Class. Quantum Grav. **17** (2000) L9–L16. Printed in the UK PII: S0264-9381(00)01857-8

LETTER TO THE EDITOR

Scalar fields as dark matter in spiral galaxies

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Received 15 February 1999, in final form 1 October 1999

Abstract. We present a model for the dark matter in spiral galaxies, which is a result of a static and axial symmetric exact solution of the Einstein–dilaton theory. We suppose that dark matter is a scalar field endowed with a scalar potential. We obtain that (a) the effective energy density goes like $1/(r^2 + r_c^2)$ and (b) the resulting circular velocity profile of test particles is in good agreement with the observed one.

PACS numbers: 9530S, 0450, 9535

One of the greatest puzzles of physics at the moment is without doubt the existence of dark matter in the cosmos. The experimental fact that the galaxy masses measured with dynamical methods do not coincide with their luminous galaxy masses gives rise to the existence of a great amount of dark matter in galaxies, galaxy clusters and superclusters. At the present time, cosmological observations indicate that the universe is filled out with about 90% dark matter, whose nature until now remains unexplained. Recently, some authors have proposed the scalar field as a candidate for dark matter in the cosmos [1, 2], in some sense the inflationary cosmological model proposes the scalar field as cosmological dark matter as well. These models consider scalar–tensor theories of gravity where one is able to add mass terms to the total energy density of the spacetime. All modern unifying field theories also contain scalar fields. For example, scalar fields are fundamental fields in Kaluza–Klein and superstring theories, because such fields appear in a natural way after dimensional reduction. In both theories the scalar field could be endowed with an exponential scalar potential [3, 4], in particular, when we deal with five-dimensional Kaluza–Klein theories, the Lagrangian density reads $\mathcal{L}_5 = R_5 + \Lambda_5$, where Λ_5 is a five-dimensional cosmological constant. After dimensional reduction and a conformal transformation one obtains the density $\mathcal{L}_4 = -R_4 + 2(\nabla\Phi)^2 + \mathrm{e}^{-2/\sqrt{3}\Phi}\Lambda$, where Φ is the scalar field which actually states that an exponential potential appears in a natural way in this theory. An analogous procedure establishes that in the low-energy limit of superstring theory one obtains a similar result [1, 3]. In general one obtains the Lagrangian from high-dimensional theories $\mathcal{L}_4 = -R_4 + 2(\nabla\Phi)^2 + \mathrm{e}^{-2\alpha\Phi}\Lambda$, therefore here we will restrict ourselves to an exponential scalar potential. In this letter we show a possible model for the dark matter in spiral galaxies, supposing that such matter is of a scalar nature.

There is a common approach to explain the rotation curves in spiral galaxies called modified Newtonian dynamics (MOND) [5, 6], which basically consists of modifying Newton's law of attraction for small accelerations by adding terms to the gravitational potential.

In this way, by adjusting some free parameters for each galaxy, one can reproduce the asymptotic behaviour of the rotation curves. However, it appears to be artificial because it is nothing but a mere correction of Newton's law, we are unable to know either where the parameters and the correction terms come from, or why nature behaves like that and therefore what Newton's law is at a cosmological scale for instance.

A convincing phenomenological model for galactic dark matter is the so-called isothermal halo model (IHM), which assumes the dark matter to be a self-gravitating ball of ideal gas (made of any kind of particle) at a uniform temperature $kT = \frac{1}{2} m_{dm} v_c$, where m_{dm} is the mass of each particle and v_c is its velocity, which eventually produces a dark matter distribution going as $\sim 1/r^2$, implying in this way an increasing mass $M(r) \sim r$. Then, by assuming that a galaxy is a system in equilibrium ($GM/r^2 = v_c^2/r$) the velocity of particles surrounding the profile above should produce flat rotation curves into a region where the dark matter dominates, i.e. at large radii when one considers an exponential distribution of luminous matter as usual [7].

Observational data show that galaxies are composed of almost 90% dark matter [6–8]. This is so because the kinematics inside the dark-matter-dominated region is not consistent with the predictions of Newtonian theory, which explains well the dynamics of the luminous sector of the galaxy but predicts a Keplerian falling off for the rotation curve. The region of the galaxy we are interested in is that in which the dark matter determines the kinematics of test particles. So we can suppose that luminous matter does not contribute in a very important way to the total energy density of the matter that determines the behaviour of particles in the mentioned region, instead the scalar matter will be the main contributor to it. Thus, as a first approximation we can neglect the baryonic matter contribution to the total energy density for the explanation of asymptotic rotation curves.

On the other hand, the exact symmetry of the dark halo is still unknown, but it is reasonable to suppose that it is symmetric with respect to the rotation axis of the galaxy. In this letter we let the symmetry of the halo be as general as we can, so we choose it to be axial symmetric. Furthermore, the rotation of the galaxy does not affect the motion of test particles around the galaxy, dragging effects in the halo of the galaxy should be too small to affect the test particles (stars) travelling around the galaxy. Hence, in our region of interest we can suppose the spacetime to be static, given that the circular velocity of stars (like the Sun) of about 230 km s^{-1} seems not to be affected by the rotation of the galaxy and we can consider a time-reversal symmetry of the spacetime. So, the model we are dealing with will be given by the gravitational interaction modified by a scalar field and a scalar potential. The model consists of the following action:

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[-\frac{R}{\kappa_0} + 2(\nabla\Phi)^2 - V(\Phi)\right], \tag{1}$$

which could be the four-dimensional action for the Kaluza–Klein or the low-energy superstings theory without an electromagnetic field, and where we have added a term which contains the scalar potential. In this action R is the scalar curvature, Φ is the scalar field, $\kappa_0 = 16\pi G/c^3$ and $\sqrt{-g}$ is the determinant of the metric. The most general static and axial symmetric line element compatible with this action, written in the Papapetrou form, is

$$\mathrm{d}s^2 = \frac{1}{f}\left[\mathrm{e}^{2k}(\mathrm{d}z\,\mathrm{d}\bar{z}) + W^2\,\mathrm{d}\phi^2\right] - f\,c^2\,\mathrm{d}t^2, \tag{2}$$

where $z := \rho + \mathrm{i}\zeta$ and $\bar{z} := \rho - \mathrm{i}\zeta$ and the functions f, W and k depend only on ρ and ζ. This metric represents the symmetries posted above. The application of the variational principle to

(1) gives rise to the field equations

$$\Phi^{;\mu}_{;\mu} + \frac{1}{4}\frac{\mathrm{d}V}{\mathrm{d}\Phi} = 0$$
$$R_{\mu\nu} = \kappa_0\big[2\Phi_{,\mu}\Phi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}V(\Phi)\big], \tag{3}$$

which are the Klein–Gordon and Einstein field equations, respectively; $\mu, \nu = 0, 1, 2, 3$. Using the harmonic maps ansatz [9–11] we find the following Poisson-like structure for the above equations [12]:

$$\begin{aligned}
&\hat{\Delta}\lambda = -\kappa_0\sqrt{-g}V(\Phi)\\
&2\hat{\Delta}\Phi = \frac{1}{4}\sqrt{-g}\frac{\mathrm{d}V}{\mathrm{d}\Phi}\\
&W_{,z\bar{z}} = -\tfrac{1}{2}\kappa_0\sqrt{-g}\,V(\Phi)\\
&k_{,z} = \frac{W_{,zz}}{2W_{,z}} + \frac{1}{4}W\lambda_{,z}^2 W_{,z} + \kappa_0 W\Phi_{,z}^2 W_{,z},
\end{aligned} \tag{4}$$

and a similar equation for $k_{,\bar{z}}$, with $\bar{z}$ instead of z, where $\hat{\Delta}$ is the Laplace operator such that for any function $h = h(z, \bar{z})$: $\hat{\Delta}h := (Wh_{,z})_{,\bar{z}} + (Wh_{,\bar{z}})_{,z}$. Moreover, $\lambda = \ln(f)$ is interpreted as the gravitational potential.

If one assumes that λ and Φ depend only on $W(z, \bar{z})$, the set of equations (4) appears in a more tractable form

$$2WW_{,z\bar{z}}D\lambda = -\kappa_0\sqrt{-g}\,V(\Phi) \tag{5}$$
$$2WW_{,z\bar{z}}D\Phi = \frac{1}{4}\sqrt{-g}\,\frac{\mathrm{d}V}{\mathrm{d}\Phi} \tag{6}$$
$$W_{,z\bar{z}} = -\tfrac{1}{2}\kappa_0\sqrt{-g}\,V(\Phi) \tag{7}$$
$$k_{,z} = \frac{W_{,zz}}{2W_{,z}} + \frac{1}{4}W\lambda'^2 W_{,z} + \kappa_0 W\Phi'^2 W_{,z} \tag{8}$$

and the corresponding expression for $k_{\bar{z}}$, where now the operator D means $Dh(W) = Wh'' + 2h'$ $\forall h = h(W)$, and $'$ denotes a derivative with respect to W. Equations (5)–(8) constitute a system of coupled differential equations because $\sqrt{-g} = W\mathrm{e}^{2k-\lambda}/2$. However, it is evident that once we have expressions for λ and Φ, k can be integrated by quadratures. Moreover, using the third of these equations λ and Φ obey differential equations where W is the independent variable.

In order to find an exact solution, we substitute $\kappa_0\sqrt{-g}\,V(\Phi)$ from (7) into (5) and (6), (remember that $\mathrm{d}V/\mathrm{d}\Phi = -2\alpha V$) and obtain two decoupled differential equations, one for λ and another for Φ. We solve these two differential equations and substitute the solution into (8). We thus find that a solution of the system (5)–(8) is given by

$$\begin{aligned}
&\lambda = \ln(M) + \ln(f_0)\\
&\Phi = \Phi_0 + \frac{1}{2\sqrt{\kappa_0}}\ln(M)\\
&V = \frac{4f_0}{\kappa_0 M}\\
&k = \tfrac{1}{2}(\ln M_{,z\bar{z}} + \ln M)
\end{aligned} \tag{9}$$

where f_0 and Φ_0 are integration constants and $W = M$ is a function restricted only by the condition

$$MM_{,z\bar{z}} = M_{,z}M_{,\bar{z}} \tag{10}$$

whose solutions are $M = Z(z)\bar{Z}(\bar{z})$, where Z is an arbitrary function. The reader can check that (9) is a solution of the field equations substituting the set (9) into (3) using the metric (2).

In what follows we study the circular trajectories of a test particle on the equatorial plane taking the spacetime (2) as the background. The motion equation of a test particle in the spacetime (2) can be derived from the Lagrangian

$$\mathcal{L} = \frac{1}{f}\left[\mathrm{e}^{2k}\left(\left(\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\right)^2 + \left(\frac{\mathrm{d}\zeta}{\mathrm{d}\tau}\right)^2\right) + W^2\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2\right] - f\,c^2\left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^2. \tag{11}$$

This Lagrangian contains two constants of motion, the angular momentum per unit of mass

$$\frac{W^2}{f}\frac{\mathrm{d}\phi}{\mathrm{d}\tau} = B, \tag{12}$$

and the total energy per unit of mass of the test particle

$$f\,c^2\frac{\mathrm{d}t}{\mathrm{d}\tau} = A, \tag{13}$$

where τ is the proper time of the test particle. An observer falling freely into the galaxy, with coordinates (ρ, ζ, ϕ, t), will have a line element given by

$$\begin{aligned}\mathrm{d}s^2 &= \left\{\frac{1}{fc^2}\left[\mathrm{e}^{2k}(\dot{\rho}^2 + \dot{\zeta}^2) + W^2\dot{\phi}^2\right] - f\right\}c^2\,\mathrm{d}t^2 \\ &= \left(\frac{v^2}{c^2} - f\right)c^2\,\mathrm{d}t^2 \\ &= -c^2\,\mathrm{d}\tau^2.\end{aligned} \tag{14}$$

The velocity $v^a = (\dot{\rho}, \dot{\zeta}, \dot{\phi})$, is the 3-velocity of the test particle, where a dot denotes a derivative with respect to t, the time measured by the free-falling observer. The squared velocity v^2 is then

$$v^2 = g_{ab}v^a v^b = \frac{\mathrm{e}^{2k}}{f}(\dot{\rho}^2 + \dot{\zeta}^2) + \frac{W^2}{f}\dot{\phi}^2, \tag{15}$$

where $a, b = 1, 2, 3$. Substituting (14) into (13) we obtain an expression for the squared energy

$$A^2 = \frac{c^4 f^2}{f - v^2/c^2}. \tag{16}$$

We are interested in test particles (stars) moving on the equatorial plane $\dot{\zeta} = 0$ and the equation of motion derived from the geodesics of metric (2) reads

$$\frac{1}{f}\mathrm{e}^{2k}\left(\frac{\mathrm{d}\rho}{\mathrm{d}\tau}\right)^2 + \frac{B^2 f}{W^2} - \frac{A^2}{c^2 f} = -c^2, \tag{17}$$

where we have used the conservation equations (12) and (13). Equation (17) determines the trajectory of a test particle around the equator of the galaxy, in this trajectory A and B remain constant. If we change the test particle, we could have other constants of motion A and B determining the trajectory of the new particle. A spiral galaxy is practically a disc of stars travelling around the equatorial plane of the galaxy in circular trajectories in the period of observation, although it had to be formed from enormous clouds of gas going around a symmetry axis with average values of A and B. Thus for a circular trajectory $\dot{\rho} = 0$, the equation of motion transforms into

$$\frac{B^2 f}{W^2} - \frac{A^2}{c^2 f} = -c^2. \tag{18}$$

This last equation determines the circular trajectories of test particles travelling on the equator of the galaxy. Using (18) and (16) we find an expression for B in terms of v^2,

$$\begin{aligned} B^2 &= \frac{v^2}{f - v^2/c^2} \frac{W^2}{f} \\ &\sim v^2 \frac{W^2}{f^2}, \end{aligned} \tag{19}$$

since $v^2 \ll c^2$. Now using (19) one concludes that for our solution (9) $v^2 = f_0^2 B^2$, i.e.

$$v_{DM} = f_0 B, \tag{20}$$

where we call $v \to v_{DM}$ the contribution of our dark matter to the circular velocity of a star.

When $Z = z$ our solution in Boyer–Lindquist coordinates $\rho = \sqrt{r^2 - 2ar + b^2} \sin\theta$, $\zeta = (r - a)\cos\theta$ reads

$$\begin{aligned} ds^2 = \frac{(1 - a/r)^2 + K^2 \cos^2\theta/r^2}{f_0 r_0} \left(\frac{dr^2}{1 - 2a/r + b^2/r^2} + r^2\, d\theta^2 \right) \\ + \frac{(r - a)^2 + K^2 \sin^2\theta}{f_0 r_0}\, d\phi^2 - f_0 c^2 \frac{(r - a)^2 + K^2 \sin^2\theta}{r_0}\, dt^2 \end{aligned} \tag{21}$$

where $K^2 = b^2 - a^2$ and r_0 only scales. The effective energy density μ_{DM} of (9) is given by the expression

$$\mu_{DM} = \frac{1}{2} V(\Phi) = \frac{2 f_0 r_0}{\kappa_0((r - a)^2 + K^2 \sin^2\theta)} \tag{22}$$

and plays the role of our dark matter density profile.

Keeping in mind that this is only the contribution of dark matter to the energy density we are in a condition to compare these results with those given by measurements. In order to do so we recall the paper by Begeman *et al* [6] where an energy density profile of the IHM $\mu(r) = \rho_0 r_c^2/(r^2 + r_c^2)$ for dark matter is used, where r_c is a core radius. It is evident that this profile is a particular case of the expression we present here, namely, for matter localized on the equator of the galaxy. So, we can fit some of the free parameters of metric (21) comparing these two profiles. We set $b = r_c$, $a = 0$ and $2 f_0 r_0/\kappa_0 = \rho_0 r_c^2$.

Let us model the circular velocity profile due to the luminous matter of the disc in a spiral galaxy by the function

$$v_L^2 = v^2(R_{opt})\beta \frac{1.97 x^{1.22}}{(x^2 + 0.78^2)^{1.43}} \tag{23}$$

which is the approximate model for the universal rotation curves (URC) as was proposed by Persic *et al* [8] for an exponential thin disc, valid for a sample of 967 galaxies; in this expression $x = r/R_{opt}$, the parameter $\beta = v_L(R_{opt})/v(R_{opt})$ where R_{opt} is the radius within which the 83% of the observable mass of the galaxy is contained and v is the observed circular velocity.

We can suppose that luminous matter near the centre of a galaxy behaves as in Newtonian mechanics. Thus with the luminous velocity (23) it is now easy to calculate the angular momentum (per unity of mass) of the test particle in the luminous matter-dominated region

$$B = v_L D, \tag{24}$$

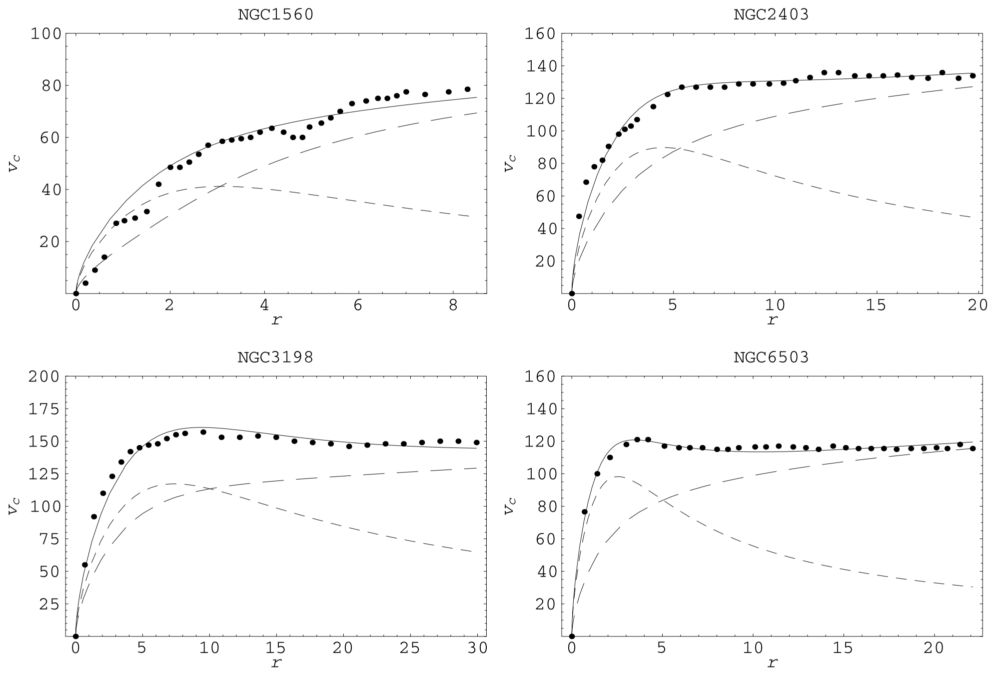

Figure 1. The circular velocity profiles of four spiral galaxies. Full curves, total circular velocity (v_C); long-broken curves, contribution of the dark matter to the total velocity (v_{DM}) and short-broken curves, contribution of luminous matter (v_L); finally, the full circles represent the observational data. The units are in km s^{-1} on the vertical axis and in kpc on the horizontal axis.

where D is the distance between the centre of the galaxy and the test particle. For our metric, $D = \int \mathrm{d}s$, keeping θ, ϕ and t constant, we obtain $D = \sqrt{(r^2 - 2ar + b^2)/f_0 r_0}$. Observe that after we have determined the dark matter energy density μ_{DM}, B is uniquely determined by v_L via (24); it is easy to show that B in (24) equals that of equations (17)–(20) by including a luminous Newtonian component in the radial geodesic equation [13]. Therefore, equations (20) and (24) imply the total circular velocity

$$v_C = \sqrt{v_L^2 + v_{DM}^2} = v_L \sqrt{1 + f_0(r^2 - 2ar + b^2)} \tag{25}$$

an expression that should fit the observed rotation curves. In order to do so, we present in figure 1 the plots containing the fits of four spiral galaxies and in table 1 the values of the parameters f_0 and b keeping $a = 0$ and the scale $r_0 = 1$. From this we see that the agreement of the resulting circular velocity profiles given by the scalar field as dark matter and the observed profiles is very good not only far away from the centre of the galaxy but inside the part of the galaxy dominated by luminous matter as well.

The criterion used to choose the sample was the ratio of the dark to luminous mass inside r_{25}, which was selected to be ~ 1 in order to test our model by using 'dark enough galaxies'. The plots shown in figure 1 would not be enough to state that our model works, it is necessary to be consistent with the phenomenological URC approach, i.e. the contribution of our dark matter should be the same as that proposed by the URC frame which is strongly luminosity

Table 1. Values of the fitting parameters f_0 and b. Also shown are the values of the quantities used.

Galaxy	f_0 (kpc^{-1})	b (kpc)	R_{opt} (kpc)	β
NGC1560	0.0726	2.119	4.6	0.344
NGC2403	0.0171	5.399	6.7	0.546
NGC3198	0.0038	12.88	11	0.547
NGC6503	0.0290	3.035	3.8	0.702

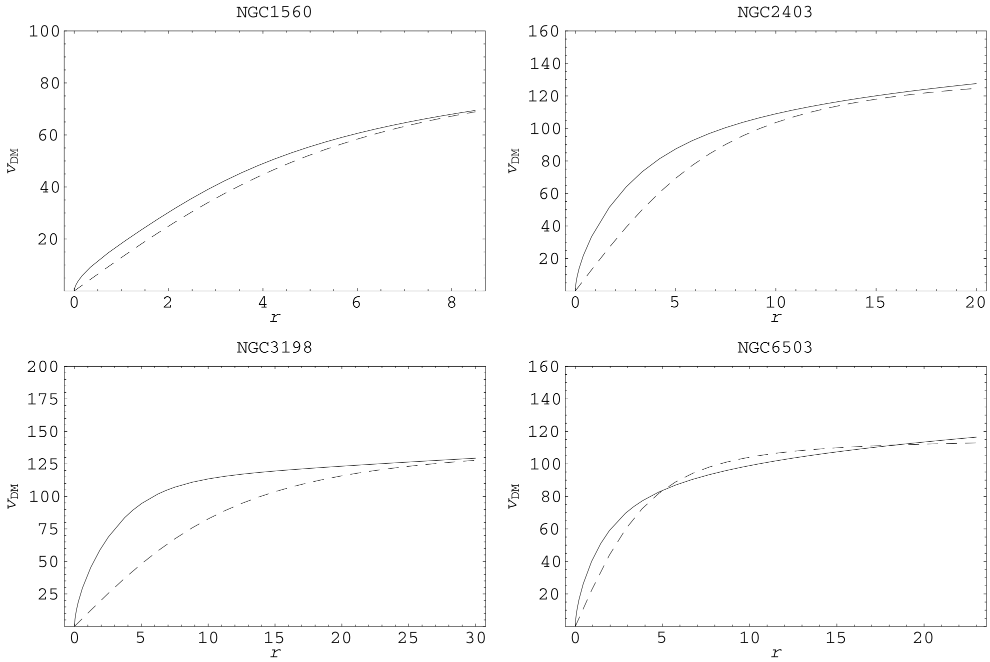

Figure 2. The contribution of dark matter to the circular velocity of test particles is shown. Full curves emerge from the model described in this letter, broken curves correspond to the URC approach. The discrepancies are of 8.3, 11, 23.3 and 5.2%, respectively.

dependent. A formula consistent with (23) is given by [8]

$$v^2_{urcDM} = v^2(R_{opt})(1-\beta)(1+\gamma^2)\frac{x^2}{x^2+\gamma^2} \tag{26}$$

where $\beta = 0.72 + 0.44 \log L/L_*$ the same parameter as in (23) and $\gamma = 1.5(L/L_*)^{1/5}$.

According to (20) and (24) the contribution of our dark matter is

$$v^2_{DM} = f_0(r^2+b^2)v^2(R_{opt})\beta\frac{1.97x^{1.22}}{(x^2+0.78^2)^{1.43}} \tag{27}$$

and after using the fitting parameters of table 1 both approaches are compared in figure 2, from which it can be concluded that our dark matter model respects the luminous matter model we have used.

Some remarks can be drawn. The energy density (22) coincides with that required for a galaxy to explain the rotation curves of test particles in its halo, but in our model, this energy density is a product of the scalar field and the scalar field potential, that is, this dark matter is produced by a Φ particle. So we have shown that there is an exact solution which describes the rotation curves of particles in a spiral galaxy. The crucial point for having the circular velocity $v_{DM} = f_0 B$ is that $f \sim W$ in the solution (9). However, this fact remains unaltered after conformal transformations in the metric $\mathrm{d}\hat{s}^2 = A(\Phi)\,\mathrm{d}s^2$, so that the circular velocity v_{DM} remains the same for all theories and frames related with metric (2) by conformal transformations.

What does our model look like in the cosmological context? When a density profile for galactic dark matter goes as the inverse of r^2 and it is supposed that the halo of a galaxy ends in the region where those of neighbouring galaxies start, the integrated amount of galactic dark matter is close to that needed for the Universe to be flat for the observed average distance between them [7], flatness being inferred from the cosmic background radiation [14] and thus permitting our model to be inside the bounds. In fact, we have developed a cosmological model that considers the same theory as here (1) with the same scalar potential [15], which has been able to explain the redshifts of type Ia supernovae, and all the parameters for structure formation lie within the ranges imposed by observations [14, 16], which make us put forward the model presented in this letter.

We want to thank Ulises Nucamendi and Hugo Villegas Brena for many helpful discussions. In particular, we thank the referee for giving us some very important suggestions about the observational section of this letter. This work was partly supported by CONACyT, Mexico, under grants 94890 (Guzmán) and 3697-E (Matos).

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