Class. Quantum Grav. **17** (2000) 1707–1712. Printed in the UK PII: S0264-9381(00)09416-8

Scalar field as dark matter in the universe

Tonatiuh Matos, F Siddhartha Guzmán and L Arturo Ureña-López

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, AP 14-740, 07000 México DF, Mexico

E-mail: `tmatos@fis.cinvestav.mx`, `siddh@fis.cinvestav.mx` and `lurena@fis.cinvestav.mx`

Received 9 November 1999

Abstract. We investigate the hypothesis that the scalar field is the dark matter and the dark energy in the cosmos, which comprises about 95% of the matter of the Universe. We show that this hypothesis explains quite well the recent observations on type Ia supernovae.

PACS numbers: 9880, 9535

There are really very few questions in science that are more interesting than that of finding the nature of the matter composing the Universe. It is amazing that even after so much effort has been dedicated to answering the question of what the Universe is composed of, it has not been possible to give a conclusive answer. From the latest observations, we do know that about 95% of the matter in the Universe is of non-baryonic nature. The old belief that matter in the cosmos is made up of quarks, leptons and gauge bosons is being abandoned due to the recent observations and inconsistencies which have arisen from this assumption [1]. Now we are convinced of the existence of an exotic non-baryonic type of matter which dominates the structure of the Universe, but its nature is still a puzzle.

Recent observations of the luminosity–redshift relation of Ia supernovae suggest that distant galaxies are moving slower than predicted by Hubble's law, that is, an accelerated expansion of the Universe seems to hold [2, 3]. Furthermore, measurements of the cosmic background radiation and the mass power spectrum also suggest that the Universe has the preferable value $\Omega_0 = 1$. There should exist a kind of missing antigravitational matter possessing a negative pressure $p/\rho = \omega < 0$ [4] which should overcome the enormous gravitational forces between galaxies. Moreover, the interaction with the rest of the matter should be very weak to pass unnoticed at the solar system level. These observations are without doubt among the most important discoveries of the end of the last century and they gave rise to the idea that the components of the Universe are matter and vacuum energy $\Omega_0 = \Omega_M + \Omega_\Lambda$. Models such as the quintessence (a slowly varying scalar field) imply $-1 < \omega < 0$ and that using a cosmological constant, requiring $\omega = -1$, appear to be strong candidates to account for the missing energy, because both of them satisfy an equation of state describing an accelerated behaviour of the Universe [5].

Observations in galaxy clusters and dynamical measurements of the mass in galaxies indicate that $\Omega_M \sim 0.4$ (see, for example, [6]). Observations of Ia supernovae indicate that $\Omega_\Lambda \sim 0.6$ [2, 3]. These observations are in very good agreement with the preferred value

$\Omega_0 \sim 1$. Everything seems to agree. Nevertheless, the matter component Ω_M decomposes itself in baryons, neutrinos, etc and dark matter. It is observed that stars and dust (baryons) represent something like 0.3% of the total matter of the Universe. The new measurements of the neutrino mass indicate that neutrinos contribute about the same quantity as matter. In other words, say $\Omega_M = \Omega_b + \Omega_\nu + \cdots \sim 0.05 + \Omega_{DM}$, where Ω_{DM} represents the dark matter part of the matter contributions which has a value $\Omega_{DM} \sim 0.35$. This value of the amount of baryonic matter is in agreement with the limits imposed by nucleosynthesis [1]. However, we do not know the nature of either the dark matter Ω_{DM} or of the dark energy Ω_Λ; we do not know the composition of $\Omega_{DM} + \Omega_\Lambda \sim 0.95$, i.e. 95% of the matter in the Universe.

In a previous work, two of the present authors have shown that the scalar field is a strong candidate to be the dark matter in spiral galaxies [7]. Using the hypothesis that the scalar field is the dark matter in galaxies, we were able to reproduce the rotation curve profiles of stars going around spiral galaxies. In fact, the scalar potential arising for the explanation of rotation curves of galaxies is exponential. Moreover, by using a Monte Carlo simulation, Huterer and Turner have been able to reconstruct an exponential potential for quintessence which brings the Universe into an accelerating epoch [8]. In this last work there is no explanation of the nature of dark matter, the value $\Omega_{DM} \sim 0.35$ is taken without further comment. Recently, there have been other papers where the late-time attractor solutions for the exponential potential have been studied [9–11]. If we are consistent with our previous work, this dark matter should also be of scalar nature, representing 35% of the matter of the Universe. In this paper we show that the hypothesis that the scalar field is the dark matter and the dark energy of the Universe is consistent with Ia supernovae observations and it could imply that the scalar field is the dominant matter in the Universe, determining its structure at a cosmological and at a galactic level. In other words, we demonstrate that the hypothesis that the scalar field represents more than 95% of the matter in the Universe is consistent with the recent observations on Ia supernovae.

We assume that the Universe is homogenous and isotropic, so we start with the Friedmann–Robertson–Walker (FRW) metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\phi^2\right)\right]. \tag{1}$$

The equations governing a Universe with a scalar field Φ and a scalar potential $V(\Phi)$ are

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{\mathrm{d}V}{\mathrm{d}\Phi} = 0, \tag{2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\kappa_0}{3}\left(\rho + \rho_\Phi\right) \tag{3}$$

where $\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$ is the density of the scalar field, ρ is the density of the baryons, plus neutrinos, plus radiation, etc, and $\kappa_0 = 8\pi G$. In order to write the field equations (2) and (3) in a more convenient form, we follow [12]. We define the function $F(a)$ such that $V(\Phi(a)) = F(a)/a^6$. Using the variable $\mathrm{d}\eta = 1/a^3\,\mathrm{d}t$, we can find a first integral of the field equation (2),

$$\tfrac{1}{2}\dot{\Phi}^2 + V(\Phi) = \frac{6}{a^6}\int \mathrm{d}a\,\frac{F}{a} + \frac{C}{a^6} = \rho_\Phi \tag{4}$$

where C is an integration constant. When the scale factor is considered as the independent variable, it is possible to integrate the field equations up to quadratures [12],

$$t - t_0 = \sqrt{3} \int \frac{\mathrm{d}a}{a\sqrt{\kappa_0(\rho_\Phi + \rho) - 3k/a^2}} \tag{5}$$

$$\Phi - \Phi_0 = \sqrt{6} \int \frac{\mathrm{d}a}{a} \left[\frac{\rho_\Phi - F/a^6}{\kappa_0(\rho_\Phi + \rho) - 3k/a^2} \right]^{1/2}. \tag{6}$$

In order to compare the data obtained from the Ia supernovae observations with a scalar-field-dominated Universe, we write the magnitude–redshift relation [2]

$$m_B^{\mathit{effective}} = \check{M}_B + 5 \log D_L(z; \Omega_i, \Omega_\Phi) \tag{7}$$

where $D_L = H_0 d_L$ is the 'Hubble-constant-free' luminosity distance and $\check{M}_B := M_B - 5 \log H_0 + 25$ is the 'Hubble-constant-free' B-band absolute magnitude at the maximum of a Ia supernovae. The luminosity distance D_L depends on the model we are working with. In what follows we compare the observational measurements obtained for $m_B^{\mathit{effective}}$ with a theory defining a scalar-field-dominated Universe. Using equation (4), the luminosity distance which depends on the geometry and on the contents of the Universe in the FRW cosmology (see, for example, [13]), reads for our case as

$$d_l(z; \Omega_i, \Omega_\Phi, H_0) = \frac{(1+z)}{H_0\sqrt{|k|}} \operatorname{sinn}\left(\sqrt{|k|} \int_{1/(1+z)}^{1} \frac{\mathrm{d}x}{\sqrt{U_\Phi}} \right) \tag{8}$$

where

$$U_\Phi := \left(\sum_i \Omega_i x^{(1-3w_i)} \right) - x^2(1 - \Omega_0) + \frac{1}{\rho_c x^2} \left(6 \int \mathrm{d}x' \, \frac{F(x')}{x'} + C \right) \tag{9}$$

and

$$\operatorname{sinn}(r) = \begin{cases} \sin(r) & (k = +1) \\ r & (k = 0) \\ \sinh(r) & (k = -1) \end{cases}$$

where i labels b (baryonic), ν (neutrinos), r (radiation), etc with equations of state $p_i = w_i \rho_i$ for each component. If we rescale $a_0 = 1$ today, then $x = a = 1/(1+z)$, where z is the redshift. Let us now compare the expression (8) with the function used to fit SNe Ia measurements [14], with an equation of state $p_x = w_x \rho_x$ for the unknown energy. In this case the luminosity distance is given by equation (8) with U_X in place of U_Φ, where

$$U_X := \left(\sum_i \Omega_i x^{(1-3w_i)} \right) - x^2(1 - \Omega_0) + x^{(1-3w_x)} \Omega_x. \tag{10}$$

Observe that both expressions (9) and (10) are very similar, the only differences are the integral term and the one containing the constant C. Thus, this comparison strongly suggests that $C = 0$ and $F(x) = V_0 x^s$, with V_0 a constant.

Within a good approximation, we can neglect the present contribution of the density of baryons, neutrinos, etc, $\rho_{om} \ll \rho_{o\Phi}$ because their contribution represents less than 5% of the matter of the Universe. The next step is to determine the scalar field potential. Fortunately a flat Universe dominated by scalar field with the function $F = V_0 a^s$ has a very important property. We can enunciate this property in the following theorem.

Theorem 1. *Let $\rho_\Phi = (6/a^6)\int(F/a)\,\mathrm{d}a$ with $F = V_0 a^s$ in a flat Universe dominated by a scalar field. Then the scalar field potential $V(\Phi)$, is essentially exponential in the regions where the scalar energy density dominates.*

Proof. If the Universe is flat, $k = 0$. From equation (6) it follows that

$$\Phi = \sqrt{\frac{6-s}{\kappa_0}}\int\frac{\mathrm{d}a}{a}\sqrt{\frac{1}{1+\rho_m/\rho_\Phi}}. \qquad (11)$$

Thus, if the scalar field dominates ($\rho_m \ll \rho_\Phi$), this implies $a \simeq \exp(\sqrt{\kappa_0/(6-s)}\Phi)$. Then, it follows that $V(\Phi) = F(a)/a^6 \simeq V_0\exp(-\sqrt{\kappa_0(6-s)}\Phi)$. □

This result strongly states that the scalar potential can only be exponential when the scalar field dominates with no other possibilities like 'power law' or 'cosine'.

The theorem fulfils the present conditions of the Universe very well with the hypothesis we are investigating. Thus, we will take an exponential potential for our model of the Universe, which implies an extraordinary agreement with the scalar potential used to explain the rotation curves of galaxies [7].

With the conditions $C = 0$ and $F = V_0 a^s$, equations (5) and (4) are easily integrated for a flat Universe. One obtains [9, 12]

$$a(t) = (K(t-t_0))^\lambda$$
$$\Phi - \Phi_0 = \sqrt{\frac{6-s}{\kappa_0}}\ln a$$

where $\lambda = 2/(6-s)$. The important quantities obtained from the solution in terms of the parameter λ are the scalar field and the scalar potential,

$$\Phi(a(t)) = \sqrt{\frac{2}{\kappa_0\lambda}}\ln(a) \qquad (12)$$

$$V(\Phi) = V_0\exp\left(-\sqrt{\frac{2\kappa_0}{\lambda}}\Phi\right), \qquad (13)$$

the energy density of the scalar field

$$\rho_\Phi = \rho_{o\Phi}a^{-2/\lambda}$$
$$\rho_{o\Phi} = \frac{6V_0}{6-2/\lambda},$$

the state equation of the scalar field

$$w_\Phi = \frac{2}{3\lambda} - 1$$

where $p_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi) = w_\Phi\rho_\Phi$. The scale factor

$$a(t) = \left(\frac{t}{t_0}\right)^\lambda,$$

where t_0 is a normalization constant. The Hubble parameter

$$H = \frac{\dot{a}}{a} = \lambda t^{-1}$$

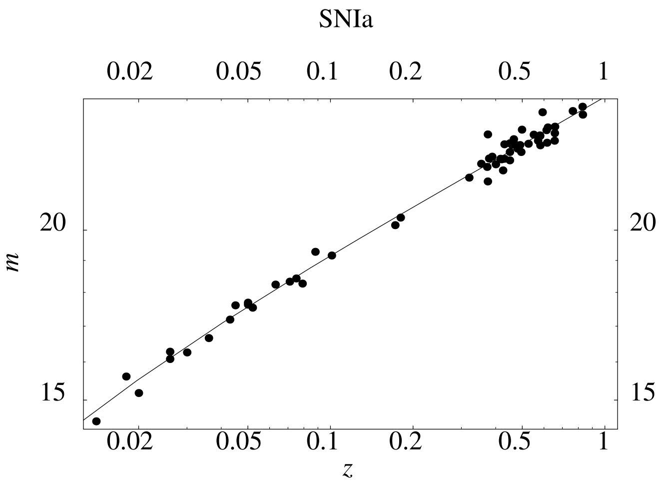

Figure 1. Fit of the solution obtained for the value $\lambda = 1.83$. The full circles represent the observational results and the full curve represents $m(z) = \check{M} + 5\log D_L$.

and the deceleration parameter

$$q = -\frac{\ddot{a}}{\dot{a}^2}a = -\frac{\lambda - 1}{\lambda}.$$

According to the solution (12) and (13), the expression for the luminosity distance now reads

$$d_l(z;\lambda, V_0, H_0) = \frac{(1+z)\lambda}{H_0(1-\lambda)}\left[\frac{6V_0}{s\rho_c}\right]^{-1/2}\left[1-(1+z)^{(1-1/\lambda)}\right] \tag{14}$$

where $w_\Phi = 1 - s/3$ and we have rescaled $a_0 = 1$ today, for $\lambda \neq 1$. Fitting (14) with the data of Ia supernovae [2, 14] we find $\lambda = 1.83$ and $V_0 = 0.78\rho_c$ for $\rho_{0\Phi} \sim 0.95\rho_c$ where ρ_c is the critical density ($\rho_c = 0.92 \times 10^{-29}$ g cm^{-3}) (see figure 1).

Now, we can calculate the deceleration parameter. We obtain $q_0 = -0.45 =$ constant, which really implies that the Universe is accelerating. For the density of the scalar field we obtain $\rho_\Phi = 0.95\rho_c a^{-1.09}$ and for its equation of state $w_\Phi = -0.636 =$ constant. Currently, we are investigating the cosmic microwave background radiation (CMBR) and the mass power spectrum. See [5] for a scalar field with the equation of state $w = -\frac{2}{3}$ and Ω_Φ up to 0.8, where it is concluded that the scalar field fits all the required observations. If we use $H_0 = 70$ km s^{-1} Mpc^{-1}, we find

$$t_0 = 25.6 \times 10^9 \text{ yr}.$$

t_0 would be the age of a Universe that was always dominated by the scalar field, which is not our case.

The good agreement of our hypotheses with experimental results suggests that the Universe lies at the current time in a scalar-field-dominated epoch. This permits us to speculate about the behaviour of the Universe for redshifts greater than $z = 1$ as restricted by SNIa observations. Observe that our results do not imply that the Universe has been dominated by a scalar field during all of its evolution. Instead, our model accepts the possibility of a Universe dominated by radiation or matter before the epoch we have analysed. In order to draw a complete history of the Universe, we consider the periods of radiation- and matter-dominated eras. A general integration of the conservation equation for a perfect fluid made of radiation (dust), indicates that the density scales as $\rho_r = \rho_{0r} a^{-4}$ ($\rho_m = \rho_{0m} a^{-3}$), with $\rho_{0r} = 10^{-5}\rho_c$ ($\rho_{0m} = 0.05\rho_c$). In the FRW standard cosmology, the Universe was radiation dominated until $a \sim 10^{-3}$, the time when the density of radiation equals the density of matter. Recalling our result $\rho_\Phi = \rho_{0\Phi} a^{-1.09}$, the Universe changed to matter dominated until $a \sim 0.21$, when the density of the scalar field equals the density of matter. At this time, the density of radiation is negligible. This corresponds to redshifts of $z = 3.7$. The implications of this model are very strong. Since this time (approximately 14×10^9 yr ago for this model), the scalar field began to dominate the expansion of the Universe and it enters in its actual acceleration phase, which includes most of the history of the Universe. Then we wonder whether the scalar field is responsible for the formation of structure too. According to [15], the formation of galaxies started at redshifts, of approximately 4.5 to 2, just when the scalar field began to be important.

Some final remarks. With our values, the solution is singular, i.e. $a(t)$ vanishes at some finite time. Moreover, the solution has no particle horizon [12] as can be seen from the expression (5) because $s > 4$. The question of why nature uses only spin-1 and spin-2 fundamental interactions over the simplest spin-0 interactions becomes clear with our result. This result tells us that, in fact, nature has preferred the spin-0 interaction over the other two and in such a case, the scalar field should thus be responsible for the structure of the cosmos.

Acknowledgments

We would like to thank Dario Nuñez and Michael Reisenberger for many helpful discussions. This work was partly supported by CONACyT, México, under grants 3697-E (TM), 94890 (FSG) and 119259 (LAU).

References

[1] Schramm D N 1998 *Nuclear and Particle Astrophysics (Cambridge Contemporary Astrophysics)* ed J G Hirsch and D Page
Shi X, Schramm D N and Dearborn D 1995 *Phys. Rev.* D **50** 2414–20
[2] Perlmutter *et al* 1999 *Astrophys. J.* **517** 565
[3] Riess A G *et al* 1998 *Astron. J.* **116** 1009–38
[4] Ostriker J P and Steinhardt P J 1995 *Nature* **377** 600
[5] Caldwell R R, Rahul D and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582–5
Zlatev I, Wang L and Steinhardt P J 1999 *Phys. Rev. Lett.* **82** 896–9
[6] Turner M S 1999 *Astron. Soc. Pac. Conf. Series* vol 666
(Turner M S 1998 *Preprint* astro-ph/9811454)
[7] Guzmán F S and Matos T 2000 *Class. Quantum Grav.* **17** L9–16
[8] Huterer D and Turner M S 1998 *Preprint* astro-ph/9808133
[9] Ferreira P G and Joyce M 1998 *Phys. Rev.* D **58** 023503
[10] de la Macorra A and Piccinelli G 1999 *Preprint* hep-ph/9909459
[11] Barreiro T, Copeland E J and Nunes N J 1999 *Preprint* astro-ph/9910214
[12] Chimento L P and Jakubi A S 1996 *Int. J. Mod. Phys.* D **5** 71
[13] Schmidth B P *et al* 1998 *Astrophys. J.* **509** 74
[14] Perlmutter *et al* 1997 *Astrophys. J.* **483** 565
[15] Madau P 1999 *Phys. Scr. , Proc. Nobel Symp. on Particle Physics and the Universe (Enkoping, Sweden, 20–25 August 1998)*
(Madau P 1999 *Preprint* astro-ph/9902228)
Turner M S 1998 Cosmology update 1998 *Proc. Wein '98 (Santa Fe, NM, June 1998)* ed J M Bowles, P Herczog and C Hoffman (Singapore: World Scientific)
(Turner M S 1999 *Preprint* astro-ph/9901168)