion as shown in Fig. 2. When $D \leq 10^{-9}$, we find

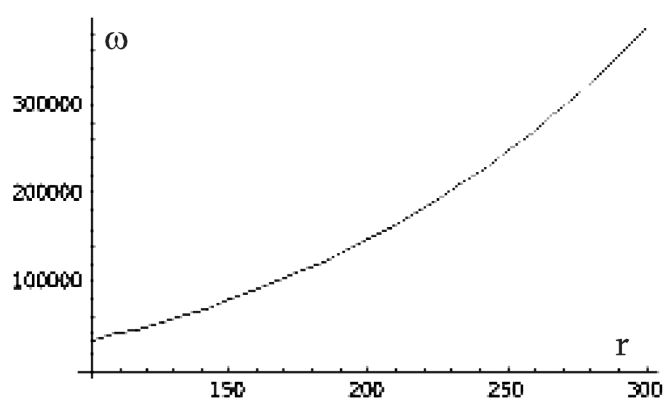

FIG. 1. Plot of $\omega(r)$ vs r in which ω is computed from either Eqs. (6)–(8) or (21) with $l = 10^{-6}$ and $D = 10^{-7}$. The distance r in the galactic halo region is taken in the range 100–300 Kpcs. The non-Newtonian values of ω are evident.

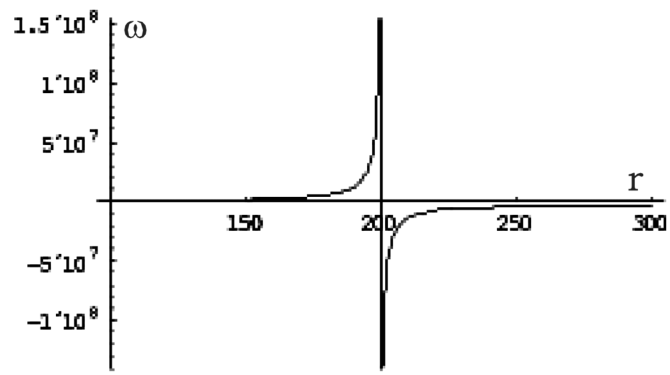

FIG. 2. Plot of $\omega(r)$ vs r in which ω is computed from either Eqs. (6)–(8) or (21) with $l = 10^{-6}$ and $D = 10^{-8}$. The distance r in the galactic halo region is taken in the range 100–300 Kpcs. The figure displays the transition behavior of ω as discussed in the text.

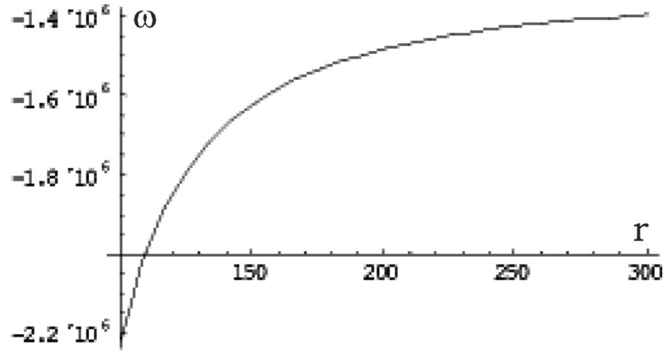

FIG. 3. Plot of $\omega(r)$ vs r in which ω is computed from either Eqs. (6)–(8) or (21) with $l = 10^{-6}$ and $D = 10^{-9}$. The distance r in the galactic halo region is taken in the range 100–300 Kpcs. The values of ω are negative indicating repulsion.

that $\rho < 0$, $\omega < -1$ (repulsive gravity), which share the woes that follow also from the choice $D = 0$ (Figs. 3 and 4). These show that we cannot decrease D below 10^{-7}, that is, we must have $D \geq 10^{-7}$. This is the condition that must be maintained in order to have a non-Newtonian halo.

The pressures are anisotropic, as is evident from Eqs. (7) and (8), which is a good feature of the solution from the point of view of exterior matching. Note that the solution cannot be matched to the Schwarzschild exterior metric at the boundary of the halo if the pressures were isotropic [12]. It can be further verified that $\rho > 0$, $\rho + p_r > 0$, $\rho + p_r + 2p_t > 0$ for $D \geq 10^{-7}$; so we can say that the halo matter is not exotic because the standard energy conditions are satisfied everywhere. Therefore, we expect attractive gravity in the halo. To confirm it, we follow the prescription by Lynden-Bell, Katz, and Bičák [13], and find that the total gravitational energy is indeed negative:

$$E_G = 4\pi \int_{r_1}^{r_2} [1 - A^{1/2}] \rho r^2 dr < 0, \tag{9}$$

due to the fact that $\rho > 0$, $1 - A^{1/2} < 0$, and $r_2 > r_1$. This

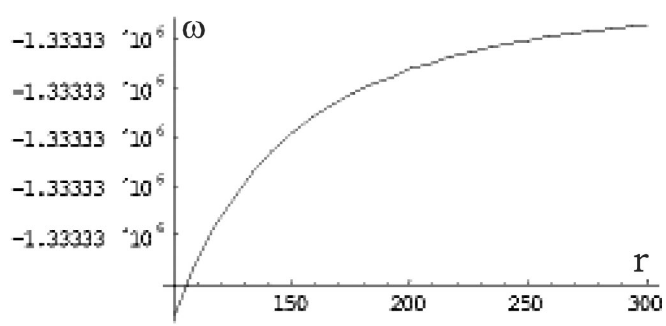

FIG. 4. Plot of $\omega(r)$ vs r in which ω is computed from either Eqs. (6)–(8) or (21) with $l = 10^{-6}$ and $D = 0$. The distance r in the galactic halo region is taken in the range 100–300 Kpcs. The values of ω are negative indicating repulsion, similar to that in Fig. 3.

prescription has been very useful in the case of scalar field wormholes too [14–16].

Certainly, the scalar field model corresponding to $D = 10^{-7}$ is highly non-Newtonian because $p_r + 2p_t \sim 10^6 \rho$. As a result, a purely Newtonian definition of mass, viz. $M(r) = 4\pi \int \rho r^2 dr$, does not apply. However, incorporating the pressure contribution, the dynamical mass in the first post-Newtonian order becomes

$$M_{pN}(r) = 4\pi \int (\rho + p_r + 2p_t) r^2 dr = 10^6 M(r), \tag{10}$$

which clearly reflects the non-Newtonian nature of the model in terms of masses.

We next focus on the observable parameters expected in this non-Newtonian halo. Whatever be the analytic model for it, there must be a way to contrast its predictions with actual measurements. The key point is that one does not directly measure the metric functions but indirectly measures gravitational potentials and masses from rotation curve and lensing observations. Faber and Visser [17] have shown how, in the first post-Newtonian approximation, the combined measurements of rotation curves and gravitational lensing allow inferences about the mass and pressure profile of the galactic halo as well as its equation of state.

The usual techniques for obtaining the potential for rotation curve (RC) measurements yield a pseudopotential (see Ref. [17] for details):

$$\Phi_{\mathrm{RC}} = \Phi \neq \Phi_N, \tag{11}$$

where Φ_N is the Newtonian potential, $\Phi = \frac{1}{2} \ln B$, and a pseudomass

$$m_{\mathrm{RC}} = r^2 \Phi'(r) \approx 4\pi \int (\rho + p_r + 2p_t) r^2 dr. \tag{12}$$

Faber and Visser also define the lensing pseudopotential as

$$\Phi_{\mathrm{lens}} = \frac{\Phi(r)}{2} + \frac{1}{2} \int \frac{m(r)}{r^2} dr, \tag{13}$$

and a pseudomass m_{lens} obtained from lensing measurements as

$$m_{\mathrm{lens}} = \tfrac{1}{2} r^2 \Phi'(r) + \tfrac{1}{2} m(r). \tag{14}$$

The first order approximations of Einstein's equations yield

$$\rho(r) \approx \frac{1}{4\pi r^2} [2m'_{\mathrm{lens}}(r) - m'_{\mathrm{RC}}(r)] \tag{15}$$

$$4\pi r^2 (p_r + 2p_t) \approx 2[m'_{\mathrm{RC}}(r) - m'_{\mathrm{lens}}(r)], \tag{16}$$

where the right-hand sides denote pseudodensity and pseudopressures. Furthermore, Faber and Visser define a dimensionless quantity,

$$\omega(r) = \frac{p_r + 2p_t}{3\rho} \approx \frac{2}{3}\frac{m'_{\rm RC} - m'_{\rm lens}}{2m'_{\rm lens} - m'_{\rm RC}}. \tag{17}$$

The pseudoquantities on the right-hand side of Eqs. (11)–(17) are actual observables from the combined measurement. If the observed pseudoprofiles reasonably match with the analytic pseudoprofiles coming from *a priori* given metric functions, one can say that the solution is physically substantiated. Otherwise, it has to be ruled out as nonviable. The impact of a small nonzero D on the analytic pseudoprofiles can now be computed. For the extreme ($D = 10^{-7}$) Matos, Guzmán, and Nuñez solution, these work out to leading order in r as

$$m_{\rm RC}(r) = \frac{lr}{2} \approx 10^{-6} r \tag{18}$$

$$m_{\rm lens}(r) \approx \frac{l(l^2 + l - 4)r}{4(l^2 - 4)} \approx 10^{-6} r \tag{19}$$

$$2(m'_{\rm RC} - m'_{\rm lens}) \approx \frac{l(l^2 - l - 4)}{2(l^2 - 4)} \approx 10^{-6}. \tag{20}$$

The dimensionless parameter ω to all orders in r with no restriction on D is

$$\omega(r) \approx \frac{2}{3}\frac{m'_{\rm RC} - m'_{\rm lens}}{2m'_{\rm lens} - m'_{\rm RC}} = \frac{l(l^2 - l - 4)r^{2+l} - D(l^3 + l^2 - 4l - 4)}{3[D(l^3 + l^2 - 4l - 4) + l^2 r^{2+l}]}, \tag{21}$$

which yields $\omega \approx 3 \times 10^5$ for $D = 10^{-7}$ within our chosen range, $r \sim 100$–300 Kpc. Note that if we straightaway put $D = 0$ in Eq. (21), we get $\omega(r) < -1$, conveying a completely wrong physical conclusion.

The pivotal result of the present article is that $D \geq 10^{-7}$ and not $D = 0$, as discussed above. Of course, the lowest limit on D is small and it is quite tempting to set it exactly to zero. But the price for it is that one gets a completely wrong picture of the halo. We have analyzed the model taking into account only the lowest value of D. Similar analysis can be carried out with other values of D as well respecting the suggested lower limit. We can say that, by and large, the conclusion of Matos, Guzmán, and Nuñez about the non-Newtonian nature of the halo is right provided the restriction on D is maintained. With this restriction in place, their model can indeed be a physically viable one. If combined measurements follow the pattern as indicated in Eqs. (18)–(21), we would say that the model is observationally supported. However, given the present uncertainties in observation, it is yet too premature to say so.

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Note added in proof.–A boson star formed by a self-interacting massive scalar field with quartic interaction potential was investigated in Ref. [18]. A similar boson star as a model of galactic halo was first investigated by Lee and Koh [19].

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