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LETTER

Wormhole cosmic censorship

Tonatiuh Matos[1] · **L. Arturo Ureña-López**[2] · **Galaxia Miranda**[1]

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Abstract We analyze the properties of a Kerr-like wormhole supported by phantom matter, which is an exact solution of the Einstein-phantom field equations. It is shown that the solution has a naked ring singularity which is unreachable to null geodesics falling freely from the outside. Similarly to Roger Penrose's cosmic censorship, that states that all naked singularities in the Universe must be protected by event horizons, here we conjecture from our results that a naked singularity can also be fully protected by the intrinsic properties of a wormhole's throat.

Keywords Wormhole · Singularity · Cosmic censorship

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✉ Galaxia Miranda
mmiranda@fis.cinvestav.mx

Tonatiuh Matos
tmatos@fis.cinvestav.mx

L. Arturo Ureña-López
lurena@ugto.mx

1 Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, A.P. 14-740, 07000 Mexico, D.F., Mexico

2 Departamento de Física, DCI, Campus León, Universidad de Guanajuato, C.P. 37150 León, Gto., Mexico

Singularities are an ubiquitous ingredient in solutions of Einstein's field equations [1]. Most of the exact solutions known up to date that represent local objects contain real singularities in their spacetime structure. However, a singularity could be causally connected with the rest of the Universe, and it is thought that any number of troubles may come out of it. Besides, the predictability of our physical theories breaks down since Einstein's equations are not valid at the singularity. This is the reason why Roger Penrose famously conjectured that spacetime singularities should be protected by an event horizon that should in turn prevent external observers from seeing them [2,3].

On the other hand, wormholes are one of the most interesting solutions of Einstein equations [4–6]. They can be seen as fast highways that connect separate places in the Universe, or even as bridges between different universes. Unfortunately, such solutions could exist provided that the matter source in Einstein's equations can violate the null energy condition [5].

More recently, phantom matter has emerged as a promising candidate to be the dark energy in the Universe [7]. At the local level, phantom matter may be the matter source of wormholes too, and then phantom wormholes are becoming again one of the most mysterious and interesting solutions of Einstein's equations [8], see also [9–12].

In this Letter, we study in some detail the physical properties of the Kerr-like wormhole found in Refs. [13,14], which is in fact an exact solution of the Einstein's equations sourced by a phantom scalar field. As we shall show, the relevant feature of the solution is that its internal ring singularity is completely unreachable because of the protection provided by the (special) wormhole's throat. This is promising evidence that not only event horizons can prevent external observers from seeing a singularity, but also can a wormhole's throat.

To begin with, we write the line element describing the spacetime around the wormholes in Boyer–Linquist coordinates as[1] [14]

$$ds^2 = -f dt^2 + \frac{K}{f} dl^2 + \frac{\Delta_1}{f}\left[K d\theta^2 + \sin^2\theta \, d\varphi^2\right], \tag{1}$$

where

$$K = \frac{\Delta}{\Delta_1}, \quad f = \exp\left(-\frac{k_1}{2\Delta}\cos\theta\right) = \exp(-\lambda), \tag{2a}$$

$$\Delta = l^2 + l_0^2\cos^2\theta, \quad \Delta_1 = l^2 + l_0^2, \tag{2b}$$

being l_0 a parameter with units of distance, and $k_1 > 0$ is a parameter with units of angular momentum. The meaning of function f can be seen from the fact that the line element (1) is an exact solution of the Einstein's equations, $R_{\mu\nu} = -8\pi G \Phi_\mu \Phi_\nu$, where

$$\Phi = \frac{1}{\sqrt{16\pi G}}\lambda, \tag{3}$$

[1] We are using units $c = 1$. Notice also that metric (1) is obtained from the original one in Refs. [13,14] and through the change of variables $l - l_1 \to l$.

is a phantom-type scalar field [13]. Moreover, the Newtonian gravitational potential associated to metric (1) is given by $\phi_g = (1/2)\ln f$, and is also directly related to the phantom field in Eq. (3).

Some properties of metric (1) are discussed in turn. Firstly, for large values of the radial coordinate, $|l| \gg l_0$, we have $\lambda \to 0$, $f \to 1$ and Δ, $\Delta_1 \to l^2$, thus, the line element (1) is asymptotically flat at large distances,

$$ds^2 \to -dt^2 + dl^2 + l^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{4}$$

However, as it usually happens with other wormholes, we must keep in mind that there are two asymptotically flat regions, for $l \to \pm\infty$, and the connection between them is called the throat of the wormhole, which must be then located at $l = 0$. Secondly, we can see that $\Delta_1 > 0$ everywhere, but that that is not the case for Δ, which can be zero at different points of the spacetime. Because of this, metric (1) has a singularity determined by the condition $\Delta = 0$, which in turns translates into a ring singularity described by $l = 0$ and $\theta = \pi/2$.

In order to verify that we have encountered a true singularity, we should take a look at the invariants of the metric. For our case, straightforward calculations show that the invariants can be generally written as

$$\text{Invariants} = \frac{F}{8k_1^2\Delta_1^{\alpha_2}k_1^{2\alpha_3}}\frac{f^{2\alpha_3}}{\Delta^{\alpha_1}}, \tag{5}$$

where F is a complicated function free of singularities that takes different forms for each invariant of the metric, and α_1, α_2, and α_3 are positive coefficients whose exact value depend upon the chosen invariant. It is nonetheless clear that the condition $f^{2\alpha_3}/\Delta^{\alpha_1} = \infty$ makes all of them diverge at the location of the ring singularity.

Two other metric quantities are affected by the ring singularity. The most troublesome of them is function f, see Eq. (2a), which is also shown in Fig. 1. Notice that f is discontinuous at the ring singularity, as can be seen from the following limits:

$$\lim_{\theta=\pi/2,|l|\to\infty} f = 1, \qquad \lim_{\theta=\pi/2,l\to 0} f = 1, \tag{6a}$$

$$\lim_{l=0,\theta\to(\pi/2)^+} f = \infty, \qquad \lim_{l=0,\theta\to(\pi/2)^-} f = 0. \tag{6b}$$

Observe that the only one function in metric (1) that could be singular is f, all other elements are regular in the whole of the space–time. Eqs. (6) also tell us that the metric function f is singular if we reach the sphere $l = 0$ through the equatorial hypersurface $\theta = \pi/2$, and then this deserves a more careful study. At $l = 0$, the function

$$f_\Delta = \frac{f^{2\alpha_3/\alpha_1}}{\Delta} = \exp\left(-\frac{k_1\alpha_3}{l_0^2\alpha_1\cos\theta}\right)\frac{1}{l_0^2\cos^2\theta} \tag{7}$$

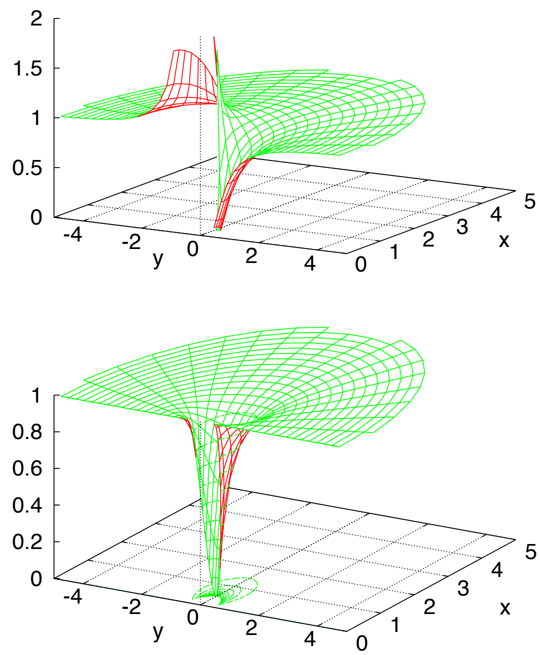

Fig. 1 The metric functions f and K, see Eqs. (12), for the values $l_0 = 0.45$ and $k_1 = 1$; the plot is given in terms of the pseudo-cartesian coordinates (x, y) defined as $x = r\sin\theta$ and $y = r\cos\theta$, where the radial coordinate r is defined as in Eq. (10). (*Top*) function f is the most troublesome metric function, as it shows quite different behaviors at the ring singularity, see also Eqs. (6). (*Bottom*) function K is regular everywhere, and it only vanishes at the ring singularity. At large distances, $|l| \gg l_0$, we recover the usual flat results for both functions, $f = 1$ and $K = 1$

has the limits

$$\lim_{\theta\to(\pi/2)^-} f_\Delta = 0, \qquad \lim_{\theta\to(\pi/2)^+} f_\Delta = \infty. \tag{8a}$$

This also means that a singularity appears in the invariants (5) only if we reach the equator for $\theta > \pi/2$. Thus, one observer can see the singularity only coming from the southern hemisphere, as from the northern hemisphere the spacetime is always regular. In conclusion, the singularity can only be reached from the southern hemisphere $\theta \to \frac{\pi}{2}^+$.

On the other hand, the quantity inside the squared brackets in Eq. (1) can be interpreted as a solid angle element modified by the metric function K:

$$d\Omega_0^2 = K d\theta^2 + \sin^2\theta\, d\varphi^2, \tag{9}$$

and such a modified solid angle indicates that the spacetime is indeed axially symmetric rather than spherically symmetric. Function K is well behaved everywhere, and it only vanishes at the location of the ring singularity, $K(l = 0, \theta = \pi/2)$ =0; at large distances, $|l| \to \infty$, it takes on the usual flat result $K = 1$ see Fig. 1.

To have a better visualization of the wormhole structure in metric (1), we define a new radial variable:

$$r^2 = \Delta_1 = l^2 + l_0^2, \tag{10}$$

and then write Eq. (1) as a conformal metric:

$$ds^2 = \frac{K}{f}\left(-\frac{f^2}{K}dt^2 + \frac{dr^2}{1 - l_0^2/r^2} + \frac{r^2}{K}d\Omega_0^2\right), \tag{11}$$

where now we have

$$K = 1 - \frac{l_0^2}{r^2}\sin^2\theta, \quad f = \exp\left(-\frac{k_1}{2}\frac{\cos\theta}{r^2 - l_0^2\sin^2\theta}\right). \tag{12}$$

It must be noticed that, in terms of the new variable, the ring singularity is located at $\theta = \pi/2$ and $r = l_0$.

As we said before, the spacetime of the wormhole is axially symmetric, and we can use this fact to draw up the structure of its throat by taking a slice of the spacetime with $\varphi = 0$. In this case, the induced metric, from Eq. (11), is of the form $ds^2 = (K/f)ds_c^2$, where

$$ds_c^2 = -\frac{f^2}{K}dt^2 + \frac{dr^2}{1 - l_0^2/r^2} + r^2 d\theta^2. \tag{13}$$

Curiously enough, ds_c^2 in Eq. (13) resembles the line element of the famous Morris–Thorne (MT) wormhole, and then we can see that there is a throat whose size is determined by the distance parameter l_0 (see for instance [5,15]). This indicates that the original metric represents a wormhole with a throat of finite size that is conformally related to the MT solution.

There is, in addition, a nice property of conformal spaces that states that the structure of null geodesics is preserved by (non-singular) conformal transformations [16]. We can then anticipate that null geodesics of metric (1) are conformally related to those of the MT wormhole (at least for the case $\varphi = 0$), through the conformal metric (13). We just need to recall that null geodesics of the MT wormhole, as drawn in terms of a typical embedding diagram, follow paths that lie on the throat's surface, and this means that our geodesics must likewise lie on such surface. In particular, at distances far from the throat, we find that the conformal factor $K/f \sim 1$, and then we shall recover exactly the case of the MT wormhole.

However, there is the issue that the conformal transformation in our case, see Eq. (11), is not well behaved everywhere, as the disturbing behavior of metric functions K, and f shows up at the ring singularity. Thus, the transformation should work well as long as the problematic points, those of the ring singularity at $r = l_0$ and $\theta = \pi/2$, are left out in our calculations.

We are to study now the null geodesics of test particles freely falling into the wormhole. It proves convenient to work with the Hamiltonian of the geodesics:

$$2\mathsf{H} = -\frac{p_t^2}{f} + \frac{f\, p_\varphi^2}{\Delta_1 \sin^2\theta} + \frac{f}{K}\left(p_l^2 + \frac{p_\theta^2}{\Delta_1}\right), \tag{14}$$

which is in itself a constant of motion, i.e. $\mathsf{H} = 0$ along any given null geodesic.

Because the line element does not depend upon t and φ explicitly, their respective momenta are conserved, $p_t = \text{const.}$ and $p_\varphi = \text{const.}$, whereas those related to variables l and θ, p_l and p_θ, respectively, will have a non-trivial dynamics. The Hamilton equations of motion are

$$\dot{l} = \frac{f}{K} p_l, \quad \dot{\theta} = \frac{f}{\Delta} p_\theta, \tag{15a}$$

$$\begin{aligned} \dot{p}_l = &-\frac{1}{2}\frac{\partial \ln f}{\partial l}\frac{p_t^2}{f} - \frac{1}{2}\frac{\partial \ln(f/\Delta_1)}{\partial l}\frac{f}{\Delta_1 \sin^2\theta} p_\varphi^2 \\ &-\frac{1}{2}\frac{\partial \ln(f/K)}{\partial l}\frac{f}{K} p_l^2 - \frac{1}{2}\frac{\partial \ln(f/\Delta)}{\partial l}\frac{f}{\Delta} p_\theta^2, \end{aligned} \tag{15b}$$

$$\begin{aligned} \dot{p}_\theta = &-\frac{1}{2}\frac{\partial \ln f}{\partial \theta}\frac{p_t^2}{f} - \frac{1}{2}\frac{\partial \ln(f/\sin^2\theta)}{\partial \theta}\frac{f}{\Delta_1 \sin^2\theta} p_\varphi^2 \\ &-\frac{1}{2}\frac{\partial \ln(f/K)}{\partial \theta}\frac{f}{K}\left(p_l^2 + \frac{p_\theta^2}{\Delta_1}\right), \end{aligned} \tag{15c}$$

where a dot denotes derivative with respect to an affine parameter. Equation (15) will be solved under different initial conditions that will be set up at large distances $l \gg l_0$. In every case the constraint $\mathsf{H} = 0$ will be strictly accomplished, and as a general rule we will choose $p_t = 1$ and $p_l \leq 0$.

To have a connection with our previous discussion about the throat of the wormhole, we find it useful to draw the paths of null geodesics over the known embedding profile of the throat of a MT wormhole. In order to achieve such a comparison, we set up $p_\varphi = 0$, and will make use of the known embedding variable of the MT wormhole [5]:

$$z(r) = l_0 \ln\left(\frac{r}{l_0} + \sqrt{\frac{r^2}{l_0^2} - 1}\right), \tag{16}$$

As we said before, in our case the geodesic paths will lie on the throat's surface too, but will show deviations because of the peculiarities induced upon them by the conformal factor K/f. Hence, geodesic paths will give us a measure of the deformations in the true throat of metric (1) with respect to that of the MT's wormhole.

The resulting trajectories are shown in Fig. 2. In general terms, we can see that null geodesics are able to avoid the naked singularity of the spacetime. This fact can be intuitively understood from the conservation equation $\mathsf{H} = 0$. For the particular case considered in our numerical solutions shown in Fig. 2, $p_t = 1$ and $p_\varphi = 0$, and then Eq. (14) reads

$$p_l^2 + \frac{p_\theta^2}{r^2} = \frac{K}{f^2}. \tag{17}$$

(Incidentally, this is the constraint equation that strictly corresponds to the conformal metric (13).) As we have seen before, at large distances $K/f^2 \sim 1$, and we recover the usual equation of motion of the MT wormhole. However, as the geodesics approach the singularity, we find that the term K/f^2 behaves discontinuously (a feature inherited from function f), and then the constraint equation (17) cannot be satisfied unless both

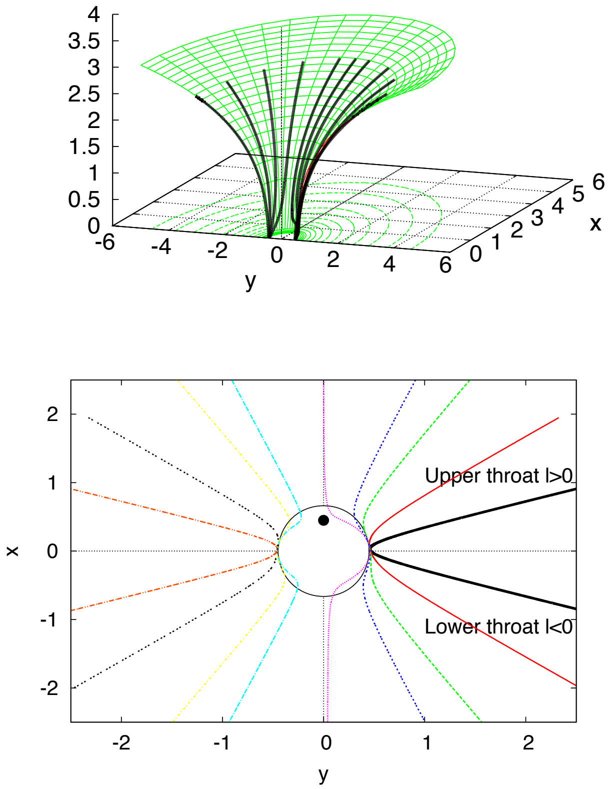

Fig. 2 Different null geodesics for different initial values of θ, given as numerical solutions of the geodesic equations (15); we are using here the pseudo-cartesian coordinates $x = r \sin\theta$ and $y = r\cos\theta$, whereas z is given by the MT function (16). (*Top*) the surface represents the MT throat's profile, the solid lines are the null geodesics as depicted on the throat, and the ring singularity is marked as a point on the xy-plane at $r = l_0$ and $\theta = \pi/2$. (*Bottom*) the paths of null geodesics projected on the xy-plane; the *upper half* represents the upper part of the throat, $l > 0$ (also shown in the *top plot*), whereas the *lower part* represents the lower part of the throat, $l < 0$. In general, geodesics are able to avoid the naked singularity as they travel on the throat's surface. The deformation seen in the geodesics paths are due to the conformal factor K/f, see Eq. (13), that changes the real throat's surface with respect to MT's at points close to the ring singularity. See also the text for more details

momenta p_l and p_θ are also discontinuous there. There is no other option, for any finite solution, but to deviate from the singularity point (see the bottom plot in Fig. 2). In particular, the angular momentum p_θ is not conserved, and then the geodesic trajectory is scattered off the singularity point: the closer the trajectory is to a singularity, the larger the change in the angular momentum it acquires, and then the larger the diversion the geodesic takes form the singularity.

At first sight, one could say that the naked singularity is protected by an angular potential barrier. This is not completely true, because the barrier is not build up from the conservation of angular momentum (as is usually the case), but rather it is dynamically induced by the same discontinuities of the metric functions. Moreover, these discontinuities must be indeed related to the deformation of the wormhole's throat in the neighborhood of the singularity, as indicated by the geodesic paths drawn upon the MT wormhole's throat in Fig. 2.

We would like to reinforce our geometrical point of view here. Even though the conformal metric (13) resembles that of the MT wormhole, there are some key differences. The first one is the presence of a non-trivial gravitational potential: $\phi_g = (1/2)\ln(f^2/K)$, which is the dynamical responsible of the deformation of geodesic paths near the singularity. Hence, if metric (13) were our only concern, then our conclusion would be that the throat's surface is exactly that of the MT wormhole, and that geodesic paths are deviated due to the presence of the gravitational potential ϕ_g.

However, the second key difference is that the conformal factor K/f has important effects on the form of the wormhole's throat too, and then we can figure out that the throat is that of the MT wormhole plus deformations in the neighborhood of the ring singularity. Therefore, we must conclude that it is the form of the throat's surface which prevents any null geodesic from reaching the ring singularity. In other words, the reason behind the deformation of geodesic paths is actually *geometrical* rather than dynamical.

It is in this regard that our case resembles the case of an event horizon surrounding the singularity at the centre of a black hole: it is not a dynamical reason (i.e. the presence of a potential barrier) that isolates the singularity, but a *geometrical* event horizon that changes the behavior of geodesic paths and separates out the external and internal parts of the black hole.

In order to see the behavior of the geodesics close to the singularity, we write the geodesic Eq. (15) explicitly under the approximations $l << l_0$ and $\theta \sim \pi/2$. We obtain that:

$$\begin{aligned} \dot{p}_l \frac{2\Delta}{f} &= F_1\, l\, M - \frac{k_1 l}{l_0^4 \cos\theta} p_\varphi^2 - \frac{k_1 l}{l_0^2 \cos\theta} \exp\left(\frac{k_1}{l_0^2 \cos\theta}\right) p_t^2, \\ \dot{p}_\theta \frac{2\Delta}{f} &= F_2\, M + \frac{k_1}{2 l_0^2 \sin\theta} p_\varphi^2 + \frac{k_1}{2} \sin\theta \exp\left(\frac{k_1}{2 l_0^2 \cos\theta}\right) p_t^2, \end{aligned} \tag{18}$$

being $M = p_l^2 l_0^2 + p_\theta^2$, and

$$F_1 = -\frac{k_1 - 2\, l_0^2 \cos\theta}{l_0^4 \cos^3\theta}, \quad F_2 = \frac{k_1 - 4 l_0^2 \cos\theta}{2 l_0^2 \cos^2\theta}.$$

Neglecting again the motion on the φ direction, so that $p_\varphi = 0$, then Eq. (18) can be integrated exactly on the southern hemisphere as follows. We observe that $F_1 l_0^2 \sim -F_{2,\theta}$, and then a solution of Eq. (18) is

$$\begin{aligned} p_l &= A_0 \sin(M_1 l l_0) \exp\left(\frac{1}{2} F_1 l^2\right) H, \\ p_\theta &= A_0 \cos(M_1 l l_0) \exp\left(\frac{1}{2} F_1 l^2\right) H, \end{aligned} \tag{19}$$

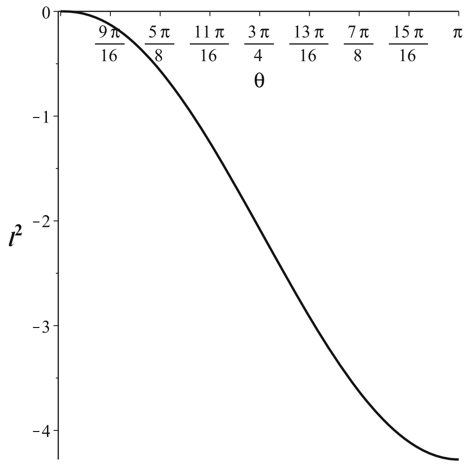

Fig. 3 Solution l^2 for the values $k_1 = 1$, $l_0 = 0.45$, $A_0 = 1$ and $p_t = 5$. In the region where the approximation is valid the function is negative. There are no solutions on the south hemisphere (near $\theta = \pi/2$) that are close to the ring singularity

where $M_{1,\theta} = F_1$, and

$$H = \exp\left(\frac{k_1}{4l_0^2 \cos\theta}\right)\cos\theta.$$

For metric (1), any geodesic must comply with the following constraint:

$$-\epsilon^2 = -\frac{p_t^2}{f} + \frac{f}{\Delta}(p_l^2 l_0^2 + p_\theta^2) + \frac{f}{\Delta_1 \sin^2\theta} p_\varphi^2, \tag{20}$$

being $\epsilon^2 = 1$ for time-like geodesics, and $\epsilon^2 = 0$ for null geodesics. Taking Eqs. (2a) and (19) into account, even for a null geodesic Eq. (20) transforms into

$$\begin{aligned} l^2 &= -\frac{k_1 l_0^2 \cos^2\theta}{k_1 - 2l_0^2\cos\theta} - \frac{l_0^4 \cos^3\theta}{k_1 - 2l_0^2\cos\theta} \ln\left(\frac{p_t^2}{A_0^2}\right) \\ &\sim -l_0^2 \cos^2\theta. \end{aligned} \tag{21}$$

for $\theta \sim \pi/2^+$. It is clear that Eq. (21) does not have solutions for points on the southern hemisphere that are also close to the ring singularity, as shown in Fig. 3. In other words, we see that it is not possible to find a geodesic trajectory that can be in contact with the singularity, and then the latter is not visible to exterior observers [17].

This leads us back again to the famous Penrose's cosmic censorship conjecture. Despite admirable attempts to prove or disprove it, its relevance in General Relativity is still a matter of debate, as the conjecture has been proved to be true under special conditions only [18–24]. On the other front line, there are counterexamples that violate the cosmic censorship, like models of dynamical collapse leading to naked singularities, in which trapped surfaces do not develop early enough to shield them [25,26]. Moreover, in [27] it was shown that naked singularities arise in gravitational collapse, a result that has been debated by Penrose himself [3]. In any case, the possible violation of the cosmic censorship has lead to new paths of study, like in the analysis of the properties of naked singularities; for instance, its observational consequences, and their differences from black holes through lensing data [28–30].

The case of the wormhole we have studied throughout this Letter is equally interesting for the discussions taking place in the specialized literature. According to our solution, it is also possible to protect a singularity if we surround it with a wormhole's throat. This may indicate a generalization of Penrose's cosmic censorship conjecture: it can be the case that naked singularities with a more involved configuration may find the formation of a wormhole's throat around them more convenient than just the appearance of an event horizon.

References

1. Stephani, H., Kramer, D., MacCallum, M.A., Hoenselaers, C., Herlt, E.: Exact Solutions of Einstein's Field Equations, 2nd edn. Cambridge University Press, Cambridge (2003)
2. Penrose, R.: Riv. Nuovo Cim. **1**, 252 (1969)
3. Penrose, R.: J. Astrophys. Astron. **20**, 233 (1999)
4. Einstein, A., Rosen, N.: Phys. Rev. **48**, 73 (1935)
5. Morris, M., Thorne, K.: Am. J. Phys. **56**, 395 (1988)
6. Visser, M.: Lorentzian wormholes: From Einstein to Hawking. AIP, Woodbury (1995)
7. Clemson, T.G., Liddle, A.R.: Mon. Not. Roy. Astron. Soc. **395**, 1585 (2009). arXiv:0811.4676 [astro-ph]
8. Lobo, F.S.N.: Phys. Rev. D **71**, 084011 (2005). arXiv:gr-qc/0502099 [gr-qc]
9. Gonzalez, J.A., Guzman, F.S., Sarbach, O.: Class. Quant. Grav. **26**, 015010 (2009). arXiv:0806.0608 [gr-qc]
10. Gonzalez, J.A., Guzman, F.S., Sarbach, O.: Class. Quant. Grav. **26**, 015011 (2009). arXiv:0806.1370 [gr-qc]
11. Gonzalez, J.A., Guzman, F.S., Sarbach, O.: AIP Conference Proceedings **1083**, 208 (2008)
12. Gonzalez, J.A., Guzman, F.S., Sarbach, O.: Phys. Rev. D **80**, 024023 (2009). arXiv:0906.0420 [gr-qc]
13. Matos, T.: Gen. Relativ. Gravit. **42**, 1969 (2010). arXiv:0902.4439 [gr-qc]
14. Miranda, G., Matos, T., Montelongo-Garcia, N.: Gen. Relativ. Gravit. **46**, 1613 (2014). arXiv:1303.2410 [gr-qc]
15. Muller, T.: Phys. Rev. D **77**, 044043 (2008)
16. Wald, R.M.: General Relativity. Univ. Pr, Chicago (1984)
17. Deshingkar, S.S.: (2010) arXiv:1012.3090 [gr-qc]
18. Etesi, G.: Int. J. Theor. Phys. **52**, 946 (2013). arXiv:1205.4550 [gr-qc]
19. Newman, R.: Gen. Relativ. Gravit. **16**, 175 (1984)
20. Israel, W.: Found. Phys. **14**, 1049 (1984)
21. Toth, G.Z.: Gen. Relativ. Gravit. **44**, 2019 (2012). arXiv:1112.2382 [gr-qc]
22. Wald, R.: Ann. Phys. **82**, 548 (1974)
23. Miyamoto, U., Jhingan, S., Harada, T.: 053E01. arXiv:1108.0248 [gr-qc] (2013)
24. Geroch, R. P., Horowitz, G.: Global structure of space- times. In: Hawking, S.W., Israel, W (eds) Published in General Relativity: An Einstein Centenary Survey. Cambridge University Press (1979)
25. Joshi, P.: Gravitational collapse: the story so far. In: Dadhich, N., Kembhavi, A. (eds) Published in The Universe: Visions and Perspectives. Kluwer Academic Publishers. pp. 161-168 (2000)
26. Bambi, C.: J. Phys. Conf. Ser. **283**, 012005 (2011). arXiv:1008.3026 [gr-qc]
27. Shapiro, S.L., Teukolsky, S.A.: Phys. Rev. Lett. **66**, 994 (1991)
28. Batic, D., Chin, D., Nowakowski, M.: Eur. Phys. J. C **71**, 1624 (2011). arXiv:1005.1561 [gr-qc]
29. Virbhadra, K.S., Keeton, C.R.: Phys. Rev. D **77**, 124014 (2008). arXiv:0710.2333 [gr-qc]
30. Virbhadra, K.S., Ellis, G.F.R.: Phys. Rev. D **65**, 103004 (2002)