Modeling galactic halos with predominantly quintessential matter

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Abstract

This paper discusses a new model for galactic dark matter by combining an anisotropic pressure field corresponding to normal matter and a quintessence dark energy field having a characteristic parameter ω_q such that $-1 < \omega_q < -\frac{1}{3}$. Stable stellar orbits together with an attractive gravity exist only if ω_q is extremely close to $-\frac{1}{3}$, a result consistent with the special case studied by Guzman et al. (2003). Less exceptional forms of quintessence dark energy do not yield the desired stable orbits and are therefore unsuitable for modeling dark matter.

1 Intoduction

The existence of dark matter has been suspected since the 1930's due to stellar motions in the outer regions of galaxies [1, 2, 3]. Zwicky [4, 5] discovered the presence of dark matter on a much larger scale by studying galactic clusters. The presence of such nonluminous matter was subsequently confirmed by observing the flatness of galactic rotation curves [6, 7, 8, 9, 10, 11]. While the dark matter provides the necessary gravitational field to account for the observation, its composition remains unknown, probably consisting of particles not included in the standard model.

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The origin of the dark-matter problem is the measurement of the tangential velocity v^{ϕ} of stable circular orbits of hydrogen clouds in the outer regions of the halo. The observed constant velocity can only be explained by assuming that the energy density decreases with the distance as r^{-2} so that the increase in the mass of galaxies is proportional to the distance r from the center.

Of the many proposed candidates for dark matter, the most favored is the standard cold dark matter (SCDM) paradigm [12, 13]. Another favorite is the Λ CDM model that is related to the accelerated expansion of the Universe [14, 15]. For a summary of some alternative theories such as scalar-tensor and brane-world models, see Rahaman et al. [16, 17, 18].

It is now recognized that on a cosmological scale, our Universe is pervaded by a dynamic dark energy that causes the expansion of the Universe to accelerate: $\ddot{a}/a = -\frac{4\pi G}{3}(\rho + 3p)$. In the equation of state $p = w\rho$, the range of values $-1 < w < -1/3$, resulting in $\ddot{a} > 0$, is usually referred to as quintessence dark energy. Moving to the galactic level, however, there is no model consistent with the cosmological one to help explain the nature of dark matter.

In this paper we propose a new model based on a combination of a quintessence-like field and a field with (possibly) anisotropic pressure, representing normal matter. We study conditions leading to stable circular orbits and attractive gravity, as well as the observed equation of state of the galactic halo, based on a combination of rotation curves and lensing measurements [19]. The conditions required for stable orbits and attractive gravity are rather stringent, but by comparing global and local effects, they proved to be consistent with earlier results obtained by Guzman et al. [20].

2 The model

In this paper the metric for a static spherically symmetric spacetime is taken as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$

where the functions of the radial coordinate r, $\nu(r)$ and $\lambda(r)$, are the metric potentials.

Now we consider a model which contains a quintessence field and a second field with an anisotropic pressure representing normal matter. Here the Einstein equations can be written as

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + \tau_{\mu\nu}), \tag{2}$$

where $\tau_{\mu\nu}$ is the energy momentum tensor of the quintessence-like field, which is characterized by a free parameter ω_q with the restriction $-1 < \omega_q < -\frac{1}{3}$. According to Kiselev [21], the components of this tensor need to satisfy the conditions of additivity and linearity. Taking into account the different signatures used in the line elements, the components can be stated as follows:

$$\tau_t^t = \tau_r^r = -\rho_q, \tag{3}$$

$$\tau_\theta^\theta = \tau_\phi^\phi = \frac{1}{2}(3\omega_q + 1)\rho_q. \tag{4}$$

Moreover, the most general energy momentum tensor compatible with spherically symmetry is

$$T_\nu^\mu = (\rho + p_t)u^\mu u_\nu - p_t g_\nu^\mu + (p_r - p_t)\eta^\mu \eta_\nu \tag{5}$$

with $u^\mu u_\mu = -1$. For our metric the Einstein field equations are stated next.

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi G(\rho + \rho_q), \tag{6}$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi G(p_r - \rho_q), \tag{7}$$

$$\begin{aligned} &\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] \\ &= 8\pi G\left(p_t + \frac{(3\omega_q + 1)}{2}\rho_q\right). \end{aligned} \tag{8}$$

3 The solutions

It is well known [17, 22] that the flat rotation condition gives the solution

$$e^\nu = B_0 r^l, \tag{9}$$

where $l = 2v^{2\phi}$, v^ϕ is the rotational velocity, and B_0 is an integration constant. According to Ref. [23], the observed rotational curve profile in the dark matter dominated region is such that the rotational velocity v^ϕ becomes more or less constant with $v^\phi \sim 300$ km/s ($\sim 10^{-3}$) for a typical galaxy. So $l = 0.000001$, as in Nandi et al. [24]. We also assume fairly large distances in Kpc. (Of course, certain galaxies, particularly disk and dwarf galaxies, are not typical in the above sense. For further discussion, see Refs. [25, 26].)

To continue the analysis, we use the following equation of state (EoS):

$$p_r = m\rho, \quad m \geq 0, \tag{10}$$

where m is a parameter corresponding to normal matter. In the case of an anisotropic pressure, the radial and lateral pressures are no longer equal. So let us assume the form

$$p_t = \alpha\rho, \quad \alpha \geq 0, \tag{11}$$

for the lateral pressure. In this manner we obtain simple linear relationships between pressure and energy-density, but with p_r not equal to p_t, unless, of course, $m = \alpha$.

Now, from equations (6) to (8) and then using equations (9), (10), and (11), one obtains the following simplified form (after some manipulation):

$$(e^{-\lambda})' + \frac{ae^{-\lambda}}{r} = \frac{c}{r}, \tag{12}$$

ω_q	m	α	a	c
-0.33333378	0.1	0.1	$-1.287692503 \times 10^{-6}$	$-1.133845791 \times 10^{-6}$
	0.1	0.3	$-1.220000166 \times 10^{-6}$	$-8.670586114 \times 10^{-7}$
	0.1	0.5	$-1.178095381 \times 10^{-6}$	$-7.019046229 \times 10^{-7}$
	0.3	0.5	$-1.192174084 \times 10^{-6}$	$-7.573912227 \times 10^{-7}$
	0.5	0.5	$-1.204000197 \times 10^{-6}$	$-8.039999743 \times 10^{-7}$
	0.5	0.8	$-1.161613061 \times 10^{-6}$	$-6.454838537 \times 10^{-7}$
	0.7	0.7	$-1.183161478 \times 10^{-6}$	$-7.315484069 \times 10^{-7}$
	0.8	0.8	$-1.176823715 \times 10^{-6}$	$-7.062353288 \times 10^{-7}$
-0.4	0.1	0.1	-0.1718752386	-0.1718749261
	0.1	0.3	-0.1309528143	-0.1309523381
	0.5	0.5	-0.1250004609	-0.1249999609
-0.5	0.1	0.1	-0.4400003664	-0.4399998064
	0.1	0.3	-0.3333338889	-0.3333332222
	0.5	0.5	-0.3333338889	-0.3333332222
-0.8	0.1	0.1	-1.327586957	-1.327585577
	0.1	0.3	-0.9871804212	-0.9871791391
	0.5	0.5	-1.166667514	-1.166666181

Table 1: Table showing typical values of a and c corresponding to various values of ω_q, m, and α, with special emphasis on $\omega_q = -0.33333378$.

where

$$a = \frac{\frac{1}{2}(l+1)(3\omega_q+1) + \frac{l^2}{4} - l[(3\omega_q+1)m + 2\alpha]/[2(1+m)]}{\frac{l}{4} + \frac{1}{2} + [(3\omega_q+1)m + 2\alpha]/[2(1+m)]} \tag{13}$$

and

$$c = \frac{\frac{1}{2}(3\omega_q+1)}{\frac{l}{4} + \frac{1}{2} + [(3\omega_q+1)m + 2\alpha]/[2(1+m)]}. \tag{14}$$

Eq. (12) yields

$$e^{-\lambda} = \frac{c}{a} + \frac{D}{r^a}, \tag{15}$$

where D is an integration constant. This constant would normally be obtained from the junction conditions, but the exact boundary of the galactic halo is unknown. In some situations D may have to be restricted for physical reasons.

Both a and c depend on ω_q, as well as on m and α. To get an overview, some typical values are listed in Table 1, using $l = 0.000001$. (The special case $\omega = -0.33333378$ will be discussed in the next section.)

From Eqs. (7) and (8), one can readily obtain the following solutions for ρ, ρ_q:

$$\rho = \frac{1}{8\pi G(1+m)}\left[\frac{(D(l+a)}{r^{a+2}} + \frac{lc}{ar^2}\right] \tag{16}$$

and

$$\rho_q = \frac{1}{8\pi G}\left[\frac{[aD - D - (lD + Da)/(1+m)]}{r^{a+2}} - \frac{1}{r^2}\left(\frac{c}{a} - 1 + \frac{lc}{a(1+m)}\right)\right]$$
$$= \frac{1}{8\pi G}\left[\frac{D(a-1)}{r^{a+2}} + \frac{1-\frac{c}{a}}{r^2} - \frac{D(a+l)}{r^{a+2}(1+m)} - \frac{lc}{ar^2(1+m)}\right]. \quad (17)$$

To complete the analysis below, we also need the following additional results:

$$\rho(effective) = \rho + \rho_q = \frac{1}{8\pi G}[G_{tt}] = \frac{1}{8\pi G}\left[\frac{D(a-1)}{r^{a+2}} + \frac{1-\frac{c}{a}}{r^2}\right], \quad (18)$$

$$p_r(effective) = p_r - \rho_q = \frac{1}{8\pi G}[G_{rr}] = \frac{1}{8\pi G}\left[\frac{(l+1)}{r^2}\left(\frac{c}{a} + \frac{D}{r^a}\right) - \frac{1}{r^2}\right], \quad (19)$$

$$p_t(effective) = \left(p_t + \frac{(3\omega_q + 1)}{2}\rho_q\right) = \frac{1}{8\pi G}[G_{\theta\theta}]$$
$$= \frac{1}{8\pi G}\left[\frac{c}{2ar^2} + \frac{D}{2r^{a+2}}\right]\left[\frac{l^2}{2} - \frac{Da^2}{cr^a + Da}\left(1 + \frac{l}{2}\right)\right], \quad (20)$$

and

$$8\pi G\left[\rho(effective) + p_r(effective) + 2p_t(effective)\right]$$

$$= \left[\frac{D(a-1)}{r^{a+2}}\right] + \frac{1}{r^2}\left[\frac{D}{r^a} + \left(\frac{c}{a} + \frac{D}{r^a}\right)\left(l + \frac{l^2}{2} - (\frac{l}{2} + 1)\frac{Da^2}{cr^a + Da}\right)\right]. \quad (21)$$

4 Stability of circular orbits

Let $U^\alpha = \frac{dx^\sigma}{d\tau}$ be the four-velocity of a test particle moving solely in the space of the halo and suppose we restrict ourselves to $\theta = \pi/2$. Then following Nandi et al. [24], the equation $g_{\nu\sigma}U^\nu U^\sigma = -m_0^2$ can be cast in a Newtonian form

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 + V(r), \quad (22)$$

which gives

$$V(r) = -\left\{E^2\left[1 - \frac{r^{-l}\left(c/a + D/r^a\right)}{B_0}\right] + \left[\frac{c}{a} + \frac{D}{r^a}\right]\left(1 + \frac{L^2}{r^2}\right)\right\}. \quad (23)$$

Here the constants

$$E = \frac{U_0}{m_0} \quad \text{and} \quad L = \frac{U_3}{m_0} \quad (24)$$

are, respectively, the conserved relativistic energy and angular momentum per unit rest mass of the test particle. For circular orbits we have $r = R$, a constant, so that $\frac{dR}{d\tau} = 0$. Also, $\frac{dV}{dr} \mid_{r=R}= 0$. From these two conditions follow the conserved parameters L and E:

$$L = \pm\sqrt{\frac{l}{2-l}}R; \tag{25}$$

using L in $V(R) = -E^2$, we get

$$E = \pm\sqrt{\frac{2B_0}{2-l}}R^{\frac{l}{2}}. \tag{26}$$

The orbits will be stable if $\frac{d^2V}{dr^2} \mid_{r=R}< 0$ and unstable if $\frac{d^2V}{dr^2} \mid_{r=R}> 0$. Putting the expressions for L and E in $\frac{d^2V}{dr^2} \mid_{r=R}$, we obtain, after straightforward calculations, the final result:

$$\begin{aligned}\left.\frac{d^2V}{dr^2}\right|_{r=R} = \frac{c}{a}\frac{2l}{2-l}\left[(l+1)\frac{1}{R^{l+1}} - \frac{3}{R^2}\right] \\ + \frac{2}{2-l}a(a+1)D\left[\frac{1}{R^{a+l+1}} - \frac{1}{R^{a+2}}\right] \\ + \frac{2l}{2-l}D\left[(l+1)\frac{1}{R^{a+l+1}} - \frac{3}{R^{a+2}}\right] \\ + \frac{2l}{2-l}(2a)D\left[\frac{1}{R^{a+l+1}} - \frac{1}{R^{a+2}}\right].\end{aligned} \tag{27}$$

To allow a comparison to ρ [Eq. (16)], $V''(R)$ has to be rewritten. Before doing so, we recall that a and c are both negative, while c/a is less than 1. Furthermore, a and c depend on ω_q and can be made small by choosing ω_q close to $-1/3$. But $|a|$ must be greater than l; otherwise the negative second row in Eq. (27) cannot catch up with the positive first row. Of course, $|a|$ must be less than 1, so that $a(a+1)$ is negative.

$$\begin{aligned}V''(R) = \frac{1}{2-l}\frac{2}{R}\frac{c}{a}l\left[(l+1)\frac{1}{R^l} - \frac{3}{R}\right] \\ + \frac{1}{2-l}\frac{2}{R}Da(a+1)\left[\frac{1}{R^l} - \frac{1}{R}\right]\frac{1}{R^a} \\ + \frac{1}{2-l}\frac{2}{R}Dl\left[(l+1)\frac{1}{R^l} - \frac{3}{R}\right]\frac{1}{R^a} \\ + \frac{1}{2-l}\frac{2}{R}D(2a)l\left[\frac{1}{R^l} - \frac{1}{R}\right]\frac{1}{R^a}.\end{aligned} \tag{28}$$

For quintessence dark energy, $-1 < \omega_q < -\frac{1}{3}$. The case where ω_q is extremely close to $-\frac{1}{3}$ will be referred to as *marginal quintessence dark energy,* otherwise as *ordinary quintessence dark energy.*

4.1 Ordinary quintessence dark energy

Suppose ω is not real close to $-\frac{1}{3}$, so that $|a|$ is not real close to l and $a+l$ is not real close to zero (Table 1). Then in Eq. (28), the second term in each bracket is relatively small and we can concentrate on the first term in each case:

$$\begin{aligned}V''(R) &\approx \frac{1}{2-l}\frac{2}{R}\frac{1}{R^l}\left[\frac{c}{a}l(l+1)+[Da(a+1)+Dl(l+1)+2alD]\frac{1}{R^a}\right] \\ &= \frac{1}{2-l}\frac{2}{R}\frac{1}{R^l}\left[\frac{D}{R^a}(a^2+a+l^2+l+2al)+\frac{c}{a}l(l+1)\right] \\ &= \frac{1}{2-l}\frac{2}{R}\frac{1}{R^l}\left[\frac{D}{R^a}[(a+l)^2+(a+l)]+\frac{c}{a}l(l+1)\right]. \qquad (29)\end{aligned}$$

Now compare this equation to Eq. (16) for $r=R$:

$$\rho = \frac{1}{8\pi G(1+m)}\frac{1}{R^2}\left[\frac{D}{R^a}(a+l)+\frac{c}{a}l\right]. \qquad (30)$$

Since $l = 0.000001$, $l(l+1)\approx l$. To make physical sense, D has to be small enough to make ρ positive. But then $V''(R)$ is *a fortiori* positive because of the extra positive term $(a+l)^2$. In other words, we do not have a stable orbit. Conversely, if the orbit is stable, then ρ has to be negative.

4.2 Marginal quintessence dark energy - global and local effects

As already noted, referring to Table 1 and Eq. (13), as ω_q gets close to $-\frac{1}{3}$, $|a|$ gets close to l and $a+l$ close to zero. Then the second term inside each bracket in Eq. (28) is no longer negligible. Collecting these terms, we get

$$\frac{1}{2-l}\frac{2}{R}\left[\frac{c}{a}l\left(-\frac{3}{R}\right)+\frac{D}{R}(-a^2-a-3l)\frac{1}{R^a}+2Dal\left(-\frac{1}{R}\right)\frac{1}{R^a}\right]. \qquad (31)$$

Observe next that both a^2 and al are negligible. Supppose that in Eq. (29), $D|a+l| < (\frac{c}{a})l$; $(a+l)^2$ is negligible and R^{-a} is close to unity for R between 100 and 500 Kpc. So in Eq. (31), since $D(-a-l) < (\frac{c}{a})l$, the quantity

$$\frac{c}{a}l(-3)+D(-a-l-2l)$$

is less than $-2(\frac{c}{a})l - 2Dl$ and hence negative. In other words, we now have $\rho > 0$ and $V''(R) < 0$, finally yielding a stable orbit.

Remark: A simple alternative argument is the following: consider the limiting case $\omega_q \to -\frac{1}{3}$. In that case, $c \to 0$ and $|a| < l$ [from Eqs. (13) and (14)] and the conclusion follows from Eqs. (30) and (31).

An example of the marginal quintessence-energy case is discussed in Guzman et al. [20]. It is assumed that the luminous matter does not contribute significantly to the energy density of the halo and may be disregarded, leaving an isotropic perfect fluid. It is shown that the equation of state may be taken to be

$$p = -\frac{1+v^{2\phi}}{3+v^{2\phi}}\rho,$$

or, equivalently, $\omega_q \approx -0.33333378$, which is indeed close to $-\frac{1}{3}$. Table 1 shows that a is small enough to meet the requirements in Eqs. (30) and (31) for both the isotropic and anisotropic cases for R between 100 and 500 Kpc.

While one usually thinks of quintessence dark energy as a cosmological phenomenon, in particular, as the cause of the accelerated expansion, the above results, taken together, point to a rather different effect on the galactic level. The marginal quintessence case, being just over the line into quintessence territory, yields stable orbits in our combined model. Ordinarily quintessence dark energy is unable to do so: evidently, its repulsive action is too strong. The difference in behavior is reminiscent of the expansion itself: while the Universe expands on a cosmological scale, bound systems, such as galaxies, are completely unaffected. In other words, the same phenomenon has different effects on the cosmological and local levels.

The idea that dark matter and dark energy are connected is not new. In discussing Chaplygin gas, it is argued in Ref. [27] that dark matter and dark energy ought to be different manifestations of the same entity.

Returning to some of the other conditions in Sec. 3, in the marginal quintessence case, both ρ_q and $\rho(effective)$ are positive as long as D is small enough, say 0.1, while $\tau_r^r = -\rho_q < 0$, as required. For the same D, $p_r(effective)$ is likely to be negative (depending on the precise values of D, a, and c), while $p_t(effective)$ is positive for any D.

5 Attraction and total gravitational energy

Suppose we now consider the question of attractive gravity by examining the geodesic equation for a test particle that is moving along a circular path of radius $r = R$:

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\gamma}\frac{dx^\mu}{d\tau}\frac{dx^\gamma}{d\tau} = 0. \tag{32}$$

This equation implies that

$$\frac{d^2r}{d\tau^2} = -\frac{1}{2}\left[\frac{c}{a}+\frac{D}{r^a}\right]\left[\frac{Da}{r^{a+1}}\left(\frac{c}{a}+\frac{D}{r^a}\right)^{-2}\left(\frac{dr}{d\tau}\right)^2 + B_0lr^{l-1}\left(\frac{dt}{d\tau}\right)^2\right]. \tag{33}$$

As before, as long as $\frac{dR}{d\tau} = 0$, the expression is negative and we conclude that objects are attracted toward the center. But this would only be relevant in the marginal quintessence case. For this case we can also determine the total gravitational energy E_g between two fixed radii, r_1 and r_2 [28]:

$$E_g = M - E_M = 4\pi \int_{r_1}^{r_2} [1 - \sqrt{e^{\lambda(r)}}]\rho r^2 dr$$
$$= 4\pi \int_{r_1}^{r_2} \left[1 - \sqrt{\frac{1}{c/a + D/r^a}}\right] \left[\frac{1}{8\pi G} \left(\frac{D(a-1)}{r^{a+2}} + \frac{(1-c/a)}{r^2}\right)\right] r^2 dr. \quad (34)$$

Here

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr \quad (35)$$

is the Newtonian mass given by

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \frac{1}{G} \left[\frac{(1-c/a)r}{2} - \frac{D}{2r^{a-1}}\right]_{r_1}^{r_2}; \quad (36)$$

E_M is the sum of the other forms of energy such as the rest energy, kinetic energy, and internal energy. So the total gravitational energy is given by

$$E_g = \frac{1}{G} \left[\frac{(1-c/a)r}{2} - \frac{D}{2r^{a-1}} - (1 - \frac{c}{a}) \frac{rF[(0.5, 1/a); (1+1/a); -ar^a D/c]}{\sqrt{c/a}}\right]_{r_1}^{r_2}$$
$$+ \frac{1}{G} \left[D(1-a) r^{(-0.5a+1)} \frac{F[(-0.5+1/a, 0.5); (0.5+1/a); -cr^a/(aD)]}{\sqrt{D} a(-0.5+1/a)}\right]_{r_1}^{r_2}. \quad (37)$$

Letting $D = 0.1$ again, let us plot the total gravitational energy, assuming a distant halo region, typically $r \sim 200$ Kpc, shown in Fig. 1. The value of E_g is small but negative, showing that gravity in the halo is indeed attractive for the marginal quintessence case. (As before, this is not the case for ordinary quintessence dark energy.)

6 The observed equation of state

For the marginal quintessence case, the equation of state of the halo fluid can be obtained from a combination of rotation curves and lensing measurements. But first we need to rewrite the metric, Eq. (1), in the following form:

$$ds^2 = -e^{2\phi(r)} dt^2 + \frac{1}{1 - 2m(r)/r} dr^2 + r^2 d\Omega_2^2, \quad (38)$$

where

$$\phi(r) = \frac{1}{2} [\ln B_0 + l \ln r] \quad (39)$$

and

$$m(r) = \frac{1}{2} \left[r - \frac{c}{a} r - \frac{D}{r^{a-1}}\right]. \quad (40)$$

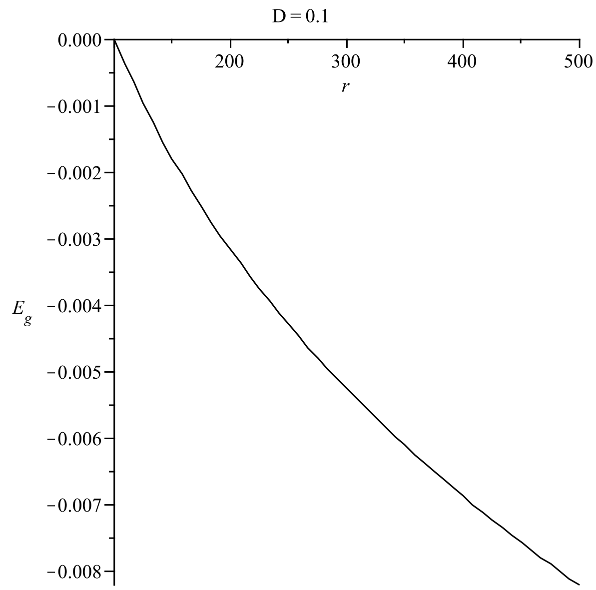

Figure 1: The variation of E_G with r in Kpc. The lower limit of integration in Eq. (24) is fixed at $r_1 = 100$ Kpc, while r_2 varies from 100 to 500 Kpc. We choose $G = 1$, $D = 0.1$, and $v^\phi \sim 10^{-3}$ (300km/s) for a typical galaxy.

As discussed in Ref. [24], the functions are determined indirectly from certain lensing measurements defined by

$$\Phi_{\text{lens}} = \frac{\Phi(r)}{2} + \frac{1}{2}\int \frac{m(r)}{r^2}dr = \frac{\ln B_0}{4} + \frac{l}{4}\ln r + \frac{1}{4}\ln r - \frac{c}{4a}\ln r + \frac{D}{4ar^a} \tag{41}$$

and

$$m_{\text{lens}} = \frac{1}{2}r^2\Phi'(r) + \frac{1}{2}m(r) = \frac{lr}{4} + \frac{1}{4}\left[r - \frac{cr}{a} - \frac{D}{r^{a-1}}\right]. \tag{42}$$

Of particular interest to us is the dimensionless quantity

$$\omega(r) = \frac{2}{3}\frac{m'_{\text{RC}} - m'_{\text{lens}}}{2m'_{\text{lens}} - m'_{\text{RC}}}, \tag{43}$$

due to Faber and Visser [19]. The subscript RC refers to the rotation curve; thus

$$\phi_{RC} = \phi(r) = \frac{1}{2}\left[\ln B_0 + l\ln r\right] \tag{44}$$

and

$$m_{RC} = r^2\Phi'(r) = \frac{lr}{2}. \tag{45}$$

The primes denote the derivatives with respect to r. The resulting expression is

$$\omega(r) = \frac{2}{3}\frac{m'_{\rm RC} - m'_{\rm lens}}{2m'_{\rm lens} - m'_{\rm RC}} = \frac{1}{3}\left[\frac{(al - a + c)r^a - Da(a-1)}{Da(a-1) + (a-c)r^a}\right]$$
$$= -\frac{1}{3}\frac{Da(a-1) + (a-c)r^a - alr^a}{Da(a-1) + (a-c)r^a}. \quad (46)$$

We conclude that for our marginal quintessence case, $\omega(r)$ is only minutely less than $-\frac{1}{3}$, which is consistent with our earlier result.

7 Conclusion

This paper discusses a new model for galactic dark matter by combining a quintessence field with a normal matter field that may have an anisotropic pressure. The quintessence field is characterized by a free parameter ω_q with the restriction $-1 < \omega_q < -\frac{1}{3}$. For convenience, the case where ω_q is extremely close to $-\frac{1}{3}$ is referred to as *marginal quintessence dark energy*, otherwise *ordinary quintessence dark energy*. It was found that for ordinary quintessence, stable orbits exists only if $\rho < 0$. For the marginal quintessence case, however, stable orbits do exist for $\rho > 0$, together with an attractive gravity. This result is consistent with that of Guzman et al. [20], who employed a perfect-fluid model for dark matter, while disregarding ordinary matter. The equation of state of the halo field, obtained from a combination of rotation curves and lensing measurements, is also consistent with the marginal case. In summary, quintessence dark energy appears to have different effects on the cosmological and local levels: while quintessence dark energy is responsible for the accelerated expansion, marginal quintessence dark energy is suitable for modeling dark matter, although ordinary quintessence dark energy is not. Finally, it has been argued that dark matter and dark energy ought to be different manifestations of the same entity.

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