Astronomische Nachrichten

Volume 320 1999 Number 3

Astron. Nachr. 320 (1999) 3, 97–104

Scalar fields as dark matter in spiral galaxies: comparison with experiments

F.S. GUZMÁN, T. MATOS and H.B. VILLEGAS, México, México

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN

Received 1998 November 26; accepted 1999 July 9

We present an exact solution for a static and axially symmetric spacetime, which is obtained from a scalar-tensor theory that comes from unification theories. As an attempt to model the dark matter (DM) in spiral galaxies we find that an exponential scalar potential is enough to explain the rotation curves in such galaxies. We also present the fitting to the rotation curve of six spiral galaxies and we find an excellent agreement between observational data and the results of our model.

Key words: cosmology: dark matter — galaxies: halos

1. Introduction

It is a fact that the near constancy of the rotation velocity of stars at different distances from the center around a considerable amount of spiral galaxies constitutes an important challenge for the theoretical machinery at our disposal. People have tried to solve this problem through different ways and has obtained some very interesting results which coincide very well with experimental data (Begeman et al. 1991). One of these approaches is the Modified Newtonian Dynamics (MOND) first proposed by Milgrom (1983) in his classical series of papers, where it is pointed out that when one considers very small accelerations (as those observed in galaxies) the Newtonian Dynamics is not valid anymore and then it would be necessary to modify the classical laws of gravitation by changing the effective gravitational potential; this modification explains the rotation curves. There is another usual approach which works in a certain logical direction: the rotation curves are measured and people reproduce them by setting the mass distribution of the whole luminous matter, from which infer a profile for dark matter distribution; under this point of view the rotation curves are currently well explained (Begeman 1991). However, there still remains a feeling of uncertainty, and it is so because the above approaches are not able to explain why nature behaves in this very curious way (in the first case) and which is the nature of dark matter (in the second one).

In the present paper we look for a correction to other already classical theory: General Relativity, which appears to be a fundamental theory, and the modification we adopt is given by unification theories. It is well known that scalar fields are one of the fundamental ingredients of high dimensional unification theories and that they appear when the dimensional reduction takes place. Phenomenologically we have to mention that the scalar fields have been successful for example in inflationary models, in this way we have some trust to the fact that scalar fields could be the solution to some other problems in the cosmos. There are some reports (Dick 1997; Cho 1998), where the possibility for the dark matter to be scalar is considered. We can not avoid to say that apart of some success of scalar fields, the models appear to be supported by the elegance and by the unification trend of the physical sciences.

Moreover, what seems to be the keypoint is the possibility to consider these unification theories with an exponential scalar potential, which appears in a natural way when higher dimensional theories are endowed with a cosmological constant. In the cases of Kaluza-Klein and Super Strings theories one finds that after the dimensional reduction, the Lagrangian density in four dimensions is of the form: $\mathcal{L}_4 = -R_4 + 2(\nabla\Phi)^2 + e^{-2\alpha\Phi}\Lambda_4$, where Λ_4 plays the role of the cosmological constant, and the value of α determines the theory one deals with (Cho 1998). This kind of potential let us to obtain an expression for the DM profile which coincides with that of an isothermal spherical halo (ISH), and in this way our model is able to explain the great number of rotational curves given by the ISH and maybe more.

The program shown in this article is not as ambitious as to try to explain the rotation curves around the whole galaxy by using an exact solution of a scalar-tensor theory, because Newtonian mechanics has proved to be nice in explaining such curves in the luminous region. In this way we only model the dark matter dominated region (DMDR), for which we obtain a certain spacetime and then we use a model for the contribution of baryonic matter to the circular velocity with our spacetime as the background for the luminous region.

Measurements of the rotation curves in a galaxy are made in the equatorial plane, so we let the symmetry of the halo as general as we can, which we choose to be axial. Moreover we consider that dragging effects in the halo of the galaxy should be too small to affect the test particles traveling on it, hence in the DMDR we suppose that the spacetime has a time reversal symmetry, *i.e.* it is static. In order to model the DMDR we can consider that the main contributor to the total energy density of the galaxy is the scalar field. So the model we work with will be given by the gravitational interaction modified by a scalar field and a scalar potential. Thus the model is well described by the following action

$$S = \int d^4x\sqrt{-g}[-\frac{R}{\kappa_0} + 2(\nabla\Phi)^2 - V(\Phi)], \tag{1}$$

which could be the four-dimensional action for the Kaluza-Klein or the Low Energy Superstings Theory without electromagnetic or matter fields, and we have added a term which contains the scalar potential. In this action R is the scalar curvature, Φ is the scalar field, $V(\Phi) = e^{-2\alpha\Phi}\Lambda$ is the scalar field potential, $\kappa_0 = \frac{16\pi G}{c^3}$, $\sqrt{-g}$ is the determinant of the metric and where we have neglected the electromagnetic field because of its very small strength in spiral galaxies ($H \sim \mu G$).

The most general static and axially symmetric metric compatible with this action written in the Papapetrou form is

$$ds^2 = \frac{1}{f}[e^{2k}(dzd\bar{z}) + W^2d\phi^2] - f\ c^2dt^2, \tag{2}$$

where $z := \rho + i\zeta$ and $\bar{z} := \rho - i\zeta$ and the functions f, W and k depend only on ρ and ζ. This metric represents the symmetries described above. The variation of the action (1) implies the following field equations

$$\begin{aligned} \Phi^{;\mu}_{;\mu} + \frac{1}{4}\frac{dV}{d\Phi} &= 0 \\ R_{\mu\nu} &= \kappa_0[2\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g_{\mu\nu}V(\Phi)] \end{aligned} \tag{3}$$

which are the Klein-Gordon and Einstein's field equations respectively; $\mu, \nu = 0, 1, 2, 3$.

In order to present systematically our model, we explain in the following section how we obtain a solution of the equations (3) and in section 3 we show how such solution does work to explain the asymptotic behavior of the rotation curves; in section 4 we combine a model for the contribution of baryonic matter to the circular velocity with our model for the DMDR; we reserve section 5 to present the comparison of the complete model with astronomical observations; finally in section 6 we draw some conclusions and comments about this model.

2. The exact solution

In order to obtain an exact solution for the set of equations (3) we use the Harmonic Maps Ansatz (Matos 1989, 1994, 1995), which basically consists of reparametrizing the metric function $f = \exp(\lambda)$, and then obtaining a comfortable form of the field equations which usually appear to have a Poisson like structure (Matos & Guzmán 1998). In this way the equations (3) read

$$\begin{aligned} \hat{\Delta}\lambda &= -\kappa_0\sqrt{-g}V(\Phi) \\ 2\hat{\Delta}\Phi &= \frac{1}{4}\sqrt{-g}\frac{dV}{d\Phi} \\ W_{,z\bar{z}} &= -\frac{1}{2}\kappa_0\sqrt{-g}V(\Phi) \\ k_{,z} &= \frac{W_{,zz}}{2W_{,z}} + \frac{1}{4}W\ \lambda_{,z}{}^2W_{,z} + \kappa_0 W\Phi_{,z}{}^2W_{,z} \end{aligned} \tag{4}$$

and a similar equation for $k_{,\bar{z}}$, with $\bar{z}$ instead of z, where $\hat{\Delta}$ is the Laplace operator such that for any function $h = h(z,\bar{z})$: $\hat{\Delta}h := (Wh_{,z})_{,\bar{z}} + (Wh_{,\bar{z}})_{,z}$. Moreover, under this approach, we can interpret $\lambda = \ln(f)$ as the gravitational potential and Φ as the scalar field.

If one assumes that λ and Φ depend only on $W(z,\bar{z})$, the set of equations (4) appears in a more tractable form

$$
\begin{aligned}
2WW_{,z\bar{z}}D\lambda &= -\kappa_0\sqrt{-g}V(\Phi) && (5)\\
2WW_{,z\bar{z}}D\Phi &= \frac{1}{4}\sqrt{-g}\frac{dV}{d\Phi} && (6)\\
W_{,z\bar{z}} &= -\frac{1}{2}\kappa_0\sqrt{-g}V(\Phi) && (7)\\
k_{,z} &= \frac{W_{,zz}}{2W_{,z}} + \frac{1}{4}W\ \lambda'^2 W_{,z} + \kappa_0 W\Phi'^2 W_{,z} && (8)
\end{aligned}
$$

and the corresponding expression for $k_{\bar{z}}$, where now the operator D means $Dh(W) = Wh'' + 2h'\ \forall h = h(W)$, and $'$ denotes derivative with respect to W. Equations (5- 8) constitute a system of coupled differential equations because $\sqrt{-g} = We^{2k-\lambda}/2$. However it is evident that once we have expressions for λ and Φ, k can be integrated by quadratures. Moreover, using the third of these equations λ and Φ obey differential equations with W being the independent variable.

We proceed as follows. We substitute $\kappa_0\sqrt{-g}V(\Phi)$ from (7) into (5) and (6), (remember that $\frac{dV}{d\Phi} = -2\alpha V$) and obtain two decoupled differential equations, one for λ and another for Φ. We solve these two differential equations and substitute the solution into (8). We thus find that a solution of the system (3) is given by

$$
\begin{aligned}
\lambda &= \ln(M) + \ln(f_0)\\
\Phi &= \Phi_0 + \frac{1}{2\sqrt{\kappa_0}}\ln(M)\\
V &= \frac{4f_0}{\kappa_0 M}\\
k &= \frac{1}{2}(\ln M_{,z\bar{z}} + \ln M) && (9)
\end{aligned}
$$

where f_0 and Φ_0 are integration constants related by $e^{-2\sqrt{\kappa_0}\Phi_0} = 4f_0\Lambda/\kappa_0$, $\alpha = \sqrt{\kappa_0}$, and $W = M$ is a function restricted only by the condition

$$MM_{,z\bar{z}} = M_{,z}M_{,\bar{z}} \tag{10}$$

whose solutions are $M = Z(z)\bar{Z}(\bar{z})$, with Z and $\bar{Z}$ arbitrary functions. Observe that f_0 and Φ_0 are integrations constants of the solution, they are different for each space-time and therefore different for each galaxy. Nevertheless, there exists a relation between them $e^{-2\sqrt{\kappa_0}\Phi_0} = 4f_0\Lambda/\kappa_0$, therefore we have only one free constant of the metric.

If one explores the simplest solution of (10) $Z = \bar{Z} = Id$ (*i.e.*, $M = z\bar{z} = \rho^2+\zeta^2$) something interesting happens. Using Swarzschild-like coordinates (Boyer-Lindquist coordinates) $\rho = \sqrt{r^2 - 2ar + b^2}\sin\theta$, $\zeta = (r-a)\cos\theta$, our solution reads

$$
\begin{aligned}
ds^2 = &\frac{(1-\frac{a}{r})^2 + \frac{K^2\cos^2\theta}{r^2}}{f_0}\left(\frac{dr^2}{1-2\frac{a}{r}+\frac{b^2}{r^2}} + r^2\,d\theta^2\right) + \frac{(r-a)^2 + K^2\sin^2\theta}{f_0}d\phi^2 -\\
&-f_0c^2((r-a)^2 + K^2\sin^2\theta)dt^2 && (11)
\end{aligned}
$$

where $K^2 = b^2 - a^2$. The reader interested in the geometry of the space-time (11) will find a brief summary of it in the appendix. Moreover one finds that the effective energy density is

$$\mu_{DM} = \frac{1}{2}V(\Phi) = \frac{2f_0}{\kappa_0((r-a)^2 + K^2\sin^2\theta)} \tag{12}$$

which for our model must represent the profile of the dark matter that determines the behavior of test particles in the DMDR. Observe that a and b are constants of the coordinates, *i.e.* gauge constants, in order to understand the space-time structure they can be removed, the space-time geometry does not depend on these constants, but for the physical interpretation they can take different values for each galaxy. Clearly when one studies this profile in the equatorial plane and takes $a = 0$ the profile coincides with that of an isothermal sphere (Begeman et al. 1991). In this way our model could explain the same range of rotation curves as an isothermal halo.

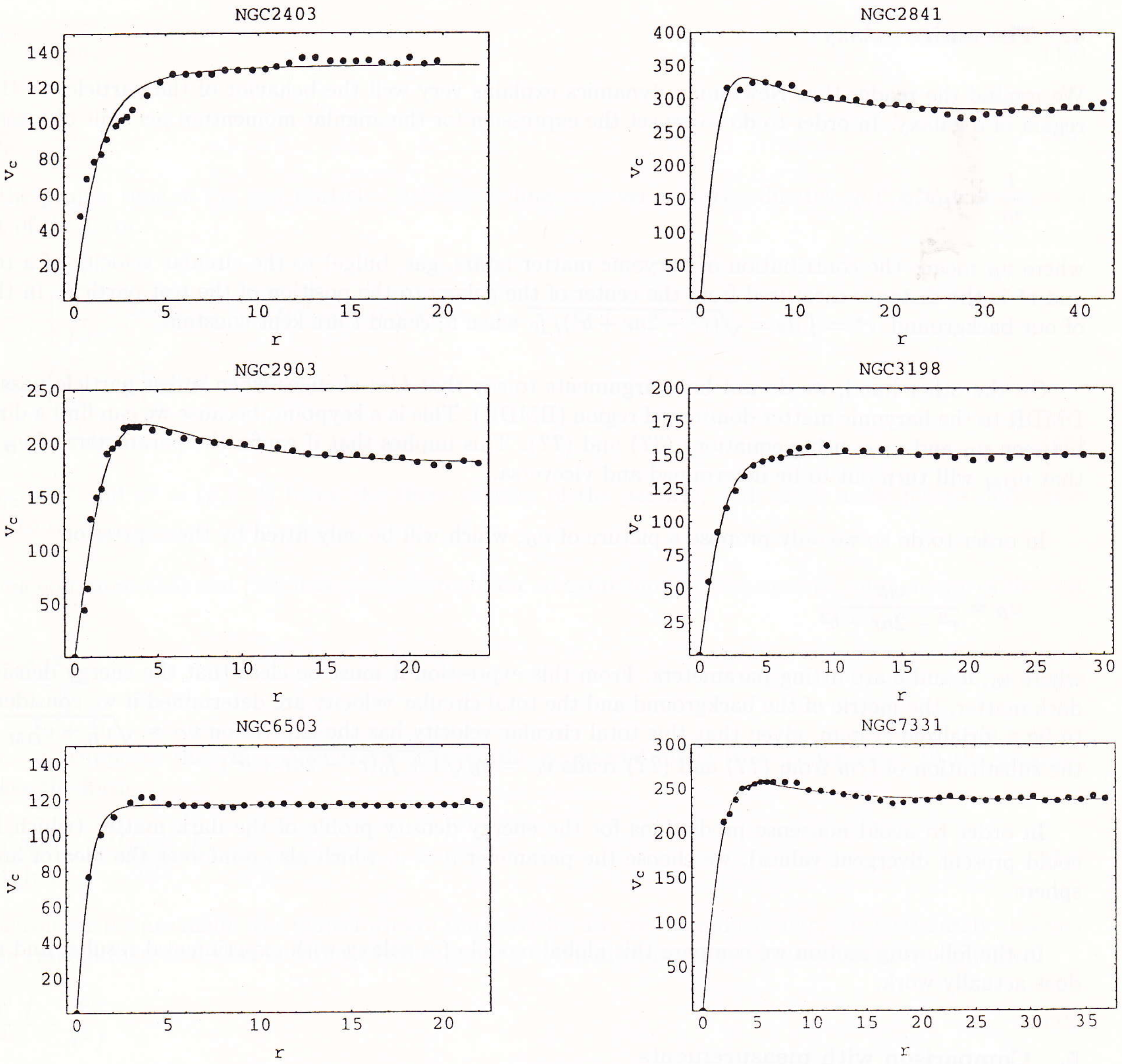

Fig. 1: Curves of v_C (km/s) vs r (kpc) and the observational data (dots) for six spiral galaxies.

Table 1: Values of the fitting parameters.

Galaxy	f_0	b	v_0	v_a (km s^{-1})
NGC2403	0.6913	2.0705	159.08	132.2
NGC2841	0.0245	3.4848	1778.6	280.1
NGC2903	0.0254	3.2419	1111.6	177.2
NGC3198	0.1081	2.9230	457.90	150.4
NGC6503	0.4312	1.3848	176.91	116.0
NGC7331	0.0417	3.2295	1152.1	235.5

6. Conclusions and comments

Starting with an action inspired in unification theories we have been able to find a spacetime background for spiral galaxies, which models the circular velocities of test particles (stars) around them.

It would be useful if we had at our disposal measurements of rotation curves outside and in other directions than those made in the equatorial plane of galaxies. In such case we could test if our model works also in that region, because, as the reader surely remembers, in some moment along this paper we fixed $\theta = \pi/2$. The price to be paid would be the difficulty to integrate the geodesics equation, but at least it could be done numerically.

Further research will point in the direction of the structure of galaxies. Our hypothesis states that while the luminous matter (stars) behaves following the Newtonian mechanics, the dark matter is of scalar origin, baryonic matter moves in a scalar field background which can only be explained by a modification of the Einstein equations and is purely relativistic. In fact, metric (11) does not have Newtonian limit and therefore the structure of the dark matter in this model can not be analyzed in this way. Nevertheless the main result of this manuscript (equation(??)) is invariant under conformal transformations, thus we have to find certain Lagrangian that together with the general solutions of the geodesics equation could let us to explain the distinct morphologies of galaxies.

References

Begeman, K.G., Broeils, A.H., Sanders, R.H.: 1991, Mon. Not. R. Astron. Soc. 249, 523
Milgrom, M.: 1983a, Astrophys. J. 270, 365
Milgrom, M.: 1983b, Astrophys. J. 270, 371
Milgrom, M.: 1983c, Astrophys. J. 270, 384
Dick, R.: 1997, in: H.V. Klapdor-Kleingrothaus and Y. Ramachers (eds.): Dark Matter in Astro- and Particle Physics, World Scientific, Singapore, pp. 395–402
Cho, Y.M., Keum, Y.Y.: 1998, Class. Quantum Grav. 15, 907
Matos, T.: 1989, Ann. Phys. 46, 462
Matos, T.: 1994, J. Math. Phys. 35, 1302
Matos, T.: 1995, Math. Notes 58, 1178
Matos, T., Guzmán, F.S.: 1998, in: D. Ruffini et al. (eds.): Proceedings of the VIII Marcel Grossman Meeting, Jerusalem, Israel, Word Scientific Singapore, (1999), in press.
Matos, T., Guzmán, F.S.: 1998, gr-qc/9810028.
Mannheim, P.D.: 1997, Astrophys. J. 479, 659

A Geometrical aspects of our space-time

The case we have dealt with ($a = 0$) permits us to see some basic properties of the space-time described by the metric (11).

The second order invariants obtained from such metric go as powers of $1/(r^2 + b^2 \sin^2\theta)$, which means that there is a singularity at $r = \theta = 0$.

On the other hand, we note that (11) is conformally flat and can be written as

$$ds^2 = \frac{e^{2x_1}}{f_0}[dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2] \tag{24}$$

being

$$x_1 = \ln\sqrt{r^2 + b^2\sin^2\theta}, x_2 = \arctan(\frac{r\cot\theta}{\sqrt{r^2+b^2}}), x_3 = \phi, x_4 = f_0 ct \tag{25}$$

Another important property of the metric is its embedded structure into $\Re^3$ for $\theta = \pi/2$ and constant t. In such case

$$f_0 ds^2 = \frac{r^2}{r^2+b^2}dr^2 + (r^2+b^2)d\phi^2 \tag{26}$$

In cylindrical coordinates ($\bar{r}$,$\bar{\phi}$,$\bar{z}$) the line element of $\Re^3$ is given by

$$d\bar{s}^2 = d\bar{r}^2 + \bar{r}^2 d\bar{\phi}^2 + d\bar{z}^2 \tag{27}$$

Thus by definning $\bar{r}^2 = r^2 + b^2$ and $\bar{\phi} = \phi$ we see that $\bar{z}$ must be a constant, which means that (??) is the metric of a plane into $\Re^3$. From here, noting that b is a gauge parameter of the Boyer-Lindquist coordinates, the interpretation of $\bar{r}$ as the radial coordinate is correct.

Address of the author:

F.S. Guzmán, T. Matos, H.B. Villegas, Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, A.P. 14-740, México 07000, D.F., México, e-mail: Francisco.Guzman@fis.cinvestav.mx