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BLACK HOLES FROM GENERALIZED CHATTERJEE SOLUTIONS IN DILATON GRAVITY

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In the framework of the dilaton gravity theory, we consider the Papapetrou metric together with the harmonic maps ansatz in order to find exact solutions for the field equations of the theory. By this means a generalized four-dimensional solution is obtained which for definite values of the dilaton coupling constant (α) reduces to solutions of the four-dimensional Einstein–Dilaton theory, low energy strings models and Kaluza–Klein theories. In particular, for the last case it contains the four-dimensional version of the well-known Chatterjee solution.

1. Introduction

One of the exact solutions of the Einstein field equations in five-dimensional gravity is the Chatterjee space-time, which is simple enough to extract information from it. It is a soliton solution and is actually a naked singularity viewed as a five-dimensional space-time.[1] The solution contains a scalar field but no electromagnetic field at all.

Five-dimensional gravity is an example of the unified theories of electromagnetism and gravitation. Einstein–Maxwell and low energy string theories are also examples of this kind of unified theories. Mathematically their effective actions in four dimensions are very similar, they differ in that the value of the scalar dilatonic field coupling constant is in each case different. Thus we can write the four-dimensional effective action for all of the abovementioned theories in the form:

$$S = \int d^4x\sqrt{-g}\,[-R + 2(\nabla\Phi)^2 + e^{-2\alpha\Phi}F_{\mu\nu}F^{\mu\nu}]\,, \tag{1.1}$$

where R is the Ricci scalar, Φ is the scalar dilaton field, $F_{\mu\nu}$ is the Faraday electromagnetic tensor and α is the dilaton coupling constant. For $\alpha = 0$ we have the

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effective action of the Einstein–Maxwell–Dilaton theory, and the scalar dilaton field appears minimaly coupled to electromagnetism, $\alpha = 1$ represents the low energy string theory where only the U(1)-vector gauge field has not been dropped out, and $\alpha = \sqrt{3}$ reduces the action (1.1) to that of the five-dimensional Kaluza–Klein theory. As is well known, all of these theories unify gravity and electromagnetism, it is interesting to note that for the string and Kaluza–Klein theories, electromagnetism cannot be decoupled from the scalar dilaton field.

The Chatterjee solution has already been studied in Refs. 1 and 2 and recently harmonic maps ansatz[3] has been applied to the action (1.1) in order to find exact solutions of its corresponding field equations.[4]

In this work we present a new exact solution of (1.1) associated field equations for arbitrary values of the α coupling constant and for $\alpha = \sqrt{3}$ reduces just to the Chatterjee solution. This new solution is the space-time of a black hole for some values of the free parameter δ with α arbitrary. In general it represents a soliton space-time of a naked singularity.

The plan of this paper is as follows. In Sec. 2 we review the five-dimensional Chatterjee solution and the properties of its four-dimensional induced matter. In Sec. 3 we present the new solution and its corresponding space-times in each of the mentioned theories and in Sec. 4 we discuss the results and present the conclusions.

2. Five-Dimensional Chatterjee Solution

In 1990 Chatterjee found[5] a five-dimensional solution given by

$$dS_5^2 = \left[1 - \frac{2M}{(\sqrt{r^2+M^2}+M)}\right] dt^2 - \frac{dr^2}{\left(1 + \dfrac{M^2}{r}\right)}$$
$$- r^2 d\Omega^2 - \left[1 - \frac{2M}{(\sqrt{r^2+M^2}+M)}\right]^{-1} d\Psi^2 \,, \tag{2.1}$$

where t is time, r the radial coordinate, θ and ϕ spherical polar angles, Ψ the fifth coordinate and as usual

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \,. \tag{2.2}$$

Although, in the limit $r \to \infty$, the constant M can be identified with the mass of an object like a star at the origin of coordinates. As we shall see later the Chatterjee metric cannot be assumed to be a black hole.

In a general set of coordinates the event horizon is defined as the surface where the norm of the timelike Killing vector vanishes. Performing a coordinate transformation to $\sqrt{r^2+M^2}+M$ and relabel, Eq. (2.1) becomes:

$$dS_5^2 = \left[1 - \frac{2M}{r}\right] dt^2 - dr^2 - \left[1 - \frac{2M}{r}\right] r^2 d\Omega^2 - \frac{d\Psi^2}{1 - \dfrac{2M}{r}} \,. \tag{2.3}$$

Looking at the first term, it seems that the metric possesses a horizon located at $r = 2M$. However, the third metric shows that two-shells about the origin shrink to a point there, so it is merely a point horizon and the Chatterjee solution is not a black hole.

On the other hand, some authors[6] have considered the matter properties associated with metrics of the form:

$$dS_5^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - R^2 d\Omega^2 - e^{\mu}d\Psi^2 \tag{2.4}$$

which includes the metric (2.1) as particular case if it is assumed that the coefficients ν, λ and R and μ depend only upon the radial coordinate and not upon the time or the fifth coordinate.

As usual, one can look at the five-dimensional Kaluza–Klein equations in vacuum as four-dimensional Einstein equations with matter. This involves the identification of the four-dimensional energy-momentum tensor as follows[6]:

$$8\pi T_0^0 = \frac{1}{R^2} - e^{-\lambda}\left[\frac{2R''}{R} + \frac{R'^2}{R^2} - \frac{R'\lambda'}{R}\right], \tag{2.5}$$

$$8\pi T_1^1 = \frac{1}{R^2} - e^{-\lambda}\left[\frac{R'^2}{R^2} + \frac{R'\lambda'}{R}\right], \tag{2.6}$$

$$8\pi T_2^2 = 8\pi T_3^3 = -\frac{1}{4}e^{-\lambda}\left[2\nu'' + \nu'^2 + \frac{4R''}{R} - \frac{2R'\lambda'}{R} + \frac{2R'\nu'}{R} - \nu'\lambda'\right], \tag{2.7}$$

or in terms of the Chatterjee solution[1]

$$8\pi T_0^0 = \frac{M^2}{r^4}, \tag{2.8}$$

$$8\pi T_1^1 = -\frac{2M}{r^2\sqrt{r^2+M^2}} - \frac{2M^3}{r^4\sqrt{r^2+M^2}} - \frac{M^2}{r^4}, \tag{2.9}$$

$$8\pi T_2^2 = \frac{M}{r^2\sqrt{r^2+M^2}} + \frac{M^3}{r^4\sqrt{r^2+M^2}}. \tag{2.10}$$

This energy-momentum tensor satisfies the equation of state for fotons or relativistic particles namely:

$$T_\mu^\mu = 0. \tag{2.11}$$

The gravitational mass associated with a three-dimensional volume is well defined[1] and given for the Chatterjee solution by:

$$M_g(r) = M\left[\frac{\sqrt{r^2+M^2} - M}{[\sqrt{R^2+M^2} - M]^{\frac{1}{2}}}\right]. \tag{2.12}$$

We see easily that $M_g(\infty) = M$ as mentioned above, agreeing with the usual metric-based definition. However, the gravitational mass goes to zero at $r = 0$.

In the next section the generalized solution, which contains the Chatterjee solution as particular case is presented.

3. Generalized Four-Dimensional Chatterjee Solution

We begin by considering the Papapetrou metric in the following parametrization:

$$dS^2 = \frac{1}{f}[e^{2k}(d\rho^2 + d\zeta^2) + \rho^2 d\varphi^2] - f dt^2 \,. \tag{3.1}$$

By solving the general field equation coming from the metric (3.1) with no electromagnetic field at all, we arrive at a solution given by[4]

$$f = e^{\lambda}, \qquad k_{,z} = \frac{\rho}{2}(4\alpha^2 a^2 + 1)(\lambda_{,z})^2 \tag{3.2}$$

with the following form for scalar dilaton field:

$$e^{2\alpha\Phi} = \kappa_0 e^{2\alpha^2 a\lambda}, \qquad a = \text{const} \tag{3.3}$$

with λ a harmonic map, i.e. a solution of the equation:

$$(\rho\lambda_{,z})_{,\bar{z}} + (\rho\lambda_{,\bar{z}})_{,z} = 0\,, \tag{3.4}$$

where $z = \rho + i\zeta$.

Let us consider the particular case where $\lambda = \delta \ln(1 - \frac{2M}{r})$ with $\delta = \text{const}$, which is a well known solution of the Lapalce equation (3.4), written in Boyer–Lindquist coordinates $\rho = \sqrt{r^2 - 2mr}\sin\theta$, $\zeta = (r - m)\cos\theta$.

The generalized Chatterjee solution is thus given as follows:

$$dS_4^2 = \frac{dr^2}{(1 - \frac{2M}{r})^{\delta}} + \left(1 - \frac{2M}{r}\right)^{1-\delta} r^2 d\Omega^2 - \left(1 - \frac{2M}{r}\right)^{\delta} dt^2 \tag{3.5}$$

and the dilaton scalar field by

$$I^3 = \kappa_0^2 \left(1 - \frac{2M}{r}\right)^{-\alpha\sqrt{|1-\delta^2|}}, \tag{3.6}$$

where α is the dilaton coupling constant. As mentioned in the introduction, for $\alpha = 0$ this is a solution in the framework of the Einstein theory, for $\alpha = 1$, (3.5) reduces to a low energy string theory solution and finally for $\alpha = \sqrt{3}$ a five-dimensional Kaluza–Klein solution is obtained in the sense that in this particular case

$$dS_5^2 = \frac{1}{I} dS_4^2 + I^2 (dx^5)^2 \,. \tag{3.7}$$

This last fact suggests that in higher dimensional theories the four-dimensional part should be multiplied by a conformal factor related to the dilaton field in order to be physically meaningful.

Looking ahead on this generalized solution one can easily recognize the following three cases:

(i) For $\delta = 1$ we have exactly a Schwarzschild solution and possesses all the well known features of the usual Schwarzschild one, which is clearly singular at $r = 0$.

(ii) For $\delta = \frac{1}{2}$, $\alpha = \sqrt{3}$ the generalized solution reduces to Kaluza–Klein soliton solution which corresponds to the Chatterjee one.

(iii) For $\delta = 2$, α arbitrary, this solution represents a black hole in the framework of the dilaton gravity theory, whose horizon is located at $r = 2M$ being twofold degenerate and the true singularity at $r = 0$ as usual for this kind of black holes. This fact can be seen from the expression for the scalar curvature given by

$$R = -\frac{6M^2}{r^4}\,. \tag{3.8}$$

In this way we have shown that our solution contains solutions in all the three theories mentioned above.

4. Discussion and Conclusions

In this work we presented the direct correspondence between the Einstein–Dilaton, low energy string and Kaluza–Klein theories from a new exact solution which can be interpreted in all of them, although this relation leads to different kinds of objects in each kind of theory.

It is interesting to note that the behavior of the scalar dilaton field differs also in each theory according to (3.3), but this behavior is always such that the scalar field grows exponentially for $r \sim 0$, and tends very quickly to a constant for $r \gg 1$. Such behavior is already found in many solutions of this kind. In particular in Kaluza–Klein exact solutions it has being encountered very often.[7–14]

If $\delta = 1$ we have Schwarzschild solution for any theory in (3.1). Nevertheless if $\delta = 2$ our solution represents again a black hole even for an arbitrary value of α. From this fact, it follows that the scalar field does not avoid the space-time singularity.

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