Class. Quantum Grav. **17** (2000) 1455–1465. Printed in the UK PII: S0264-9381(00)06315-2

Possible astrophysical signatures of scalar fields

Tonatiuh Matos and Hugo V Brena
Depto de Física, CINVESTAV IPN, AP 14-740, 07000 México DF, Mexico

Received 23 July 1999

Abstract. An exact, axially symmetric solution for gravity minimally coupled to a scalar field is employed to calculate several effects that might be interesting for astrophysical detection of scalar fields. It is found that, as the ratio Q_s/m of the scalar charge to the mass parameter of the metric increases, the difference with general relativity predictions becomes larger, which can be used to set an upper limit for this ratio, both from the weak- and the strong-field regimes. For the latter, we reanalysed the strong light deflection near a compact star, previously studied in the context of general relativity, for which we found a discrepancy between a few and 80%, depending on the value of Q_s/m; although this effect is very difficult to observe even for general relativity, it would put a bound on Q_s from a strong-field situation. Brief mention is made of what these effects look like in other scalar–tensor theories of gravity.

PACS numbers: 0420J, 0450, 0480, 9530S

1. Introduction

Besides its simplicity and elegance, general relativity (GR) has the virtue of having passed every experimental and observational test it has been confronted with. At the same time, there have been many alternative theories which have tried to account for the gravitational interaction, each with its own motivation and with predictions which depart from those of general relativity in different ways. It is very useful then to have a standard set of tests that every gravitational theory must pass.

Until the mid-1970s, all the known effects to test a theory of gravity dealt with measurements within the solar system, where the gravitational fields, velocities and mass densities are very small compared with some fiducial parameter. On the other hand, the discovery of the binary pulsar in 1975 [1] opened up a new era in observational gravitation, allowing one to measure for the first time relativistic effects in the presence of a strong gravitational field. Moreover, in the 20 years of continuous monitoring PSR1913+16, and with later sky surveys revealing new binary pulsar systems, it has been possible both to validate the predictions of GR in this new, strong-field domain, and to rule out or severely constrain alternative theories of gravity for which solar system predictions do not differ from those of GR.

On the other hand, among the most popular and best explored alternatives to GR are scalar–tensor (ST) theories. The scalar field, both minimally and conformally coupled to gravity, has been the subject of intensive research in recent years, and in particular the dilaton has generated a growing interest because of its importance in string theory. Although these matters could seem to be of little astrophysical relevance, the so-called 'spontaneous scalarization' phenomenon in neutron stars has been recently described [3]; it was found that, when a very compact body has a mass-to-radius ratio greater than some critical value, it turns out that a scalar field could help the star to reach a state with a lower energy than the state without the scalar field. In different contexts, the scalar field has also been proposed as a candidate for gravitational lensing [4] and for the dark matter at cosmological scales [5, 6]. Therefore, we consider as interesting the study of possible observational signatures involving exact solutions to the field equations of gravity minimally coupled to a scalar field; they will hopefully allow us to find some other non-perturbative effects.

In this work we shall study a particular axially symmetric solution to gravity minimally coupled to a scalar field in the context of astrophysical observations involving both the weak and strong gravitational field regimes. This paper is organized as follows. In section 2 we introduce the metric studied and comment on some interesting features it possesses; in section 3, some considerations regarding the asymptotic expansions (weak-field domain) of the metric are made, along with the classical, solar system tests; section 4 presents some results concerning the strong-field behaviour, and finally, in section 5 a discussion of these results is made.

2. The metric

The metric we shall use to describe the spacetime near the companion in a binary pulsar system is an axially symmetric solution to the action for gravity minimally coupled to a scalar field (here and throughout, units in which $c = G = 1$ are used):

$$S = \int \mathrm{d}^4x\, \sqrt{-g}(-R + 2\nabla^\mu \Phi \nabla_\mu \Phi), \tag{1}$$

where Φ is the scalar field. From the above action, the following field equations are obtained after variations with respect to the metric $g_{\mu\nu}$ and the scalar field Φ:

$$\Phi^{;\mu}{}_{;\mu} = 0, \tag{2}$$

$$R_{\mu\nu} = 2\Phi_{,\mu}\Phi_{,\nu}.$$

On the other hand, it can be shown [7] that it is possible to find axially symmetric solutions which are rather simple. In fact, the metric we shall study was actually obtained in [7] as a solution to dilatonic gravity, which is a theory originally derived from superstring-inspired models and features an exponential coupling between the scalar and any matter fields; for instance, for an electromagnetic field $F_{\mu\nu}$, an additional term $\mathrm{e}^{-2\alpha\Phi} F^{\mu\nu} F_{\mu\nu}$ should appear in the action, where α is a coupling constant. However, the equivalence principle tests put a very stringent bound on α, namely $\alpha < 10^{-12}$, while for string theory $\alpha = 1$. This implies that the full dilatonic theory is not well suited to model the gravitational interaction between macroscopic bodies. Therefore, we shall drop the dilatonic coupling (which does not eliminate the scalar field in the solutions found in [7]) and concentrate on the above, simpler theory. We believe that it is worth trying to use these solutions to model the exterior field for macroscopic bodies, in order to look for any possible difference in their predictions with those of GR or other scalar–tensor theories.

The solution we shall consider is then

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r}\right)\mathrm{d}t^2 + \mathrm{e}^{2k_s}\frac{\mathrm{d}r^2}{1 - 2m/r} + r^2(\mathrm{e}^{2k_s}\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2), \tag{3}$$

with

$$e^{2k_s} = \left(1 + \frac{m^2 \sin^2\theta}{r^2(1 - 2m/r)}\right)^{-1/b^2},$$

$$\Phi = \frac{1}{2b} \ln\left(1 - \frac{2m}{r}\right),$$

where b is a constant of integration. It can be observed that, in the limit when $b \to \infty$, we recover the Schwarzschild solution, as can also be noted if we calculate the scalar charge for this metric; following [2] we obtain

$$Q_s = \frac{1}{4\pi} \oint_S d^2 S^\mu \nabla_\mu \Phi = \frac{m}{b},$$

where the integration is over a 2-sphere of radius R and the limit is taken when $R \to \infty$ (Q_s can also be simply read off the $1/r$ term in the expansion of Φ in powers of $1/r$); in this case, when $b \to \infty$ we have $Q_s \to 0$, corresponding to GR.

Let us finally note that this solution does not describe a spacetime with rotation, which is what usually breaks the spherical symmetry; instead, its axial character arises from the presence of the scalar field, even though this field is spherically symmetric. While axially symmetric solutions with spherical sources are known in GR (see, e.g., [8]), the description of suitable sources with a scalar field is not straightforward; therefore, and because of its similarity with the Schwarzschild solution, we shall assume that the above metric can be employed to model the exterior field of a macroscopic object endowed with a minimally coupled scalar field, and that it can be matched to a regular interior solution. The analysis of this interior problem is currently being carried out numerically [9].

3. First and second post-Newtonian limits of the metric

The first consideration to be made about the metric in equation (3) when studied in an astrophysical context, is that it must satisfy the solar system tests; namely, light deflection, radar echo delay and the perihelion shift of Mercury. Since relevant physical quantities such as distances, velocities and gravitational fields are small everywhere in the solar system (with the possible exception of the surface of the Sun), one needs only to consider approximations involving the first or second powers of such quantities [10]. To do this for the exact solution, one takes the metric elements and performs a series expansion in powers of $1/r$; the Newtonian limit corresponds then to g_{tt} to order $O(1/r)$ and the rest of the elements to order $O(1)$, and including additional terms in the expansions the post-Newtonian approximations to different orders are obtained. As is well known, in the case of the solar system tests, the observational accuracy is within the first post-Newtonian (1PN) level.

It turns out that the 1PN expansion of the metric in equation (3) is identical to that of GR, which implies, among other things, that the scalar field makes no difference up to this limit. Nonetheless, some additional terms do appear at the second post-Newtonian (2PN) order, and they involve the constant b; since the contributions at this order must be within the observational errors, this gives the possibility to limit in some way the value of b, which in turn sets bounds on the ratio Q_s/m. Note also that usual computations make use of perturbative techniques starting from the field equations, instead of an exact solution.

3.1. Null geodesics

To illustrate the above, consider the light deflection effect: take a photon travelling from very far away, being deflected near the Sun and going away again. In order to find the deflection experienced by the photon, we must write down the null geodesic equation; following standard methods [11], and setting $\theta = \pi/2$ for simplicity (i.e. considering only equatorial trajectories), one is led to

$$\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 + \mathrm{e}^{-2k_s}\left(\frac{B^2}{r^2}\right)\left(1 - \frac{2m}{r}\right) = \mathrm{e}^{-2k_s}A^2 \tag{4}$$

where A and B are the constants of motion corresponding to t and φ, respectively,

$$A = \left(1 - \frac{2m}{r}\right)\frac{\mathrm{d}t}{\mathrm{d}s},$$

$$B = r^2\frac{\mathrm{d}\varphi}{\mathrm{d}s}.$$

Next, noting that $\mathrm{d}r/\mathrm{d}s = (\mathrm{d}r/\mathrm{d}\varphi)(\mathrm{d}\varphi/\mathrm{d}s)$, and using the fact that, at the minimum distance (say r_0) to the Sun, $\mathrm{d}r/\mathrm{d}\varphi = 0$, we obtain, as is the case for general relativity:

$$B = \frac{Ar_0}{\sqrt{1 - 2m/r_0}}.$$

Substituting back into the geodesic equation (4), the following expression can be derived:

$$\varphi(r_2) - \varphi(r_1) = \int_{r_1}^{r_2} \frac{\mathrm{d}r}{r\left((1-m/r)^2/(1-2m/r)\right)^{1/2b^2}\left[(r^2/r_0^2)\,(1-2m/r_0) - (1-2m/r)\right]^{1/2}}. \tag{5}$$

The total deflected angle for the ST theory $\Delta\varphi_{ST}$ is then, up to 2PN order

$$\Delta\varphi_{ST} = 2|\varphi(r) - \varphi(\infty)|_{r=r_0} - \pi \approx \frac{4m}{r_0} + \frac{m^2}{r_0^2}\left(\frac{15\pi}{4} - 4 - \frac{\pi}{4b^2}\right).$$

The result for GR can be obtained by taking the limit $b \to \infty$; denoting it by $\Delta\varphi_{GR}$, and making m and r_0 equal to the Sun's mass and radius, respectively, we arrive at

$$\epsilon_\varphi \equiv \left|\frac{\Delta\varphi_{ST} - \Delta\varphi_{GR}}{\Delta\varphi_{GR}}\right| \approx \frac{m\pi}{16r_0a^2}. \tag{6}$$

Now, as mentioned above, for the metric in equation (3) to represent approximately the Sun, the result from equation (6) must be within the observational error, which in turn limits the value of b. Taking an error of $\epsilon_\varphi \leqslant 0.1\%$ [12], one has $b > 2.1 \times 10^{-2}$. This in turn means that, as long as the coupling constant satisfies this inequality, the scalar field will still make no difference, as far as the light deflection in the solar system is concerned.

3.2. Test particles and other alternative ST theories

The presence of the scalar field introduces some ambiguity about which is the conformal frame representing physical quantities (i.e. measurable quantities); using a conformal transformation depending on the scalar field, we can rewrite our results for some specific theory (e.g. the Brans–Dicke theory). This leads us to wonder whether any difference appears if we consider effects such as those studied above in the frame of some of these conformally equivalent theories. Of course, it can be readily seen that the tests involving photons will look exactly the same, since null geodesics are conformally invariant. For massive test particles, however, this is not the case, so we shall analyse the situation with more care.

We use the following general result, which can be stated by somewhat cumbersome but straightforward calculations [13]: let $\mathcal{L}^*$ be a Lagrangian for a particular scalar–tensor theory (e.g. equation (1)), and let $\tilde{\mathcal{L}}$ be a Lagrangian obtained from $\mathcal{L}^*$ by a conformal transformation, say $\mathrm{d}\tilde{s}^2 = \Omega\,\mathrm{d}s_*^2$. It then follows that the equations of motion derived from $\tilde{\mathcal{L}}$ are identical to those obtained by applying the conformal transformation directly to the equations of motion derived from $\mathcal{L}^*$. This is important because, once we have a solution $\mathrm{d}s_*^2$ for one theory, a conformal transformation allows us to write a solution $\mathrm{d}\tilde{s}^2$ for some other theory. More specifically, we shall take $\Omega \equiv \Omega(\Phi)$, where Φ is the scalar field introduced in equation (1).

Let us continue taking the spacetime defined by equation (3) as a model for the Sun, and consider the perihelion shift of a test particle moving along a bounded trajectory, which is the usual way to treat Mercury. The geodesic equation is very similar to the null case, equation (4)

$$\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 + \mathrm{e}^{-2k_s}\left(1+\frac{B^2}{r^2}\right)\left(1-\frac{2m}{r}\right) = \mathrm{e}^{-2k_s}A^2. \tag{7}$$

Here we put $(\mathrm{d}r/\mathrm{d}\varphi)(r = r_\pm) = 0$, where r_+ and r_- are the distances from Mercury to the Sun at perihelion and aphelion, respectively. From this, the following expressions are obtained:

$$B^2 = \frac{2m\,(1/r_- - 1/r_+)}{(1/r_-^2)\,(1-2m/r_-) - (1/r_+^2)\,(1-2m/r_+)},$$
$$\frac{A^2}{B^2} = \frac{1}{2m}\left(1-\frac{2m}{r_+}\right)\left(1-\frac{2m}{r_-}\right)\left(\frac{1}{r_-}+\frac{1}{r_+}\right),$$

which, together with equation (7), allows us to write

$$\varphi(r_+) - \varphi(r_-) = \int_{r_-}^{r} \frac{\mathrm{d}x}{x^2\left((1-m/x)^2/(1-2m/x)\right)^{1/2b^2}\left[A^2/B^2 - \left(1/B^2 + 1/x^2\right)(1-2m/x)\right]^{1/2}}.$$

A direct expansion in powers of $1/r$ up to 2PN order leads to

$$\varphi(r_+) - \varphi(r_-) \approx \left[1 + m\left(\frac{1}{r_+}+\frac{1}{r_-}\right) + \frac{3}{2}m^2\left(\frac{1}{r_+}+\frac{1}{r_-}\right)^2\right] \times \int_{r_-}^{r} \frac{\mathrm{d}x}{x^2\left[(1/r_- - 1/x)\,(1/x - 1/r_+)\right]^{1/2}} \times \left[1 + \frac{m}{x} + \left(\frac{3}{2} - \frac{1}{2b^2}\right)\frac{m^2}{x^2}\right],$$

and the following expression is finally obtained for the perihelion shift, $\Delta\omega_{ST}$, up to 2PN order:

$$\Delta\omega_{ST} = 2|\varphi(r_+) - \varphi(r_-)| - 2\pi \approx \frac{6\pi m}{a(1-e^2)} + \left(19 - \frac{1}{b^2}\right)\frac{m^2\pi}{a^2(1-e^2)^2} + \left(3 - \frac{1}{b^2}\right)\frac{m^2\pi e^2}{2a^2(1-e^2)^2},$$

where e and a are the eccentricity and semimajor axis of the orbit of Mercury, respectively, and m is the Sun mass. Again, the result from GR, $\Delta\omega_{GR}$, can be obtained by taking $b \to \infty$, and putting $L \equiv (1-e^2)a$ one obtains

$$\epsilon_\omega \equiv \left|\frac{\Delta\omega_{ST} - \Delta\omega_{GR}}{\Delta\omega_{GR}}\right| \approx \frac{(2+e^2)m}{12Lb^2}. \tag{8}$$

When the error of 0.5% in the observations is taken into account [10], the result from equation (8) sets the limit $b > 1.2 \times 10^{-3}$, which is consistent with equation (6), as mentioned earlier.

Now, regarding the conformal transformation, we assume that, for $r \to \infty$, the conformal factor Ω can also be expanded in powers of $1/r$, i.e. $\Omega = b_0(1 + b_1/r + b_2/r^2 + \cdots)$, with $a_0 \neq 0$ and b_i constants. From this,

$$\begin{aligned}\Delta\omega_{ST(\Omega)} &= 2|\varphi(r_+) - \varphi(r_-)| - 2\pi \\ &\approx (3m + d_2)\frac{2\pi}{L} + \left[\left(19 - \frac{1}{b^2}\right)m^2 + (6m - d_2 + 4d_3)\right]\frac{\pi}{L^2} \\ &\quad + \frac{m^2\pi}{2}\left(3 - \frac{1}{b^2}\right)\frac{e^2}{L^2} + 2m\pi d_2\frac{1}{bL},\end{aligned}$$

where $d_2 \equiv (b_2 - 2mb_1)/(b_1 - 2m)$, $d_3 \equiv (b_3 - 2mb_2)/(b_1 - 2m)$. Let us specialize Ω for the following interesting cases.

(a) We can put $\Omega = \exp(-2a\Phi)$ and consider the metric $\mathrm{d}\tilde{s}^2$ obtained from equation (3) to be a solution for the Brans–Dicke theory, in which case we obtain the values $d_2 = d_3 = 0/0$. This result can be traced back to the constants of motion, since they have the values $A^2 = 1$, $B^2 = 0$; this in turn implies $\mathrm{d}\varphi/\mathrm{d}s = 0$, $(\mathrm{d}t/\mathrm{d}s)^2 = 1$, and there seems to be no dynamics.

(b) We can take into account the fact that the field equations obtained from the action (1) are left unchanged under the replacement $\Phi \to -\Phi$, and we can use this new Φ to transform again to the Brans–Dicke frame. Now we have $d_2 = -m$ and $d_3 = 0$. In this new frame, the mass of the spacetime, as measured by the test particle orbit, can be read off the $1/r$ term in the expansion of $\tilde{g}_{tt}$; putting $\tilde{g}_{tt} = -1 + 2M/r + \mathrm{O}(r^{-2})$, we see that $M \equiv m/2$, and obtain

$$\Delta\omega_{ST(\Omega)} \approx \frac{2\pi M}{L},$$

from which it can be noted that even the 1PN term differs from that obtained in GR.

(c) Finally, let us take a non-minimally coupled theory, using $\Omega = (1 + 2\xi\Phi)^{-1}$, where $\xi > 0$ is the coupling constant; in this case, $d_2 = m\zeta\,(1 - \zeta/(1 - \zeta))$, $d_3 = (2m^2\zeta\left(1/3 + \zeta - 2\zeta^2\right))/(1 - \zeta)$, where $\zeta \equiv \xi/b$; here the mass of the spacetime, as measured by the test particle orbit, is obtained by taking $M \equiv m(1 - \zeta)$ so that we have again $\tilde{g}_{tt} = -1 + 2M/r + \mathrm{O}(r^{-2})$; thus

$$\Delta\omega_{ST(\Omega)} \approx \frac{6\pi M}{L}\left(1 + \tfrac{4}{3}\zeta + \zeta^2\right),$$

from which only a lower bound for the ratio ζ can be found. Taking the aforementioned error of 0.5%, we see that $\zeta \leqslant 3 \times 10^{-3}$; for instance, if $b \sim 1$, then $\xi \leqslant 10^{-3}$.

4. Light deviation near a compact star

We now turn our attention to the strong-field domain, and in particular to the binary pulsar. The Shapiro delay and the periastron shift are two of the most impressive tests that GR has passed, but they are still concerned with perturbative expansions, since the quantities involved are functions of the ratio of the pulsar radius R_{PSR} to the binary pulsar orbit radius R_{ORB}, which is of the order of $R_{PSR}/R_{ORB} \sim 10^{-5}$ (see, e.g., [12]).

In recent years, a number of tests involving non-perturbative, strong-field effects have been proposed. For instance, the strong deflection of the pulsar signal by the companion, in systems where the latter is also a compact star (neutron star or black hole), was analysed in [14]; similarly, a differential measurement of the Shapiro time delay in pulsar–black-hole systems was suggested in [15]. In both cases, the metric near the companion is assumed to be either the Schwarzschild [14] or the Kerr [15] metric, i.e. use is made of general relativity. However, as mentioned in the introduction, the spontaneous scalarization effect suggests that a scalar field might actually be present in a neutron star, and we shall try to explore this hypothesis.

In what follows, we shall consider the test proposed in [14] in the context of our solution with a scalar field. The test can be summarized as follows: let us take a binary pulsar system in which the companion to the pulsar is a neutron star, and assume that the spacetime near the former can be described by the metric (3). Consider next the 'semiclassical' picture in which the pulsar emits a photon bundle which travels towards the companion in a straight path, is deflected instantaneously at the companion, and then travels away from the system, again in a straight path, reaching an observer at the Earth. Evidently, as the pulsar moves on its orbit, the angle θ that the photon must be deflected by to reach the Earth will vary, and the modulation of this 'indirect' signal with orbital phase would be a signature of a strong-field effect. In this approximation, the ratio between the intensity I of a bundle of strongly deflected photons and the intensity I_0 of the same bundle before deflection is, according to [14],

$$\frac{I}{I_0} = \left(1 - d\cos\chi\frac{\mathrm{d}\theta}{\mathrm{d}r_0}\right)^{-1}\frac{\sin\chi}{\sin\theta}, \tag{9}$$

where χ is the angle between the initial direction of the photon and the line connecting the two stars and d is the distance between them. To be observed at the Earth, the indirect beam will have to be deflected by an angle θ given by

$$\theta = \arccos(\sin i\,\sin(\phi + \omega)), \tag{10}$$

where ϕ is the angular position of the emitting star measured from periastron, ω the longitude of periastron and i the inclination of the binary pulsar system.

Now, to obtain the actual deflection angle in the semiclassical picture described above, we consider the simplified case where the photon comes from infinity, gets to a minimum distance r_0 and goes to infinity (to the Earth) again. The total deflection is given again by

$$\Delta\varphi = 2|\varphi(r_0) - \varphi(\infty)| - \pi, \tag{11}$$

where $\varphi(r)$ is given by equation (5). However, in this case r_0 will be of the order of the companion radius, which can be comparable to the radius R_{PSR} of the pulsar itself for pulsar–neutron star systems; because of this, we cannot expand the integral in powers of m/r to take only the first or second post-Newtonian terms, since $R_{PSR} \sim 10M_{PSR}$, where M_{PSR} is the pulsar mass. Instead, we shall integrate numerically equation (11), and from this the orbital modulation of the indirect signal, equation (9), can be obtained directly and plotted for some particular system. Moreover, since we want to compare this modulation for different values of b, it is convenient to 'normalize' it with respect to some reference, e.g. the same quantity obtained from the Schwarzschild metric, which was the original proposal for this test.

As is discussed in detail in [14], the system PSR1534+12 is one of the most promising binary pulsar candidates in which to measure this kind of strong deflection effect. If the photons are to be deflected by large angles, the pulsar beam must sweep sufficiently close to the companion, and there is evidence for the presence of a relatively broad beam in PSR1534+12 which is likely to fulfil this condition. Accordingly, using the orbital Keplerian parameters for this system, as well as standard numerical methods (see, e.g., [16]), we perform the following steps for several values of b:

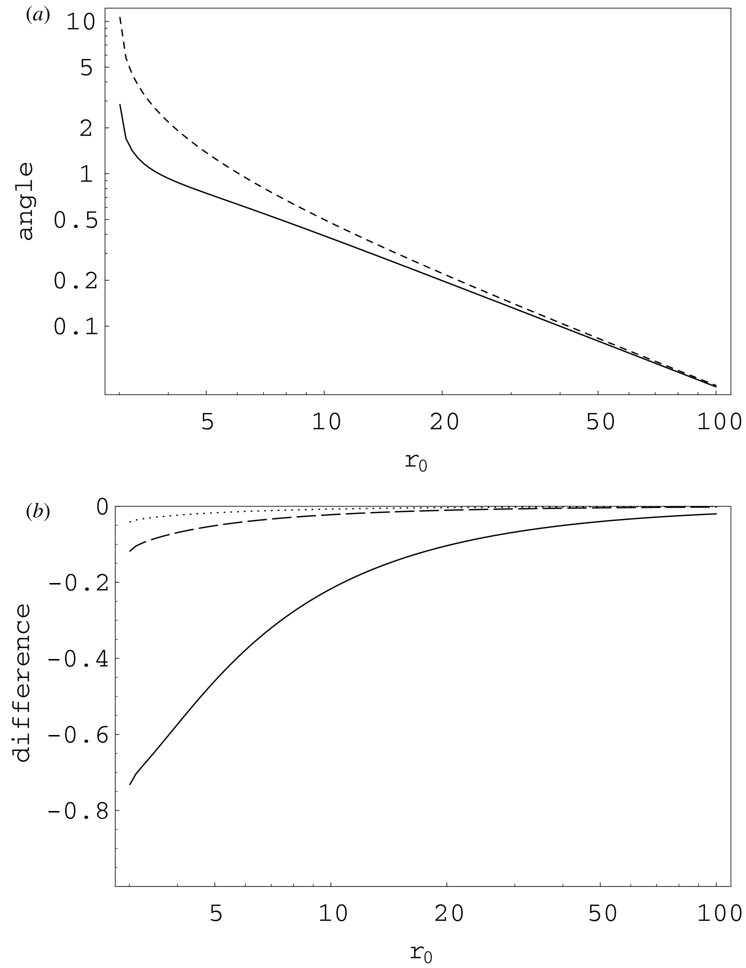

Figure 1. Comparison of the deflected angle for different values of b^2. (*a*) Values calculated from the Schwarzschild metric (broken) and for $b^2 = 0.1$ (full); (*b*) fractional difference, equation (12), for $b^2 = 3$ (full), $b^2 = 1$ (broken) and $b^2 = 0.1$ (dotted). The radius r_0 is expressed in units of m, with $G = c = 1$.

(a) integrate equation (11), or equivalently equation (5), for some range of r_0;
(b) find the derivative $\mathrm{d}\theta/\mathrm{d}r_0$ for the same range of r_0;
(c) using equation (10), calculate the deflection θ that the incident photon must experience to be observed at the Earth, as a function of the angular position of the emitting star ϕ along its orbit; if we now equate $\Delta\varphi$ from equation (11) with this deflection angle θ, we can determine the value of r_0 necessary for the photons to reach the observer, also as a function of ϕ;
(d) finally, given θ and r_0 from the previous step, we can numerically evaluate $\mathrm{d}\theta/\mathrm{d}r_0$, and obtain the intensity modulation, equation (9), also as a function of the angular position of the companion.

This procedure was carried out for $b^2 = 3$, 1 and 0.1, which are values allowed by the weak-field measurements in the solar system, as mentioned in section 3; the results are shown in figures 1 and 2. In figure 1(*a*), the deflection angle θ for the Schwarzschild metric and for

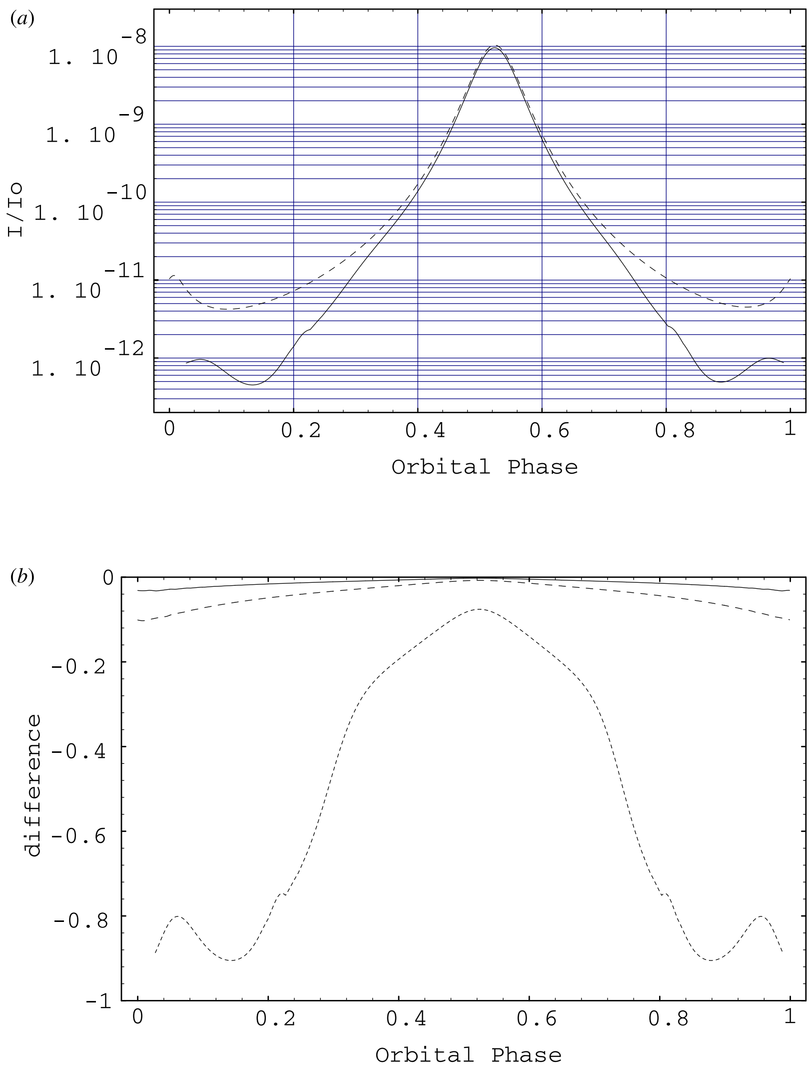

Figure 2. Comparison of the intensity reduction I/I_0 for different values of b^2 as a function of orbital phase (defined as $(t - T_0)/P_{ORB}$) for PSR 1534+12. (*a*) Values calculated from the Schwarzschild metric (broken) and for $b^2 = 0.1$ (full); (*b*) fractional difference, equation (13), for $b^2 = 3$ (full), $b^2 = 1$ (broken) and $b^2 = 0.1$ (dotted). The radius r_0 is expressed in units of m, with $G = c = 1$.

$b^2 = 0.1$ are plotted as a function of r_0 in the range $3m < r_0 < 100m$; figure 1(*b*) shows the quantity

$$\frac{\Delta\varphi_{ST} - \Delta\varphi_{Schwarzschild}}{\Delta\varphi_{Schwarzschild}} \tag{12}$$

for $b^2 = 3$, 1 and 0.1 in the same range of r_0.

Figure 2(*a*) shows the reduction I/I_0 of the intensity of the strongly deflected beam as a function of orbital phase for the Schwarzschild metric and for $b^2 = 0.1$, while figure 2(*b*) depicts the fractional difference

$$\frac{(I/I_0)_{ST} - (I/I_0)_{Schwarzschild}}{(I/I_0)_{Schwarzschild}} \tag{13}$$

for $b^2 = 3, 1$ and 0.1. The signal is strongly modulated with the orbital phase, although the ratio (9) is very small, of the order of 10^{-8}. For the scalar coupling we are considering here, the intensity ratio is even smaller as b decreases, being as far as one order of magnitude below the value from general relativity for $b^2 = 0.1$.

5. Discussion

In this work we have employed an exact, axially symmetric solution to the field equations of gravity minimally coupled to a scalar field to model an astrophysical object in the two different regimes in which gravitational tests are made, namely, the weak- and the strong-field regimes. In the case of the weak-field domain, we employed the metric (3) to model the Sun and calculated the deflection of light rays passing nearby. It turns out that the current observational errors for this effect set the limit $b > 0.02$ on the single free parameter appearing in the solution. A lower bound consistent with the above can be obtained from the perihelion shift of Mercury if we perform the calculations in the original Einstein–Fierz frame; it can be concluded that, even if a scalar field of the form (3) were actually present in the solar system, it cannot be presently detected. However, the results concerning the conformal transformation to other ST theories are interesting because this type of transformation is used in string theory, where a dilaton field coupled to curvature appears in the low-energy effective action [17]: it has been argued [18, 19] that one can, by this kind of field redefinition, work in the 'string frame' or in the Einstein frame. However, as has already been pointed out (see, e.g., [5]), one should be very careful when selecting a particular frame as the 'physical' one.

On the other hand, we have also investigated whether a scalar field of this type would exhibit some particular signature in a strong-field setting. To this end we have reanalysed the strong light deflection effect in a binary pulsar with a compact companion (neutron star); in this case we are interested in the fraction of the pulsar beam which is deflected by a large angle when passing near the companion. The pulsar PSR1534+12 seems to be one of the best candidates for this kind of measurement, and the ratio of the 'indirect' to the 'direct' beams I/I_0, as given by equation (9), was calculated for this system. In figure 2(*b*) we can note that this ratio decreases as b becomes smaller, and it is around one order of magnitude below the Schwarzschild value for $b^2 = 0.1$. This is to be expected, since smaller values of b correspond to larger values for the scalar charge, Q_s, and a correspondingly larger deviation from GR. Moreover, as figure 1 shows, the deflected angle for the same value of r_0 is smaller than the Schwarzschild angle as b decreases. This would imply that the incoming photon would experience a smaller deflection as $b \to 0$, i.e. it would 'feel' a weaker gravitational field with respect to the case without the scalar field. In this sense, it can be said that the scalar field has a negative contribution to the gravitational field, i.e. to the spacetime curvature around the compact object.

As was stressed in [14], due to the fact that the intensity of the strongly deflected part of the pulsar beam would be around eight orders of magnitude smaller than the direct beam, this effect seems quite difficult to measure even for the case discussed of PSR1534+12, and it would require the coherent summation of pulse data over a very long interval of time. Another drawback is the fact that the inclination of the orbital plane must be very close to 90°, which severely limits the number of suitable candidates. If this test is ever carried out, however, it would provide a way to set a limit on b from the strong-field regime, for instance by comparing the measured intensity with the intensity calculated for different values of b, such as the cases considered here.

Finally, it should be noted that binary pulsar effects involving 1PN quantities, such as the usual Shapiro delay or the frequency shift of the indirect signal, will be identical for the scalar field solution, and they have already been calculated in [14].

Acknowledgments

This work was partly supported by CONACyT, México, under grants 85975 (HVB), 3697-E (TM) and a co-joint project with the Physikalisches Institut of the Jena Universität, Germany. HV also wishes to thank Professor S Kopeikin for useful comments.

References

[1] Hulse R A and Taylor J H 1975 *Astrophys. J.* **195** L51
[2] Garfinkle D, Horowitz G T and Strominger A 1991 *Phys. Rev.* D **43** 3140
[3] Damour T and Esposito-Farèse G 1993 *Phys. Rev. Lett.* **70** 2220
[4] Virbhadra K S, Narasimha D and Chitre S M 1998 *Astron. Astrophys.* **337** 1
[5] Cho Y M and Keum Y Y 1998 *Class. Quantum Grav.* **15** 907
[6] Huterer D and Turner M S 1998 *Preprint* astro-ph/9808133
[7] Matos T, Núñez D and Quevedo H 1995 *Phys. Rev.* D **51** R310
[8] Matos T, Núñez D and Quevedo H 1998 *Phys. Lett.* A **249** 271
[9] Matos T and Villegas-Brena H in preparation
[10] Will C M 1981 *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge University Press)
[11] Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)
[12] Damour T and Taylor J H 1992 *Phys. Rev.* D **45** 1840
[13] Wald R 1984 *General Relativity* (Chicago, IL: University of Chicago Press)
[14] Campana S, Parodi A and Stella L 1995 *Mon. Not. R. Astron. Soc.* **277** 1162
[15] Laguna P and Wolszczan A 1997 *Preprint* astro-ph/9705054
[16] Press W H, Flannery B P, Teukolsky S A and Vetterling W T 1988 *Numerical Recipes in C* (Cambridge: Cambridge University Press)
[17] Fradkin E S and Tseytlin A A 1985 *Nucl. Phys.* B **261** 1
[18] 't Hooft G 1975 *Trends in Elementary Particle Theory* ed H Rollnik and K Dietz (Berlin: Springer)
[19] Damour T and Vilenkin A 1996 *Phys. Rev.* D **53** 2981