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INFLATION FROM AN EFFECTIVE STANDARD MODEL OF PARTICLE PHYSICS FOR CURVED SPACETIME

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Beginning from an effective theory in eight dimensions, in Ref. 1, Macias, Camacho and Matos proposed an effective model for the electroweak part of the Standard Model of particles in curved spacetime. Using this model, we investigate the cosmological consequences of the electroweak interaction in the early universe. We use the approximation that, near the Planck epoch, the Yang–Mills fields behave like a perfect fluid. Then we recover the field equations of inflationary cosmology, with the Higgs field directly related to the inflaton. We present some qualitative discussion about this and analyze the behavior of isospin space using some known exact solutions.

1. Introduction

Higher-dimensional theories are one of the most old and interesting unification theories considered by physicists and mathematicians unifying gravitation with Yang–Mills interactions. The history of these theories began with Theodore Kaluza and Oscar Klein, when they formulated a five-dimensional theory unifying gravitation and electromagnetism, but this theory used a great amount of hypothesis on the structure of spacetime and on the functional dependence of physical quantities. The first real geometrical formulation of these theories was given by Kerner[2] and Cho[3] (see also Ref. 4). They formulated the theory as a principal fiber bundle, being the four-dimensional spacetime the basis space of the bundle, and the gauge group of the Yang–Mills fields they wanted to unify with gravitation, the typical fiber. This formulation pretended to be a fundamental theory of all interactions, because it unified all the Yang–Mills gauge theories with gravitation. Nevertheless, it is now believed that a superstrings theory could be the fundamental theory of all interactions, among other reasons, because it contains the Kaluza–Klein theories (KK) in its low energy limit. In this case, KK become effective theories for Yang–Mills and gravitational fields instead of fundamental ones.

In Ref. 1, Macias, Camacho and Matos (MCM) considered an effective eight-dimensional theory, supplemented with the Yukawa and the fermionic sectors as in the Standard Model and, as expected, the Weinberg, Glashow and Salam Model was recovered (see also Refs. 5 and 6). Then, some questions arose. How is the cosmology predicted by this Standard Model in curved spacetime? Is there inflation in this cosmology? Which scalar field plays the role of the inflaton? What is its relation with the Higgs field? In this paper we begin with the eight-dimensional Kaluza–Klein–Dirac theory of Ref. 1 in order to construct the cosmology of an effective particle theory for the electroweak Standard Model in a curved spacetime, and answer at least partially these questions.

Following Kerner[2] and Cho,[3] who established the appropiated general structure of the metric on the bundle in order to consider a non-Abelian Lie gauge Group G, MCM set up the metric

$$\begin{aligned} ds^2 &= g_{\hat{\mu}\hat{\nu}}\, dx^{\hat{\mu}} \otimes dx^{\hat{\nu}} \\ &= g_{\mu\nu}\, dx^{\mu} \otimes dx^{\nu} - I_1^2(x)[dx^5 + kA_\mu(x)dx^\mu] \\ &\quad \otimes [dx^5 + kA_\nu(x)dx^\nu] \\ &\quad - I^\dagger(x)I(x)\gamma_{ij}(y)\left[dy^i + \frac{k}{L}K^i_\alpha(y)A^\alpha_\mu(x)dx^\mu\right] \\ &\quad \otimes \left[dy^j + \frac{k}{L}K^j_\beta(y)A^\beta_\nu(x)dx^\nu\right], \end{aligned} \tag{1}$$

where $\gamma_{ij}(y)$ is the metric of the isospace manifold, $K^i_\alpha(y)$ are its Killing vectors, $A_\mu(x)$ and $A^\alpha_\mu(x)$ are the gauge potentials. MCM considered $\mathrm{SU}(2) \times \mathrm{U}(1)$ as a structure group of the eletroweak interactions. Here $I_1(x)$ plays the role of the dilaton of the theory and also gives the radius of the S^1 part of the isospace. The field $I(x)$ is endowed with a three-dimensional spinorial structure and plays the role of a Higgs field in the effective theory. This field enters into the metric through the scalar combination $I^\dagger(x)I(x)$ and is associated to the radius of the S^3 part of the isospace. In Eq. (1), k and L are scaling constants.

They used the following action

$$I_8 = \int d^4x\, d^4y \sqrt{-\hat{g}}\left(\frac{1}{16\pi GV}[\hat{R} + V + Y_u] + \mathcal{L}_D\right). \tag{2}$$

In Eq. (2), V is the Higgs potential and Y_u represents the terms for the Yukawa couplings. From this theory, they obtained the four-dimensional effective eletroweak part of the Standard Model. It was shown in Ref. 1 that the masses of the gauge bosons and of the first two fermion families were correctly given by this theory. Here, we are interested in studying the cosmological implications of this kind of approach. We can represent the matter content of the universe as a perfect fluid describing a highly energetic and isotropic radiation. Then, instead of the Dirac Lagrangian, $\mathcal{L}_D$, we will use a perfect fluid Lagrangian. We will also neglect the Yukawa interactions terms Y_μ.

Finally, we will assume that the metric $g_{\mu\nu}$ for the base spacetime $\mathcal{M}$ is given by the Robertson–Walker metric and that both the scalar $I_1(x)$ and spinorial $I(x)$ fields depend only on time, $x^0 = t$; otherwise, they could be used to distinguish between different directions in spacetime, invalidating our assumption of the isotropy of the early universe. In this way, all of the terms involving the gauge fields, A_μ, A_μ^α, do not have to be explicitly accounted for in writing the final equations of motion.

From this model we obtain that the inflaton σ is produced in a natural way as a combination of the scalar fields $I_1(t)$ and $I^\dagger(t)I(t)$; in this way, we asign a geometrical origin to the inflaton: it is related to the radius of the internal space, the isospin space. On the other hand, inflation takes place by a phase transition of the (electroweak) Higgs field, i.e., the scalar field that generates inflation and the scalar field that gives mass to particles are related. So, depending on the particular inflationary potential that is used, this model allows that these two roles, of the same field, may be played in two different steps. According to this model, a part of the internal space deflates while the spacetime inflates; after some time, the scalar field settles to a constant, while the spacetime keeps expanding. Then another question arises: how can the same scalar field be responsible for these two different phenomena at the origin of the Universe? In the concluding remarks we give a partial answer to this problem. Using the simplification of modelling the matter contribution as a perfect fluid, we obtain an approximation of the behavior of the internal space and the well-known evolution of spacetime.

2. Effective Four-Dimensional Action

We start with metric (1). Following MCM, (see also J. Toms in his review essay on the Kaluza–Klein theory,[7]) in order to easily calculate geometrical quantities of this metric, we change to a horizontal lift basis $\hat{e}_{\hat{\mu}}$ defined by the relations

$$\hat{e}_{\hat{\mu}} \equiv \begin{cases} e_\mu = \partial_\mu - kA_\mu\partial_5 - \dfrac{k}{L}A_\mu^\alpha K_\alpha^i\partial_i\,, \\ e_5 = \partial_5\,, \\ e_i = \partial_i\,. \end{cases} \tag{3}$$

Evidently, this basis is not holonomic; its commutation relations can be easily derived (see Ref. 4). One finds

$$\begin{aligned} [e_\mu, e_\nu] &= -kF_{\mu\nu}e_5 - \frac{k}{L}K_\alpha^i F_{\mu\nu}^\alpha e_i\,, \\ [e_\mu, e_i] &= \frac{k}{L}A_\mu^\alpha(\partial_i K_\alpha^j)e_j\,, \\ [e_\mu, e_5] &= 0\,, \end{aligned} \tag{4}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,,$$
$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + \frac{k}{L} f_{\beta\gamma}^{\ \ \alpha} A^\beta_\mu A^\gamma_\nu \tag{5}$$

are the field strengths for the potentials A_μ and A^α_μ, respectively. In this basis, the metric takes a diagonal form. Finally, the space Ricci scalar curvature reads

$$\begin{aligned}\hat{R} = R &- \frac{1}{I^\dagger I}\tilde{R} - \frac{1}{4}\left(\frac{k}{L}\right)^2 I^\dagger I \gamma_{ij} K^i_\alpha K^j_\beta F^{\alpha\mu\nu} F^\beta_{\mu\nu} \\ &- \frac{1}{4}k^2 I_1^2 F^{\mu\nu} F_{\mu\nu} + 3(\partial^\mu \log I^\dagger I)_{;\mu} \\ &+ 3\partial_\mu \log I^\dagger I \partial^\mu \log I^\dagger I + 2(\partial^\mu \log I_1)_{;\mu} \\ &+ 2\partial_\mu \log I_1 \partial^\mu \log I_1 + 3\partial_\mu \log I^\dagger I \partial^\mu \log I_1 \\ &+ \frac{3}{2} g^{\mu\nu}{}_{,\nu} \partial_\mu \log I^\dagger I + g^{\mu\nu}{}_{,\nu} \partial_\mu \log I_1 \,,\end{aligned} \tag{6}$$

where R is the scalar curvature of the spacetime manifold $\mathcal{M}$ and $\tilde{R}$, which is a constant in our case, is the scalar curvature of the isospace manifold $S^1 \times S^3$.

It should be noticed that we have used an effective version of Kaluza–Klein theory. It seems to us that this is the most appropiate way to include the Standard Model into a theory of gravitation (see Refs. 5 and 6).

Let us consider the following action defined in the eight-dimensional double fiber bundle space $\mathcal{P} \sim \mathcal{M} \times \mathrm{SU}(2) \times \mathrm{U}(1)$:

$$\mathcal{S} = \int d^4x\, d^4y \sqrt{-\hat{g}}\, \frac{1}{16\pi G V L_1} (\hat{R} + \hat{V}(I_1, I) + \hat{\mathcal{L}}) \,, \tag{7}$$

where we introduce L_1 on dimensional grounds. We begin by using the eight-dimensional theory with a potential of the form

$$\hat{V}(I_1, I) \equiv \hat{V}_1\left(\sqrt{\frac{3}{8\pi G}} \log I^\dagger I\right) + \hat{V}_2\left(-\sqrt{\frac{1}{6\pi G}} \log I_1\right) \tag{8}$$

and we include the Lagrangian $\hat{\mathcal{L}}$ to represent, when the dimensional compactification has been achieved, the matter content of the universe.

According to Eq. (1), the determinant of the metric, $\hat{g}$, is given by

$$\sqrt{-\hat{g}} = \sqrt{-g}\sqrt{\gamma} I_1 (I^\dagger I)^{3/2} \,, \tag{9}$$

where $g \equiv \det(g_{\mu\nu})$ and $\gamma \equiv \det(\gamma_{ij})$. The coupling between the curvature scalar and the fields $I_1(x)$ and $I^\dagger I(x)$ will lead us, after dimensional reduction, to a Brans–Dicke theory. We prefer to make a conformal transformation in order to avoid this coupling.

Let $f(x)$ be a function which depends only on spacetime coordinates of $\mathcal{M}$. We multiply the internal part of metric $\hat{g}_{\hat{\mu}\hat{\nu}}$, Eq. (1), by the function $f^2(x)$, so we can define new scale functions for the $S^1 \times S^3$ group manifold, by means of

$$K(x) \equiv f(x)I_1(x) \quad \text{and} F(x) \equiv f(x)I(x) . \tag{10}$$

We can obtain the Ricci curvature associated with this conformal transformed metric $\hat{g}'_{\hat{\mu}\hat{\nu}}$ by substituting $I_1 \to K$ and $I \to F$ in the expression previously found for Ricci scalar curvature, Eq. (6).

If we set $f = [I_1(I^\dagger I)^{3/2}]^{-\frac{1}{4}}$, we obtain that the determinat $\hat{g}'$ is given by the equation

$$\sqrt{-\hat{g}'} = \sqrt{-g}\sqrt{\gamma} . \tag{11}$$

The relations between the old and the new fields are

$$\begin{aligned} \log F^\dagger F &= -\frac{1}{2}\log I_1 + \frac{1}{4}\log I^\dagger I , \\ \log K &= \frac{3}{4}\log I_1 - \frac{3}{8}\log I^\dagger I . \end{aligned} \tag{12}$$

Let us now introduce the following useful definitions for the fields Φ and Ψ

$$\begin{aligned} \sqrt{\frac{3}{16}}\log I^\dagger I &= \frac{1}{\sqrt{2}}\Phi , \\ \sqrt{\frac{3}{4}}\log I_1 &= \frac{1}{\sqrt{2}}\Psi . \end{aligned} \tag{13}$$

We can further make the identification of the scalar field, σ, with a combination of the fields previously defined, and two additional definitions for an effective cosmological constant and for the dimensional factor k^2

$$\begin{aligned} \sigma &= \frac{\Phi - \Psi}{\sqrt{16\pi G}} , \\ \hat{\Lambda} &= \frac{1}{2}\tilde{R}\exp\left(-\sqrt{\frac{8\pi G}{3}}\sigma\right) , \\ k^2 &\to 16\pi G . \end{aligned} \tag{14}$$

We finally get an effective four-dimensional spacetime action, i.e.,

$$\begin{aligned} \mathcal{A} = \int d^4x\sqrt{-g}\Big\{ &\frac{1}{16\pi G}[R - 2\hat{\Lambda}] - \frac{1}{4}\exp\left(-\sqrt{24\pi G}\sigma\right)F_{\mu\nu}F^{\mu\nu} \\ &- \frac{1}{4}\exp\left(\sqrt{\frac{8\pi G}{3}}\sigma\right)F^\alpha_{\mu\nu}F^{\alpha\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma) + \mathcal{L}_{\text{matter}}\Big\} . \end{aligned} \tag{15}$$

Then, we are left with an Einstein–Hilbert–Yang–Mills effective action, which presents an interaction between the scalar field and the Yang–Mills fields. This action is the main result of this section.

It should be noticed that two surface terms have been dropped out in obtaining the effective action (15). This can actually be done by imposing the following boundary conditions

$$\begin{aligned} &\lim_{x\to\infty} \partial^\mu \log K \to 0\,, \\ &\lim_{x\to\infty} \partial^\mu \log F^\dagger F \to 0\,. \end{aligned} \tag{16}$$

Summarizing, the process we have followed to get an effective four-dimensional action $\mathcal{A}$ from the eight-dimensional action $\mathcal{S}$, is

$$\mathcal{S} \overset{\text{conf. trans.}}{\longrightarrow} \mathcal{S}' \overset{\text{integration}}{\longrightarrow} \mathcal{A} \tag{17}$$

which is usually called a dimensional compactification procedure by isometry, see Ref. 8.

3. Equations of Motion and Some Examples

It is straightforward to get the equation of motion for the different fields of this theory from the effective four-dimensional action $\mathcal{A}$ of Eq.(15). The corresponding Einstein equations read

$$\begin{aligned} G_{\mu\nu} = -8\pi G \Big\{ &\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2} g_{\mu\nu}\partial_\rho\sigma\partial^\rho\sigma + g_{\mu\nu} V_{\text{eff}}(\sigma) \\ &- \exp\left(-\sqrt{24\pi G}\,\sigma\right)\left(F^\rho_\mu F_{\nu\rho} - \frac{1}{4} g_{\mu\nu} F^{\tau\rho} F_{\tau\rho}\right) \\ &- \exp\left(\sqrt{\frac{8\pi G}{3}}\,\sigma\right)\left(F^{\alpha\rho}_\mu F^\alpha_{\nu\rho} - \frac{1}{4} g_{\mu\nu} F^{\alpha\tau\rho} F_{\alpha\tau\rho}\right) + T_{\mu\nu} \Big\}\,, \end{aligned} \tag{18}$$

where we have made use of the following definition

$$V_{\text{eff}}(\sigma) \equiv V(\sigma) + \frac{\hat\Lambda(\sigma)}{8\pi G} \tag{19}$$

and $T_{\mu\nu}$ stands for the energy–momentum tensor of the matter content of the universe.

In turn, the equation for the scalar field σ becomes

$$\begin{aligned} &\partial_\mu\partial^\mu\sigma + \Gamma^\lambda_{\mu\lambda}\partial^\mu\sigma + \frac{\partial V_{\text{eff}}}{\partial\sigma} - \frac{1}{4}\sqrt{24\pi G}\exp\left(-\sqrt{24\pi G}\,\sigma\right) F^{\mu\nu} F_{\mu\nu} \\ &\quad + \frac{1}{4}\sqrt{\frac{8\pi G}{3}}\exp\left(\sqrt{\frac{8\pi G}{3}}\sigma\right) F^{\alpha\mu\nu} F_{\alpha\mu\nu} = 0\,. \end{aligned} \tag{20}$$

Finally the equations for the Yang–Mills fields are

$$\partial_\nu F^{\mu\nu} + \Gamma^\lambda_{\nu\lambda} F^{\mu\nu} - \sqrt{24\pi G}(\partial_\nu\sigma) F^{\mu\nu} = 0 \tag{21}$$

and

$$\partial_\nu F^{\mu\nu}_{\alpha} + \Gamma^{\lambda}_{\nu\lambda} F^{\mu\nu}_{\alpha} + \sqrt{\frac{8\pi G}{3}} (\partial_\nu \sigma) F^{\mu\nu}_{\alpha} - \frac{k}{L} \delta_{\gamma\beta} f^{\gamma}_{\alpha\delta} F^{\beta\mu\sigma} A^{\delta}_{\sigma} = 0 . \tag{22}$$

There is, additionally, a conservation equation $G^{\mu\nu}{}_{;\mu} = 0$.

It is interesting to note the role played in this system of equations by the isospace scale factors K^2 and $F^\dagger F$ of the conformally transformed metric $\hat{g}'_{\hat{\mu}\hat{\nu}}$. They appear explicitly through the combination

$$F^\dagger F = \exp\left(\sqrt{\frac{8\pi G}{3}}\,\sigma\right) , \qquad K^2 = \exp\left(-\sqrt{24\pi G}\,\sigma\right) , \tag{23}$$

which can be deduced at once from Eqs. (12)–(14). We see, in Einstein's equations (18) that K^2 appears as an effective coupling constant for the electromagnetic field. Analogously, the scale factor $F^\dagger F$ appears in front of the Yang–Mills energy–momentum tensor density. Both these factors appear again in the scalar field equation of motion, Eq. (20).

It should be noted that the scale factors K^2 and $F^\dagger F$ are not independent but are related through the equation

$$(F^\dagger F)^3 K^2 = 1 . \tag{24}$$

It is also interesting that the scalar field σ should satisfy the boundary condition

$$\lim_{x\to\infty} \partial_\mu \sigma \to 0 , \tag{25}$$

which is a consequence of the boundary condition we imposed on the fields K^2 and $F^\dagger F$ of Eq. (16). This condition translates in a limitation on the asymptotic behavior of the scalar field σ and therefore, on the whole theory we are constructing.

Let us consider a very special case of the system of equations derived from the effective four-dimesional action $\mathcal{A}$, Eq. (15). If we consider that the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ of Eq. (1), describes the very early universe, then we can assume the following statements:

(1) We can represent the matter content of the universe as a perfect fluid of a highly energetic and isotropic radiation, which presents no interaction. The energy momentum tensor associated with the Lagrangian $\mathcal{L}_{\text{matter}}$ is then given by $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$.
(2) In this way, all the terms involving the gauge fields, A_μ, A^α_μ do not have to be taken explicitly into account.
(3) It is assumed that the universe was isotropical in its early stages of evolution. Consequently, the metric $g_{\mu\nu}$ for the base space $\mathcal{M}$ is taken as the Robertson–Walker metric and both the scalar $I_1(x)$ and spinorial $I(x)$ fields are taken to depend only on time.

In this case, the system of differential equations, Eqs. (18)–(22) becomes

$$
\begin{aligned}
&3\left(\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}\right)=8\pi G\left(\rho+\frac{1}{2}\dot{\sigma}^2+V_{\text{eff}}(\sigma)\right),\\
&\ddot{\sigma}+3\frac{\dot{R}}{R}\dot{\sigma}+\frac{dV_{\text{eff}}}{d\sigma}=0\,,\\
&\dot{\rho}+3\frac{\dot{R}}{R}(\rho+p)=0\,,\\
&p=\omega\rho\,,
\end{aligned}
\tag{26}
$$

which is clearly the system of equations for the standard cosmological model filled with a perfect fluid and endowed with a scalar field.

The potential term V_{eff} has been taken arbitrarily until now but, with an appropriate choice, this system of equations can lead us to inflationary solutions; in this sense, we can argue that the scalar field σ may play the role of the inflaton of the very early universe.

There exist well known inflationary solutions for this system of equations (see for e.g. Ref. 9 for a review of the inflationary paradigm). Most of such inflationay solutions are in agreement with the boundary conditions we imposed for the inflaton σ in Eq. (25). What this condition means concerning the inflaton is that, after a long enough time, say t_0, the inflaton settles down to its *vacuum value* σ_0.

Let us follow the evolution of the internal space resulting from some specific behavior of the scalar field σ. We choose a couple of exact inflationary solutions for the system of Eqs. (26), which were found in Ref. 10 (see also, for e.g. Refs. 11 and 12 for a singular-free cosmology). These solutions are not realistic since they do not describe radiation or matter dominated epochs, but have the advantage that they are exact and follow the evolution of the scalar field through several epochs of expansion of a multi component universe. The first case corresponds to the potential $V_{\text{eff}}=V(\sigma)=\lambda(\sigma^2+\delta^2)^2$, with a negative cosmological constant. The solution describes a closed universe, with a scale factor given by $a(t)=a_0\sin^2(\sqrt{\lambda}\delta t)$. The exact solution for the scalar field is $\sigma(t)=\delta\cot(\sqrt{\lambda}\delta t)$ (see Fig. 1). When t goes to zero, the scalar field σ_1 goes to infinity, then the universe inflates and the scalar field experiences a rapid decay followed by an epoch of slow variation, during the standard decelarated expansion. The behavior of the isospace scale factors is: $F^\dagger F$ initiates at infinity and K^2 begins from zero and eventually both fields remain almost constant (see Fig. 1). Since this solution is periodic, when the spacetime starts to contract again the scalar field decreases and goes to $-\infty$ after a deflationary epoch.

Another interesting solution from which we can see the qualitative behavior of the fields, is given by $\sigma(t)=\delta\coth(\sqrt{\lambda}\delta t)$; $a_2(t)=a_0\sinh^2(\sqrt{\lambda}\delta t)$, for the potential $V_{\text{eff}}(\sigma)=\lambda(\sigma^2-\delta^2)^2$. Here, the scalar field goes to infinity as t approaches zero, but it quickly settles in the constant value δ, which corresponds to the minimum of the scalar potential V_{eff}, i.e., the system experiences a phase transition as t

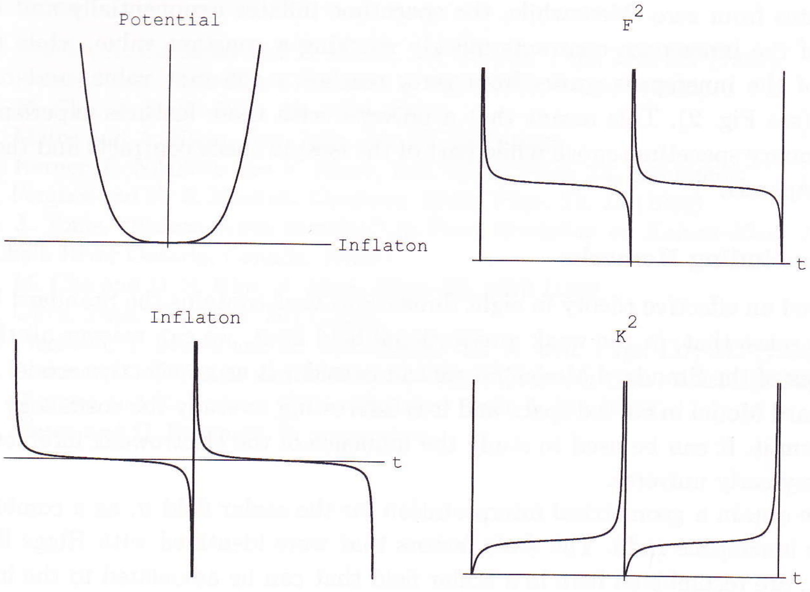

Fig. 1. The potential $V(\sigma) = \lambda(\sigma^2 + \delta^2)^2$ is shown in the top left panel and the inflaton evolution $\sigma = \delta \cot(\sqrt{\lambda}\delta t)$ in the bottom left. In the right panels, the scale factors $F^\dagger F = \exp\left(\sqrt{\frac{8\pi G}{3}}\sigma\right)$ and $K^2 = \exp(-\sqrt{24\pi G}\sigma)$ of the manifolds S^3 and S^1 are plotted as a function of cosmic time.

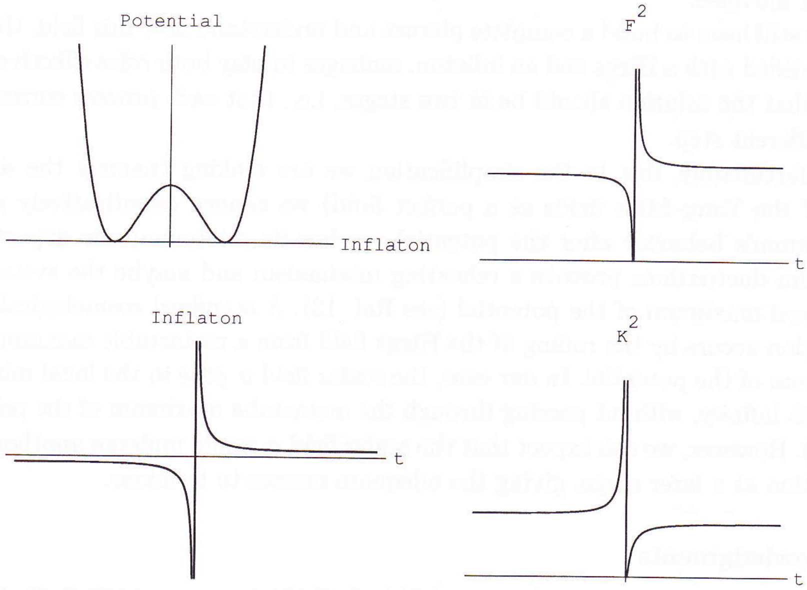

Fig. 2. The potential $U(\rho) = \lambda(\rho^2 - \delta^2)^2$ and the time evolution of the inflaton $\rho = \delta \coth(\sqrt{\lambda}\delta t)$ are displayed in the upper and lower left panels, respectively.The behavior of the scale factors of manifolds S^3 and S^1, $F^\dagger F = \exp\left(\sqrt{\frac{8\pi G}{3}}\rho\right)$ and $K^2 = \exp(-\sqrt{24\pi G}\rho)$, is shown in the right panels.

separates from zero. Meanwhile, the spacetime inflates exponentially and the S^3 part of the innerspace contracts quickly reaching a constant value, while the S^1 part of the innerspace grows from zero, reaches a constant value, and remains there (see Fig. 2). This means that a universe with these features experiences an inflationary spacetime epoch while part of the isospin space contracts and the other part expands.

4. Concluding Remarks

We used an effective theory in eight dimensions that contains the Standard Model, in the sense that, in the weak gravitational field limit, we can recover all the predictions of the Standard Model. So we can consider it as an effective model for the Standard Model in curved space and it is interesting to study the cosmology resulting from it. It can be used to study the influence of the electroweak interaction on the very early universe.

We obtain a geometrical interpretation for the scalar field σ, as a combination of the innerspace radii. The scale factors that were identified with Higgs fields in Ref. 1, are recombined here in a scalar field that can be associated to the inflaton. An important issue for this idendification is that the inflationary evolution leads to a reasonable behavior for the internal radii. We have proved that, for two particular exact solutions, the two innerspace radii effectively tend to a constant value soon after the inflationary regime. In such a way, they are rendered invisible in the present universe.

We still have to build a complete picture and understand how this field, that can be identified with a Higgs and an inflaton, manages to play both roles effectively. We think that the solution should be in two stages, i.e., that each process corresponds to a different step.

Unfortunately, due to the simplification we are making (namely the description of the Yang–Mills fields as a perfect fluid) we cannot quantitatively predict the system's behavior after the potential reaches its minimum; one expects that quantum fluctuations provoke a reheating mechanism and maybe the system gets to a local maximum of the potential (see Ref. 13). A standard cosmological phase transition occurs by the rolling of the Higgs field from a metastable maximum to a minimum of the potential. In our case, the scalar field σ goes to the local minimum through infinity, without passing through the metastabe maximum of the potential $V_{\rm eff}(\sigma)$. However, we can expect that the scalar field σ would undergo another phase transition at a later stage, giving the adequate masses to fermions.

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