*General Relativity and Gravitation, Vol. 30, No. 1, 1998*

# Can Dilaton Fields in Astrophysical Objects be Detected?

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*Received September 26, 1996. Rev. version May 14, 1997*

We analyze a new class of static exact solutions of Einstein–Maxwell–Dilaton gravity with arbitrary scalar coupling constant $\alpha$, representing a gravitational body endowed with electromagnetic dipole moment. This class possesses mass, dipole and scalar charge parameters. A discussion of the geodesic motion shows that the scalar field interaction is so weak that it cannot be measured in gravitational fields like the sun, but it could perhaps be detected in gravitational fields like pulsars. The scalar force can be attractive or repulsive. This gives rise to the hypothesis that the magnetic field of some astrophysical objects could be fundamental.

KEY WORDS : Einstein–Maxwell–Dilaton gravity

A great number of astrophysical objects in the cosmos are gravitational bodies with magnetic dipole fields. One would suppose that the Einstein–Maxwell (EM) theory predicts the existence of gravitational objects endowed with magnetic dipoles. In fact there is a set of exact solutions of the EM equations representing exterior fields of gravitational objects endowed with magnetic dipoles [1]. Some of them are reasonably small, but they do not give the correct behavior of the gravitational field far away from the sources; the other ones are acceptable in their behavior at infinity [2], but the number of terms of them is so enormous that it makes them

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unmanageable. On the other hand, EM theory actually is not a unification theory, but rather a superposition one, Einstein-plus-Maxwell. Here the electromagnetic field appears as the energy-momentum tensor (there is in fact no explanation of its existence) and the electromagnetic field appears as a model. For other theories like Kaluza–Klein (KK) and Low Energy Superstring (LESS) theories, the electromagnetic field is a component of a more general field, and the existence of gravitation and electromagnetism follows from its decomposition. In these theories the electromagnetic field is a consequence of a more general unified field, and is not a model. In [3] and [4] it is shown that the existence of electromagnetic dipoles is natural for LESS and KK but not so natural for EM. A class of solutions given in [3] possesses a gravitational field with the behavior of the Schwarzschild solution coupled with a magnetic dipole. They are reasonably small, but they possess a scalar field interaction, the so-called dilaton. Of course we have not observed astrophysical objects with a scalar field interaction, but its prediction in KK and LESS theories should be established at classical level if such theories are to be taken as realistic. In fact there are enough classical objects in nature with manifest gravitational-electromagnetical interactions. KK and LESS predict the existence of the dilaton at this level. In this work we will show that the dilaton interaction cannot be measured in weak gravitational fields like that of the sun, even if the sun did possess one, but it will be perhaps possible to measure it in stronger gravitational fields like that of a pulsar. According to KK and LESS theories, since these solutions posses a magnetic dipole moment parameter and a newtonian behavior at infinity, this gives rise to the hypothesis that the magnetic field of some astrophysical objects could be of fundamental origin, *i.e.*, the magnetic field could be a consequence of a more general scalar-gravito-electromagnetic field.

In a previous work [3] we presented a method for finding exact solutions of the KK field equations. These solutions represent exterior fields of a gravitational body, endowed with arbitrary electromagnetic fields such as monopoles, dipoles, etc. or the superposition of them, from the five-dimensional point of view. Here there exists a coupling between the electromagnetic and a scalar field, parametrized by a coupling constant $\alpha^2 = 3$. In another work [4] we generalized this method for arbitrary $\alpha$ in order to incorporate all the most important theories unifying gravitation and electromagnetism; KK, LESS and EM. In the present work we analyze explicit solutions of the field equations of the Lagrangian

$$\mathcal{L} = \sqrt{-g}[-R + 2(\nabla\Phi)^2 + e^{-2\alpha\Phi}F^2] \tag{1}$$

obtained by this method. In (1) $R$ is the curvature scalar, $F$ is the Faraday tensor, and $\Phi$ is the scalar field, the dilaton. The coupling between the dilaton and the electromagnetic field is parametrized by $\alpha$. If $\alpha = 0$, (1) is the Lagrangian of the EM theory, if $\alpha = 1$, (1) is the Lagrangian of the LESS theory and for $\alpha^2 = 3$, (1) is the Lagrangian of the KK theory.

The class of solutions we want to deal with in this work, written in Boyer–Lindquist coordinates, reads [3,4]

$$ds^2 = e^{2(k_s + k_e)} g^{\gamma} \frac{dr^2}{1 - (2m/r)} + g^{\gamma} r^2 (e^{2(k_s + k_e)} d\theta^2 + \sin^2\theta \, d\varphi^2) - \frac{1 - (2m/r)}{g^{\gamma}} dt^2,$$

$$A_{3,\zeta} = Q\rho\tau_{,\zeta}, \qquad A_{3,\bar{\zeta}} = -Q\rho\tau_{,\bar{\zeta}}, \qquad e^{-2\alpha\Phi} = \frac{\kappa_0^2 e^{\tau_0 \tau}}{(1 - (2m/r))g^{\beta}} . \qquad (2)$$

This class of solutions can be divided into two subclasses, the subclass (a),

$$g = a_1 e^{q_1 \tau} + a_2 e^{q_2 \tau},$$

$$k_{s,\zeta} = \frac{\rho}{2\alpha^2}(\lambda_{,\zeta} - \tau_0 \tau_{,\zeta})^2, \qquad k_{e,\zeta} = -\rho\gamma q_1 q_2 (\tau_{\zeta})^2, \qquad \tau_0 = q_1 + q_2,$$

and the subclass (b),

$$g = a_1 \tau + 1, \qquad k_{s,\zeta} = \frac{\rho}{2\alpha^2}(\lambda_{,\zeta})^2, \qquad k_e = 0, \qquad \tau_0 = 0,$$

where $\zeta = \rho + i\,z = \sqrt{r^2 - 2mr}\,\sin\theta + i\,(r - m)\cos\theta$. $\mathbf{A} = A_i dx^i$, $i = 1...4$ is the electromagnetic four potential, $m$ the mass parameter, $\gamma = 2/(1 + \alpha^2)$, $\beta = (2\alpha^2)/(1 + \alpha^2)$; $Q$, $a_1 + a_2 = 1$, $q_1$ and $q_2$ are constants subjected to the restrictions

$$2\gamma a_1 a_2 (q_1 - q_2)^2 + \kappa_0^2 Q^2 = 0$$

for the subclass (a), and

$$2\gamma a_1^2 - \kappa_0^2 Q^2 = 0$$

for the subclass (b). The class of solutions (2) can be interpreted as a magnetized Schwarzschild solution in dilaton gravity for $\alpha \neq 0$, while for $\alpha = 0$ the construction of dipoles is different and the form of the metric is not more similar to the Schwarzschild solution [4]. In the following we will assume $\alpha \neq 0$. $\lambda$ and $\tau$ are harmonic maps in a two-dimensional flat space, i.e., they are solutions of the Laplace equation

$$(\rho\lambda_{,\zeta})_{,\bar{\zeta}} + (\rho\lambda_{,\bar{\zeta}})_{,\zeta} = 0, \qquad (\rho\tau_{,\zeta})_{,\bar{\zeta}} + (\rho\tau_{,\bar{\zeta}})_{,\zeta} = 0 . \qquad (3)$$

In this work we have fixed $\lambda = \ln(1 - (2m/r))$. $\tau$ determines uniquely the electromagnetic potential. Two examples are the magnetic monopole

$$\tau = \ln\left(1 - \frac{2m}{r}\right), \qquad A_3 = 2mQ(1 - \cos\theta), \tag{4}$$

and the magnetic dipole

$$\tau = \frac{\cos\theta}{(r-m)^2 - m^2\cos^2\theta}, \qquad A_3 = \frac{Q(r-m)\sin^2\theta}{(r-m)^2 - m^2\cos^2\theta}. \tag{5}$$

Nevertheless, we can substitute an arbitrary electromagnetic field in (2); eqs. (4) and (5) correspond to the two first spherical harmonics, solutions of the Laplace equation (3). If $\tau = 0$, (2) reduces to the Schwarzschild space time coupled to the scalar field $\Phi$, which is manifested only through $k_s$. We interpret the function g as the contribution of the electromagnetic field to the curvature of the space time.

If $g \longrightarrow 1$, and $k_s + k_e \longrightarrow 0$ for $r \longrightarrow \infty$, the solutions are asymptotically flat, and they are flat for $m = Q = 0$, at least for the examples given in (4) and (5). A general study of the solutions contained in (2) will be given elsewhere [5].

In this work we are interested in extracting some physics from dilaton theories. In order to do so, we study the geodesic motion of test particles traveling around the space time (2). Since $e^{2(k_s+k_e)} - 1 \sim 10^{-11}$ for a star like the sun, metric (2) is spherically symmetric in this aproximation. We start from the Lagrangian

$$\mathcal{L} = e^{2(k_s+k_e)} g^\gamma \frac{(dr/ds)^2}{1-(2m/r)} + g^\gamma r^2\left(\frac{d\varphi}{ds}\right)^2 - \frac{1-(2m/r)}{g^\gamma}\left(\frac{dt}{ds}\right)^2, \tag{6}$$

where $s$ is the proper time of the test particle. In (6) we have set $\theta = \pi/2$; in this case the function $\tau$ for the dipole field does not contribute and g= 1. But in general, for any value of $\theta$, the function g changes only very near to the Schwarzschild radius $r_s = 2m$, but it tends very rapidly to one far away from $r_s$, for any value of $\theta$. In any case, in the following we will set g in all the equations where it appears. Following any standard text book on gravitation, we first write the motion equations. We have two constants of motion,

$$\frac{\delta\mathcal{L}}{\delta t} = 0 \Rightarrow \frac{1-\frac{2m}{r}}{g^\gamma}\left(\frac{dt}{ds}\right) = A$$

$$\frac{\delta\mathcal{L}}{\delta\varphi} = 0 \Rightarrow g^\gamma r^2\frac{d\varphi}{ds} = B$$

so $dt/ds$ and $d\varphi/ds$ can be put in terms of $A$ and $B$. Using the equation of motion

$$P_\mu P^\mu = -c^2$$

one obtains

$$-\varepsilon = e^{2(k_e+k_s)}\, g^\gamma \frac{(dr/ds)^2}{1-(2m/r)} + g^\gamma r^2\left(\frac{d\varphi}{ds}\right)^2 - \frac{1-(2m/r)}{g^\gamma}\left(\frac{dt}{ds}\right)^2, \quad (7)$$

where $\varepsilon = c^2, 0, -c^2$ for particles, photons and tachyons respectively. We rewrite eq. (7) in the more familiar form

$$\left(\frac{dr}{ds}\right)^2 + \frac{e^{-2(k_s+k_e)}}{g^\gamma}\left[\frac{B^2}{r^2 g^\gamma} + \varepsilon\right]\left(1-\frac{2m}{r}\right) = e^{-2(k_s+k_e)} A^2. \quad (8)$$

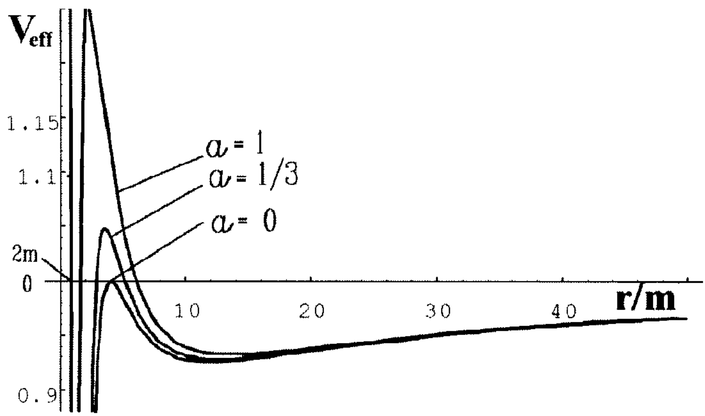


Figure 1. The effective potentials for the magnetized Schwarzschild solutions for the KK ($a = \frac{1}{3}$), LESS ($a = 1$) theories and the Schwarzschild solution ($a = 0$). The plot is drawn using $m$ units on the horizontal axis.

Here we have separated the part of the motion equation related to the constant $B$ from the part related to the constant $A$ obtained from the variation with respect to the coordinate $t$. Let us define an effective potential by

$$V_{\rm eff} = \frac{e^{-2(k_s+k_e)}}{2\, g^\gamma}\left[\frac{B^2}{r^2 g^\gamma} + \varepsilon\right]\left(1-\frac{2m}{r}\right) \quad (9)$$

and an effective energy by

$$E_{\rm eff} = \frac{1}{2} e^{-2(k_e+k_s)} A^2, \quad (10)$$

in order to obtain the familiar form for the motion equation

$$\frac{1}{2}\left(\frac{dr}{ds}\right)^2 + V_{\rm eff} = E_{\rm eff}\ .$$

This interpretation is suggested by performing a series expansion for $r \gg 2m$.

In the following we will take only the subcase (b) of (2), where the function $k_e = 0$ and the constant $\tau_0 = 0$ as well. If $\theta = \pi/2$, the effective potential $V_{\rm eff}$ and the effective energy $E_{\rm eff}$ reduce to

$$V_{\rm eff} = \left(\frac{(1-(m/r))^2}{1-(2m/r)}\right)^a\left(\frac{\varepsilon}{2} - \frac{m\varepsilon}{r} + \frac{B^2}{2r^2} - \frac{mB^2}{r^3}\right),$$

$$E_{\rm eff} = \left(\frac{(1-(m/r))^2}{1-(2m/r)}\right)^a\frac{A^2}{2},$$

where $a = 0$ for the Schwarzschild space time and $a = 1/\alpha^2$ for the dilatonic case. We interpret the factor $[(1-(2m/r))/(1-(m/r))^2]^a$ as the contribution of the dilaton field to the effective potential $V_{\rm eff}$ and to the effective energy $E_{\rm eff}$, and the function g as the contribution of the electromagnetic field. In Figure 1 we have plotted the effective potential for the different theories. The qualitative behavior is very similar in all of them. In Figure 2 we see the effective energy for the same values of $\alpha$; the behavior is here very violent; not so far away from the Schwarzschild radius, the effective energy is constant.

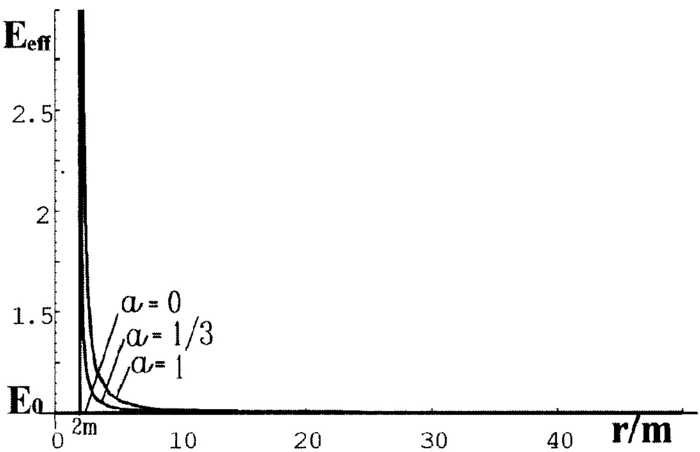


Figure 2. The effective energy for the magnetized Schwarzschild solutions. The plot is made in $m$ units on the horizontal axis and in $E_0$ units on the vertical axis.

In order to obtain the trajectories of a test particle travelling around a star of the sun's size, we make the standard transformation $u(\varphi) = 1/[r(\varphi(s))]$. The geodesic equation (8) transforms into

$$B^2(u')^2 + \left(\frac{(1-mu)^2}{1-2mu}\right)^a [(1-2mu)(B^2u^2+\varepsilon) - A^2] = 0, \qquad (11)$$

where a prime means derivative with respect to $\varphi$. This is a first-order differential equation of the form

$$\tfrac{1}{2}(u')^2 + V(u) = 0 \qquad (12)$$

which defines naturally the function $V(u)$. After derivation with respect to $\varphi$, eq. (12) transforms into a equation of the form $u'' + \partial_u V(u) = 0$. This differential equation is very difficult to solve and we will not try to solve it here. But for a trajectory around a star like the sun, the mass parameter $m \sim 1.5$ Km, while $r \sim 10^6$ Km. Therefore $u^3 \sim 0$ is a good approximation, conserving the rest of the terms. In that case the geodesic equation transforms into

$$u'' + \omega^2 u = \frac{m\varepsilon}{B^2} + 3mKu^2, \qquad (13)$$

where $\omega = \sqrt{1-(am^2/B^2)(A^2-\varepsilon)}$ and $K = 1+(am^2A^2/B^2)$. The difference with the Schwarzschild geodesic equation is that for the Schwarzschild case $\omega = K = 1$. Following the standard procedure, we find that the trajectories are ellipses with a perihelia precession given by

$$\Delta\varphi_p = 6\pi\frac{m^2c^2K}{B^2\omega^3} = \frac{6\pi m}{b(1-e^2)}\frac{K}{\omega}, \qquad (14)$$

where $b$ is the semimajor axis of the ellipse and $e$ is its eccentricity. Again the difference with the Schwarzschild solution is the $\omega$ and $K$ multiplying the perihelia precession of the Schwarzschild solution in (14). In the first approximation in $m$, there is no difference between eq. (13) and the one obtained from the Schwarzschild solution, since $\omega$ and $K$ depend only on $m^2$. Therefore there is no difference between (13) and the equation for the Schwarzschild solution for the calculation of null geodesics, since this is always made in the first approximation in $m$. For the calculation of the trajectories of particles, there is some difference only in the second approximation in $m$, given by $\omega$ and $K$. It can be checked numerically that for Mercury $A \sim c/\sqrt{1-\beta^2}$ where $\beta = v^2/c^2$, and then $A^2$ must be of order of the energy of the test particle at infinity, $A^2 \sim c^2$. The term $K - 1 \sim (am^2/\beta^2 r^2) \sim 2.6 \times 10^{-8} a$ and $\omega^2 - 1 = -(am^2/r^2) \sim -0.7 \times 10^{-16}\, a$ for Mercury. Here $G$ is the universal gravitation's constant, $B \sim vr/\sqrt{1-\beta^2} \sim 2.78 \times 10^{15}\ \mathrm{m^2/sec}$ is the angular momentum of Mercury per unity of mass and $M_{\odot}$ is the mass of the sun. This means that the difference between the Schwarzschild geodesics and the geodesics for stars like the sun [eq. (11)] is too small to be measured. Let us assume for a moment that we could take these theories as realistic. Then we conclude that if a star of the size of the sun contains a scalar field inherent in it, we would not know, because its interaction with the rest of the world is too small to be detected. Nevertheless, for a pulsar of mass $M = 2M_{\odot}$, which matter is typically contained in a radius of $r = 10$ Km$\sim 3m$, the scalar interaction cannot be neglected. Thus, such interactions should be detectable in stronger gravitational fields like pulsars, where the gravitational field is much stronger.

We have seen that the KK and the LESS theories naturally predict the existence of magnetic dipoles coupled with gravitational objects. Here, though, the electromagnetic field is a consequence of the natural coupling predicted by the theory. If we would like to model a pulsar by such a theory, we would not need to explain the origin of the magnetic dipole in it using an internal hypothesis, since this magnetic dipole would then be a consequence of some more general interaction between gravitation and electromagnetism. The price we must pay is the existence of a scalar field which has not been measured till now. Nevertheless, the KK and the LESS theories predict that even if the magnetic dipole field can be felt around the body, the scalar field interaction is so weak that it can be measured only near to a distance of order of the Schwarzschild radius $r_s$. This is so because of the behavior of the scalar field (see Fig. 3)

$$\Phi = \frac{1}{2\alpha}\left[\ln\left(1 - \frac{2m}{r}\right) + \beta \ln\left(\frac{Q\cos\theta}{(r-m)^2 - m^2\cos^2\theta} + 1\right)\right].$$

Near to $r_s$, $\Phi$ grows, but it is constant after a few times the value of $r_s$. Thus the scalar interaction vanishes very rapidly far away from a distance $r_s = 2m$, and it is attractive or repulsive depending on whether $\alpha$ is positive or negative. Hence, according to these theories, there exist objects which possess a fundamental magnetic dipole moment, which is a consequence of a more general gravito-electromagnetic interaction which posses a scalar field. Otherwise, according to these theories, even if an astrophysical object like the sun would posses a scalar field inherent in it, we would not be able to measure it because of the small force provoked by

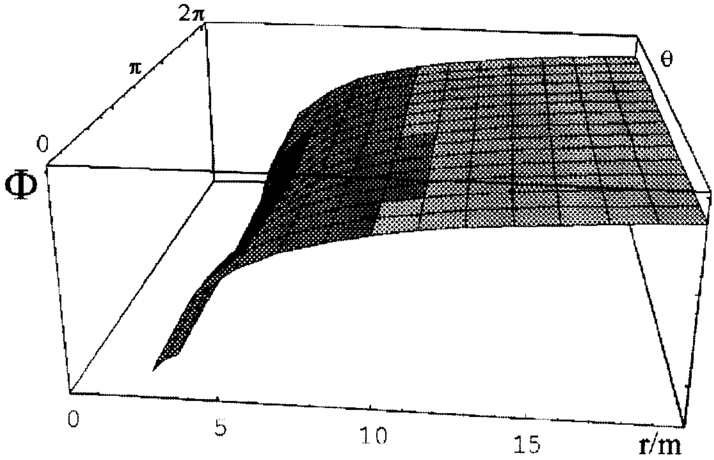


Figure 3. The scalar field (dilaton) for the magnetized Schwarzschild solutions. The plot is made in $m$ units on the horizontal axis.

it. Nevertheless, this attractive or repulsive scalar force could have effects in stronger gravitational fields that we should see in astrophysical bodies, but to predict them, we must solve the geodesic equation (8) near to $r_S$.

## ACKNOWLEDGEMENT

This work is partially supported by CONACYT-Mexico.

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