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LETTER TO THE EDITOR

Fermion mass gap in the loop representation of quantum gravity

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Abstract. An essential step towards the identification of a fermion mass generation mechanism at the Planck scale is to analyse massive fermions in a given quantum gravity framework. In this letter the two mass terms entering the Hamiltonian constraint for the Einstein–Majorana system are studied in the loop representation of quantum gravity and fermions. One term resembles a bare mass gap because it is not zero for states with zero (fermion) kinetic energy, unlike the other term which is interpreted as 'dressing' the mass. The former contribution originates from (at least) triple intersections of the loop states acted on whilst the latter is traced back to every pair of coinciding end points, where fermions are located. Thus, fermion mass terms get encoded in the combinatorics of loop states. Finally, the possibility is discussed of relating fermion masses to the topology of space.

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Physics at the Planck length $\ell_P := \sqrt{G\hbar/c^3} = 1.6 \times 10^{-33}$ cm, where a quantum notion of spacetime is called for, is acquiring deeper significance due to a number of new results. Amongst the most striking we find: (i) a definition of a gravitational Hamiltonian [1] and, more recently, a skein-relation interpretation of the Hamiltonian constraint [2]; (ii) a determination of area and volume spectra [3]; and (iii) an insight into the origin of black hole entropy [4]. All of these were obtained within a non-perturbative approach to quantum gravity [5–7] and they present discrete and combinatorial features that seem to encode fundamental quantum aspects of spacetime at ℓ_P. For instance, by coupling a clock scalar field to gravity, a Hamiltonian was built up in [1] that evolves the gravitational field itself. The action of this Hamiltonian on loop states is concentrated at intersection points of the loops. More recently, the Hamiltonian constraint was interpreted as a skein relation when acting on the space of knots [2]; showing that knot polynomials satisfying the skein relation solve the full quantum Einstein equations. In [3] it was shown that area and volume operators can be defined at the quantum level. Their spectra are discrete and related to the way intersections occur between loops and the surface whose area should be determined or among loops inside the region whose volume is under study. This former notion of area

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is further exploited in relation to the horizon of a black hole in [4]. Hence a value for its entropy can be estimated that agrees with the standard proportionality between entropy and horizon area. It should be stressed that each of these results was obtained after a suitable regularization procedure to make them well defined; the resulting operators are finite, diffeomorphism invariant and (regularization-) background independent.

Now it is natural to wonder how compatible are the continuous picture of space we are used to and the above discreteness. It turns out that smooth space can be thought of as a large length limit of certain loopy states or weaves [8]. Moreover, the notion of gravitons also emerges here: they are associated to embroideries on weave states [9]. Appealing as this idea is, it cannot describe nature as a whole; one must learn first what the notion of matter is, if any, consistently with the above discrete picture. Indeed such a consistency must be looked for in any given quantum gravity scenario.

It has been proposed in the past that wormholes at the Planck scale might behave as charged particles [10] and that quantum gravity states could have half-integral angular momentum, when the space 3-manifold has non-trivial topology [11]. Also, by following the path integral approach to quantum gravity, it was realized that one might include the contribution of non-orientable spacetimes (non-orientable foam) in the corresponding amplitude. This, together with CP invariance, could produce an effective mass for the otherwise massless fermionic fields living in such spacetimes [12]. The standard model of electroweak interactions has been studied along similar lines by taking a random Planck lattice as an effective theory coming from a Planck-scale foam spacetime [13].

In the loop representation of non-perturbative quantum gravity, some steps have been made towards unravelling the notion of matter. Coupled electromagnetic field and gravity were considered in [14] as a simple unified description of gravitational and electromagnetic interactions. Physical states were found parametrized by two loops, each of which carries information about both gravity and electromagnetism—the Chern–Simons functional and Jones polynomial playing a role in the analysis. In [15] massless spin-$\frac{1}{2}$ fields and gravity were studied. A (clock-) scalar field was coupled to gravity and a Hamiltonian evolving both fermion and gravity fields was introduced. The fermionic contribution becomes concentrated at the end points of the curves (where fermions are located) of the loop states acted upon. More recently, in [16] a kinematical analysis was developed for the Einstein–Maxwell–Dirac theory.

Crucial to the notion of matter is the understanding of the origin of fermion masses, given an unsatisfactory status in this regard in the standard model of electroweak interactions [13]. To gain this understanding a compulsory step is to find the analogue of the term $m\overline{\Psi(x)}\Psi(x)$ that reveals the fermion mass in the Lagrangian form of field theory. In this letter an analysis is given of the mass of a spin-$\frac{1}{2}$ field of Majorana type coupled to gravity, using the loop representation for canonical quantum fermions and gravity of [15]. Specifically, we study the features inherited from the discreteness and combinatorial aspects appearing in such an approach.

To begin with, we recall the outcome of the canonical analysis for the Einstein–Majorana (EM) system using Ashtekar variables. There are three first-class constraints, namely the Gauss, vector and Hamiltonian ones [17]

$$
\begin{aligned}
\mathcal{G}_{AB} &:= -\mathcal{D}_a\widetilde{\sigma}^a{}_{AB} - \eta_{(A}\widetilde{\theta}_{B)},\\
\mathcal{V}_a &:= \widetilde{\sigma}^b{}_{AB}F_{ab}{}^{BA} - \widetilde{\theta}_A\mathcal{D}_a\eta^A\\
\mathcal{H} &:= -\tfrac{1}{2}\widetilde{\sigma}^a{}_A{}^C\,\widetilde{\sigma}^b{}_C{}^B\,F_{abB}{}^A - \widetilde{\sigma}^a{}_A{}^B\widetilde{\theta}_B\mathcal{D}_a\eta^A + m\big((\widetilde{\sigma})^2\eta_A\eta^A - \tfrac{1}{4}\widetilde{\theta}^A\widetilde{\theta}_A\big)\\
&\equiv \mathcal{H}_{\text{Einstein}} + \mathcal{H}_{\text{Weyl}} + \mathcal{M}_1 + \mathcal{M}_2\,,
\end{aligned}
\tag{1}
$$

$A_a{}^{AB}(x), \eta^A(x)$ being the configuration variables and $\widetilde{\sigma}^a{}_{AB}(x), \widetilde{\theta}_A(x)$ the corresponding canonical momenta†. Here the Majorana spin-$\frac{1}{2}$ field contribution to $\mathcal{H}$ consists of the last three terms in (1), of which $\mathcal{M}_1$ and $\mathcal{M}_2$ are related to mass. To proceed to the quantum theory, one has to solve the problem of constructing their (regularized) quantum operator version. This is achieved by adopting loop variables, where the Gauss law is automatically fulfilled, following [1, 15]. For spin-$\frac{1}{2}$ and gravity, loop variables were built up in [15] as

$$X[\alpha] := \psi^A(\alpha_i)U_A{}^B[\alpha]\psi_B(\alpha_f), \qquad Y[\alpha] := \widetilde{\pi}^A(\alpha_i)U_A{}^B[\alpha]\psi_B(\alpha_f),$$
$$Y^a[\alpha](s) := \widetilde{\pi}^A(\alpha_i)U_A{}^B[\alpha](0,s)\widetilde{\sigma}^a_B{}^C(\alpha(s))U_C{}^D[\alpha](s,1)\psi_D(\alpha_f)\,. \tag{2}$$

Among their properties it is worth mentioning the fermionic (Grassmann) identity: if (α, β, γ are open curves) $\alpha_i = \beta_i = \gamma_i$ then $X[\alpha]\,X[\beta]\,X[\gamma] = 0$. No three fermions can exist at the same point simultaneously. The loop variable $Y^a[\alpha](s)$ was used to define the kinetic fermion term of the Hamiltonian constraint of the Einstein–Weyl theory [15]. Next, the first mass term $\mathcal{M}_1$ is translated into loop variables. Consider a closed loop γ with three gravitational hands inserted in it and an open loop α with the fermion field $\eta(x)$ placed at its ends. That is to say

$$V^{abc}[\gamma,\alpha] := T^{abc}[\gamma](s,t,r)\,X[\alpha], \tag{3}$$

where $T^{abc}[\gamma](s,t,r) := \mathrm{Tr}\big\{\widetilde{\sigma}^a(\gamma(s))U_\gamma(s,t)\widetilde{\sigma}^b(\gamma(t))U_\gamma(t,r)\widetilde{\sigma}^c(\gamma(r))U_\gamma(r,s)\big\}$ is the loop variable used in the construction of the volume operator [3] and $X[\alpha]$ is as given above. It is straightforward to show that when the two loops shrink down to a *common* point x, we have the local quantity

$$\mathcal{M}_1 = \left(\frac{m}{3\sqrt{2}}\right)\lim_{\gamma,\alpha\to x}\eta_{abc}V^{abc}[\gamma,\alpha] = m(\sigma)^2\eta_A\eta^A. \tag{4}$$

The construction becomes more transparent if (3) is rewritten as follows, then use is made of the fundamental spinor identity $\epsilon_{AB}\epsilon_{CD} + \epsilon_{AC}\epsilon_{DB} = \epsilon_{AD}\epsilon_{CB}$ inserted at the intersection point of the two loops. The result is the difference of further loop variables

$$T^{abc}[\gamma](s,t,r)\,X[\alpha] = N^{abc}[\alpha\cdot\gamma](s^*,t^*,r^*) - N^{cba}[\alpha\cdot\gamma^{-1}](1-s^*,1-t^*,1-r^*) \tag{5}$$

$$N^{abc}[\alpha\cdot\gamma](s^*,t^*,r^*) := \mathrm{Tr}\big\{\eta(\alpha_i)U[\alpha](0,p)U[\gamma](0,s)\widetilde{\sigma}^a(\gamma(s))U[\gamma](s,t)\widetilde{\sigma}^b(\gamma(t))$$
$$\times\, U[\gamma](t,r)\widetilde{\sigma}^c(\gamma(r))U[\gamma](r,1)U[\alpha](p,1)\eta(\alpha_f)\big\}, \tag{6}$$

with $\alpha(p) = \gamma(0) = \gamma(1) = x$ and s^*, t^*, r^* being the values of the parameter of $\alpha\cdot\gamma$, where the gravitational hands are inserted. Here $\mathrm{Tr}\big\{\psi\mathcal{O}^{(1)}\cdots\mathcal{O}^{(n)}\psi\big\} := \psi^{A_1}\mathcal{O}^{(1)A_2}_{A_1}\cdots\mathcal{O}^{(n)A_{n+1}}_{A_n}\psi_{A_{n+1}}$.

Regarding $\mathcal{M}_2$, this can be expressed as

$$\mathcal{M}_2 := -\left(\tfrac{1}{4}m\right)\lim_{\alpha\to x} Z[\alpha] \qquad \text{with} \qquad Z[\alpha] := \widetilde{\theta}^A(\alpha_i)\,U_A{}^B[\alpha]\,\widetilde{\theta}_B(\alpha_f), \tag{7}$$

when α shrinks down to the point x.

† Notice that the two fermionic mass terms are non-vanishing because the *2*-spinor field $\eta^A(x)$ is *Grassmann* valued (e.g. $\eta_A\eta^A = -2\eta^0\eta^1$); as opposed to the incorrect remark in [16].

Based on the loop transform of [15], the action of the (non-regularized) operators (3) and (7) can be defined on loop states as

$$\widehat{Z}[\alpha]\,\Psi\,[\beta] = \delta^3(\alpha_f,\beta_i)\,\delta^3(\alpha_i,\beta_f)\,\Psi[\alpha\cdot\beta] + \delta^3(\alpha_f,\beta_f)\,\delta^3(\alpha_i,\beta_i)\,\Psi[\alpha\cdot\beta^{-1}], \tag{8}$$

$$\begin{aligned}\widehat{V}^{abc}[\gamma,\alpha]\Psi[\beta] = &\sum_{\mu=\pm1}\sum_{j=1}^{8}\sum_{i=1}^{6}\left(\frac{1}{\sqrt{2}}\right)^3\\ &\times\left(\Delta^a[\alpha\cdot\gamma^\mu(s^*),\beta]\Delta^b[\alpha\cdot\gamma^\mu(t^*),\beta]\Delta^c[\alpha\cdot\gamma^\mu(r^*),\beta]\right)_i\\ &\times(-1)^{r_{ij}}c_{ij}(-1)^{(1-\mu)/2}\Psi[(\alpha\cdot\gamma^\mu\cdot\beta)_{ij}]\,,\end{aligned} \tag{9}$$

$$\Delta^a[\alpha\cdot\gamma^\mu(s^*),\beta] = \tfrac{1}{2}\int_0^1 \mathrm{d}u\,\dot{\beta}^a(u)\delta^3(\alpha\cdot\gamma^\mu(s^*),\beta(u))\,.$$

$\widehat{V}^{abc}[\gamma,\alpha]$ produces 16 multiloop states† (indices μ and j) for each exclusive configuration labelled by i. In other words, $(\Delta^a[\alpha\cdot\gamma^\mu(s),\beta]\Delta^a[\alpha\cdot\gamma^\mu(t),\beta]\Delta^a[\alpha\cdot\gamma^\mu(r),\beta])_i$ represent the six different ways in which the open loop β is attached to the open loop $\alpha\cdot\gamma^\mu$. $\Psi[(\alpha\cdot\gamma^\mu\cdot\beta)_{ij}]$ denote the multiloop states resulting from rerouting $\alpha\cdot\gamma^\mu$ and β. r_{ij} is the number of orientation-reversed segments of any $\alpha\cdot\gamma^\mu$ or β-loop segments between intersections required to get a consistent overall orientation, while the parameter c_{ij} is such that $c_{ij}=-1$ if the multiloop $\Psi[(\alpha\cdot\gamma^\mu\cdot\beta)_{ij}]$ has an open component starting at α_i and ending at β_f; otherwise $c_{ij}=+1$.

Regularizing the mass terms $\mathcal{M}_1$ and $\mathcal{M}_2$ amounts to regularizing (8) and (9). To this aim, use is made of an auxiliary background flat metric and a preferred set of coordinates in which this metric is Euclidean. We take the partition of the three-dimensional fictitious space in cubes of sides L. The regularized quantum versions of the mass operators are readily found by introducing [1, 15]

$$\widehat{\mathcal{M}} := \lim_{L,\xi\to0}\sum_I L^3\sqrt{-\widehat{\mathcal{M}}^L_{1\,I}-\widehat{\mathcal{M}}^{L,\xi}_{2\,I}}\,. \tag{10}$$

We describe $\widehat{\mathcal{M}}^L_{1\,I}$ ($\widehat{\mathcal{M}}^{L,\xi}_{2\,I}$ will be analysed below)

$$\begin{aligned}\widehat{\mathcal{M}}^L_{1\,I} &:= \frac{\sqrt{2}m}{8(3!)L^6}\int_{\partial I}\mathrm{d}\sigma^2\int_{\partial I}\mathrm{d}\tau^2\int_{\partial I}\mathrm{d}\rho^2\,(-1)^{r_a+r_b+r_c}\eta^{abc}n_a(\sigma)n_b(\tau)n_c(\rho)\widehat{V}^{abc}[\gamma,\alpha],\\ &= \frac{\sqrt{2}m}{8(3!)L^6}\int_{\partial I}\mathrm{d}\sigma^2\int_{\partial I}\mathrm{d}\tau^2\int_{\partial I}\mathrm{d}\rho^2\,(-1)^{r_a+r_b+r_c}\eta^{abc}n_a(\sigma)n_b(\tau)n_c(\rho)\left\{\widehat{N}^{abc}-\widehat{N}^{cba}\right\},\end{aligned} \tag{11}$$

where one particular box has been considered (in the fictitious metric). ∂I indicates its boundary, namely the union of the six faces of the cube oriented outwards and γ and α are, respectively, closed and open with a common point of intersection. The closed loop γ has three gravitational hands inserted, lying on the boundary of the box at the points σ, τ, and ρ. The loop γ is the triangle formed by the three segments that connect the three points σ, τ and ρ and $\gamma(s)=\sigma$, $\gamma(t)=\tau$, $\gamma(r)=\rho$. The open loop has two fermions at its ends,

† Of these 16 contributions, eight come from attaching the open loop β to the loop $\alpha\cdot\gamma$ and the other eight by attaching the open loop β to $\alpha\cdot\gamma^{-1}$.

$\eta(\alpha_i)$ and $\eta(\alpha_f)$, respectively. These fermions are in the box; n_a is the normal 1-form to the box boundary. No summation convention is applied to r_a and η^{abc}; $r_a = 0$ at the front and $r_a = 1$ at the back of the boundary.

The action of $\widehat{\mathcal{M}}^L_{1\,I}$ on the loop states is as follows. The three surface integrals on the boundary of the box I and the three line integrals along the loop β that parametrize the loop state, combine to give three numbers related to the intersections of the open loop with the boundary of the cube. The non-vanishing contribution can be traced back to the intersection of the open loop β simultaneously with three different faces of the cube. That is to say when the open loop β has *at least* a triple point of intersection with the cube. The cube shrinks down to that point in the limit $L \to 0$. Thereby the gravitational hands are smeared on the boundary of the cube, for each permutation of them there are 8×16 terms which correspond to all the possible ways in which the hands can lie on the faces of the cube. Each one of these terms has, in general, a different weight for two reasons: the first is due to the orientation of the open loop β when it intersects one of the three faces of the cube. The second comes from the factor $+1$ or -1, depending on whether the hands lie at the front or back faces. Note that of these 8×16 terms only one particular permutation of the hands enters once: 16 of them contribute for a specific loop β, since each hand intersects the open loop once. By taking into account the six permutations of the hands there are, for a specific loop, $16 \times 3!$ terms and for a general situation $8 \times 16 \times 3!$ terms.

It is important to mention that the prescription given above depends in a way on the open loop β labelling the loop state. More precisely, one needs to know some topological information (intersections and end points) about β in order to calculate its specific contribution. Observe that the prefactor in (11) is finite in the $L \to 0$ limit because the surface integrals produce a factor L^6 that cancels out the one in the denominator. Hence one gets a finite action for the operator. This is analogous to the case of the volume operator [3].

Due to the fact that $\widehat{\mathcal{M}}^L_{1\,I}$ has only gravitational hands, it can even 'see' loop states without fermionic excitations, namely loop states parametrized by closed loops with at least a triple point of intersection. This is precisely the difference from the kinetic fermion term [15] (i.e. $\mathcal{H}_{\text{Weyl}}$ above) which is 'blind' to this type of loop state (it yields zero on such states). Hence, the interpretation proposed here for $\widehat{\mathcal{M}}_1$ is that it forms the *gap fermion mass*. This is the major result presented here.

For the case of $\widehat{\mathcal{M}}_2$, and hence $\widehat{Z}[\alpha]$, consider $(\alpha_{\boldsymbol{x},\boldsymbol{y}})(s)$, a straight line (in the background metric) that starts at $\boldsymbol{x}$ and points in the $\boldsymbol{y}$ direction

$$(\alpha_{\boldsymbol{x},\boldsymbol{y}})(s) = \boldsymbol{x} + s\boldsymbol{y}, \qquad (\alpha_{\boldsymbol{x},\boldsymbol{y}})(0) = \boldsymbol{x}, \qquad (\alpha_{\boldsymbol{x},\boldsymbol{y}})(1) = \boldsymbol{x} + \boldsymbol{y}\,. \tag{12}$$

Then define the following

$$\widehat{\mathcal{M}}^{L,\xi}_{2\,I} := \frac{1}{L^3}\int_I \mathrm{d}^3x\, \widehat{\mathcal{M}}^{\xi}_2(\boldsymbol{x}), \tag{13}$$

$$\widehat{\mathcal{M}}^{\xi}_2(\boldsymbol{x}) := \frac{-\frac{1}{4}m_D}{\frac{4}{3}\pi\xi^3}\int \mathrm{d}^3y\,\theta(\xi - |\boldsymbol{y}|)\,\widehat{Z}[\alpha_{\boldsymbol{x},\boldsymbol{y}}], \tag{14}$$

where $\xi < L$ and θ is the step function.

Now, if the end points of the open loop β, β_i and β_f, coincide with the end points of the open loop α inside the ball centred at $\boldsymbol{x}$ and with radius ξ, one has

$$\lim_{L,\xi\to 0}\sum_I L^3\sqrt{-\widehat{\mathcal{M}}^{L,\xi}_{2\,I}}\,\Psi\,[\beta] = \lim_{L\to 0,\xi\to 0}\sqrt{\frac{\frac{1}{4}m}{\frac{4}{3}\pi\xi^3}\,L^3\,(\widehat{\mathcal{F}}_{\beta_i}+\widehat{\mathcal{F}}_{\beta_f})^{1/2}}\,\Psi\,[\beta], \tag{15}$$

$$\widehat{\mathcal{F}}_e\,\Psi\,[\beta] = \begin{cases}\Psi\,[\alpha_{\boldsymbol{x},\beta_i-\boldsymbol{x}}\cdot\beta] & \text{if } e=\beta_f\\ \Psi\,[\alpha_{\boldsymbol{x},\beta_f-\boldsymbol{x}}\cdot\beta^{-1}] & \text{if } e=\beta_i\,.\end{cases} \tag{16}$$

The regularization parameters can be chosen as† $L(\epsilon)=b\epsilon$ and $\xi(\epsilon)=b\,\sin\epsilon$, where b is an arbitrary length. Remarkably, the prefactor in (15) is finite and is given by

$$\lim_{\epsilon\to 0} C(L(\epsilon),\xi(\epsilon)) = \sqrt{\frac{\frac{1}{4}m}{\frac{4}{3}\pi\xi^3}L^3} = \sqrt{\frac{3m}{8\pi}}. \tag{17}$$

In this way, in contrast to $\widehat{\mathcal{M}}_1$, only loops with pairs of coinciding point-like fermion excitations contribute to $\widehat{\mathcal{M}}_2$, which is quadratic in the fermion momentum variables. In this respect, it is rather similar to the kinetic energy fermion contribution to the Hamiltonian constraint $\mathcal{H}_{\text{Weyl}}$. However, since it modifies the fermion mass, a 'dressing' interpretation seems more appropriate.

In summary, the Majorana-type mass for fermions has been studied in the loop representation of non-perturbative quantum gravity and fermions. There are two contributions, one of which resembles a mass gap, whereas the other seems to dress the corresponding mass. The former is non-zero, even for loop states lacking fermion excitations, which contain at least triple intersections. The latter requires the presence of coinciding pairs of end points characterizing the loop states. Setting the Einstein and kinetic fermion terms to zero, the Majorana mass operator turns out to be

$$\widehat{M}_{\text{Majorana}} = \sum_{\text{i,e}}\sqrt{\widehat{\mathcal{M}}^{(\text{i})}_1+\widehat{\mathcal{M}}^{(\text{e})}_2} \tag{18}$$

where the sum runs over (at least triple) intersections i of the loop states (with and without fermionic excitations) and end points e for open loops with pairs of coinciding point-like fermionic excitations. The limit in which the regularization parameters go to zero is understood on the r.h.s. of (18). Some further comments are in order.

Topology of space. Recently Smolin put forward evidence for the equivalence between minimalist quantum wormholes (i.e. the identification of pairs of space points) without matter and quantum Einstein–Weyl theory expressed in loop variables [23]. In this picture, the fermionic character of the Weyl field is associated with the antisymmetrization of the mouths of the wormholes. In this way the fermionic matter gets encoded in the topological properties of the space. Also, non-minimalist wormholes (that is smooth manifolds) can be considered as having the results of the minimalist ones as their low-energy limit [23]. This in turn suggests a scenario where a Weyl field living on a space foam, like the one considered by Friedman *et al* [12], yields an effective theory in which the fermion field becomes massive! Nevertheless, the analysis of [12] relies on a perturbative approach, which it is better to avoid in the loop representation. A way out consists in following Smolin's strategy of studying the equivalence of the Einstein–Majorana theory, as given in the present work, to non-minimalist quantum wormholes. Further work is needed to settle

† This choice corrects the one in [15].

the issue of generating fermion masses from the topology of the space in the context of non-perturbative quantum gravity.

A mechanism of mass generation would, of course, allow one to calculate the values of the masses, but realistic values, i.e. related to nature, might well only come from the incorporation of the other non-gravitational fundamental interactions. This possibility was left open and some steps are in progress [24] along the lines presented here combined with those of [13]. Also, a massive Dirac field is currently under study.

Reality conditions and spin networks. Relying on the Ashtekar approach for gravity and spin-$\frac{1}{2}$ fields involves two complex local degrees of freedom for the gravitational field unless reality conditions are supplemented [20]. Of course, the question remains open as to whether such reality conditions will single out the correct inner product at the quantum level. Nevertheless, the present analysis is expected to be robust enough to encompass real variables along the lines of [21]. This will be possible after extending the spin-network framework to include spin-$\frac{1}{2}$ fields, as in [22].

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