Hydrodynamical description of Galactic Dark Matter*

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We consider simple hydrodynamical models of galactic dark matter in which the galactic halo is a self-gravitating and self-interacting gas that dominates the dynamics of the galaxy. Modeling this halo as a sphericaly symmetric and static perfect fluid satisfying the field equations of General Relativity, visible barionic matter can be treated as "test particles" in the geometry of this field. We show that the assumption of an empirical "universal rotation curve" that fits a wide variety of galaxies is compatible, under suitable approximations, with state variables characteristic of a non-relativistic Maxwell-Boltzmann gas that becomes an isothermal sphere in the Newtonian limit. Consistency criteria lead to a minimal bound for particle masses in the range 30 eV $\leq m \leq$ 60 eV and to a constraint between the central temperature and the particles mass. The allowed mass range includes popular supersymmetric particle candidates, such as the neutralino, axino and gravitino, as well as lighter particles ($m \sim$ keV) proposed by numerical N-body simulations associated with self-interactive "cold" and "warm" dark matter structure formation theories.

Keywords: Hydrodynamics; dark matter; thermodynamic properties; models beyond the standard model

Consideramos modelos hidrodinámicos simples de materia oscura galáctica en los cuales el halo galáctico es un gas autogravitante y autointeractivo que domina la dinámica de la galaxia. Modelando este halo como un fluido perfecto estático y esféricamente simétrico que satisface las ecuaciones de relatividad general, la materia visible bariónica puede tratarse como partículas de prueba en la geometría de este campo. Mostramos que la suposición de una curva de rotación universal que ajusta una amplia variedad de galaxias es compatible, bajo aproximaciones apropiadas, con variables de estado características de un gas no relativista de Maxwell-Boltzmann que en el límite Newtoniano se convierte en una esfera isoterma. Criterios de consistencia dan una cota mínima para las masas de las partículas de 30 eV $\leq m \leq$ 60 eV y una constricción entre la temperatura central y la temperatura de las partículas. El rango de masas permitido incluye candidatos supersimétricos populares como el neutralino, axino y gravitino, así como partículas más ligeras ($m \sim$ keV) propuestas por simulaciones numéricas asociadas con teorías de formación de estructura de materia oscura fría y tibia.

Descriptores: Hidrodinámica; materia oscura; propiedades termodinámicas; modelos alternativos del modelo standard

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1. Introduction

The presence of large amounts of dark matter at the galactic lengthscale is already an established fact. It is currently thought that this dark matter is made of relique self-gravitating gases which are labeled as "cold" (CDM) or "hot" (HDM), depending on the relativistic or non-relativistic nature of the particles energetic spectrum at their decoupling from the cosmic mixture [1–3]. HDM scenarios are not favoured, as they seem to be incompatible with current theories of structure formation [2–4]. CDM, usualy examined within a newtonian framework, can be considered as non-interactive (collisionless particles) or self-interactive [5]. N-body numerical simmulations are often used for modeling CDM gases [6–9]. However, in recent numerical simulations (see Refs. 7–9) non-interactive CDM models present the following discrepancies with observations at the galactic scale [10, 11]: (a) the "substructure problem" related to excess clustering on sub-galactic scales, (b) the "cusp problem" characterized by a monotonic increase of density towards the center of halos, leading to excessively concentrated cores. In order to deal with these problems, the possibility of self-interactive dark matter has been considered, so that nonzero pressure or thermal effects can emerge, thus leading to self-interactive models of CDM (*i.e.* SCDM) [12–16] and "warm" dark matter (WDM) models [17–22] that challenges the duality CDM *vs.* HDM. Other proposed dark matter sources consist replacing the gas of particles approach by scalar fields [23, 24] and even more "exotic" sources [25].

Whether based on SCDM or WDM, current theories of structure formation point towards dark matter characterized by particles having a mass of the order of at least keV's (see Ref. 12–22), thus suggesting that massive but light particles, such as electron neutrinos and axions (see Table I), should be

TABLE I. Particle candidates for a MB Dark Matter gas.

SCDM/WDM Light Candidates	mass in keV	References
Light Gravitino	~ 0.5	[35]
”	$\sim 0.75 - 1.5$	[36, 37]
Sterile Neutrino	$\sim 2.6 - 5$	[38]
”	< 40	[39]
”	$1 - 100$	[40]
Standard Neutrinos	~ 1	[41, 42]
Light Dilaton	~ 0.5	[43]
Light Axino	~ 100	[44]
Majoron	~ 1	[45–47]
Mirror Neutrinos	~ 1	[48, 49]
CDM Heavy Candidates	**mass in GeV**	**References**
Neutralino	> 32.3	[50]
	> 46	[51]
Axino	~ 10	[52, 53]
Gravitino	$\lesssim 100$	[54]

eliminated as primary dark matter candidates (though there is no reason to assume that these particles would be absent in galactic halos). Of all possible weakly interactive massive particles (WIMPS), complying with the required mass value of relique gases, only the massive Neutrinos (the muon or tau neutrinos), have been detected, whereas other WIMPS (neutralino, gravitino, photino, sterile neutrino, axino, etc.) are speculative. See Ref. 26–28 and Table I for a list of candidate particles and appropriate references.

In this paper we develop an alternative description of galactic DM. Since the dark matter halo constitutes about 90% of the galactic mass, we consider the galactic garvitational field as a spacetime whose sole, self-gravitating, source is this halo, described as a perfect fluid. Assuming this galactic spacetime to be static and sphericaly symmetric, the barionic dark matter can be though of as test particles following stable and circular geodesics of this spacetime curvature. Since the tangential velocity, v, along these geodesics can be calculated for such a spacetime, we can express the field equations in terms of of v. However, the velocity profile $v = v(r)$ has been observed for a wide range of galaxies, leading to “Universal Rotation Curves” (URC’s) that provide an empiric fit to these rotation velocities [31]. Therefore, by inserting the empiric formula for the URC derived by Persic and Salucci [30] into the field equations [given in terms of $v(r)$], we obtain a constraint on one of the metric coefficients. Once this constraint is solved, we can obtain the state variables characterizing the galactic dark fluid associated with this URC. We solve this constraint assuming that the velocities are non-relativistic, thus expanding around $v_0/c \ll 1$, where v_0 is the terminal velocity associated with the flattening of the URC.

2. Field equations

Considering the line element of a static spherically symmetric space time

$$ds^2 = -A^2(r)\, c^2\, dt^2 + \frac{d\,r^2}{1 - 2\,M(r)/r} + r^2\,\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad (1)$$

the tangential velocity of test particles along stable circular geodesic orbits can be expressed in terms of the metric coefficients as

$$\frac{V^2}{c^2} \equiv v^2(r) = \frac{r\,A'}{A}. \quad (2)$$

Becomes a dynamical variable replacing $A(r)$. Assuming as source of (1) a perfect fluid momentum energy tensor: $T^{ab} = (\rho + p)\, u^a u^b + p\, g^{ab}$, with $u^a = A^{-1}\,\delta^a{}_{ct}$, the following field equations in terms of (2) become

$$M' + \frac{\left(-3 - 5\,v^2 + 4\,vv'\,r + 2\,v^4\right) M}{r\,(1+v^2)} - \frac{v\,\left(-2\,v + 2\,v'\,r + v^3\right)}{1+v^2} = 0. \quad (3)$$

$$\kappa\, p \;=\; 2\,\frac{-M - 2\,Mv^2 + v^2 r}{r^3}, \quad (4)$$

$$\kappa\,\rho = \frac{\left[-8\,vv'\,r - 2\,\left(2\,v^2+1\right)\left(v^2 - 3\right)\right] M}{r^3\,(1+v^2)} + \frac{4\,r^2 vv' + 2\,v^2\left(-2 + v^2\right) r}{r^3\,(1+v^2)}, \quad (5)$$

where $\kappa = 8\pi G/c^4$ and a prime denotes derivative with respect to r. Writing the field equations in terms of the orbital velocity, v, provides a useful insight into how an (in principle) observable quantity relates to spacetime curvature and with physical quantities (state variables) which characterize the source of spacetime.

3. Thermodynamics

If we assume that the self gravitating ideal “dark” gas exists in physical conditions far from those in which the quantum properties of the gas particles are relevant, we would be demanding that these particles comply with Maxwell-Boltzmann (MB) statistics. Following Ref. 34, the condition that justifies an MB distribution is given by

$$\frac{n\,\hbar^3}{(m\,k_B\,T)^{3/2}} \ll 1, \quad (6)$$

where n, T, $\hbar$ and k_B are, respectively, the particle number density, absolute temperature, Planck’s and Boltzmann’s constants. If the constraint (6) holds and we further assume thermodynamical equilibrium and non-relativistic conditions, the ideal dark gas must satisfy the equation of state of a non-relativistic monatomic ideal gas

$$\rho = mc^2 n + \frac{3}{2} n k_{_B} T, \qquad p = n\, k_{_B} T, \tag{7}$$

whose macroscopic state variables can be obtained from a MB distribution function under an equilibrium Kinetic theory approach (the non-relativistic and non-degenerate limit of the Jüttner distribution) [32]. An equilibrium MB distribution restricts the geometry of spacetime [33], resulting in the existence of a timelike Killing vector field $\beta^a = \beta\, u^a$, where $\beta \equiv mc^2/k_{_B}T$, as well as the following relation (Tolman's law) between the 4-acceleration and the temperature gradient

$$\dot{u}_a + h_a^b (\ln T)_{,b} = 0, \qquad h_a^b = u_a u^b + \delta_a^b, \tag{8}$$

leading to

$$\frac{A'}{A} + \frac{T'}{T} = 0 \qquad \Rightarrow \qquad T \propto A^{-1}. \tag{9}$$

The particle number density n trivialy satisfies the conservation law $J^a{}_{;a} = 0$ where $J^a = n\, u^a$, thus the number of dark particles is conserved. Notice that given (4) and (5), the equation of state (7) and the temperature from the Tolman law (9), we have two different expressions for n

$$n = \frac{p}{k_{_B} T} \;\propto\; p\, A, \tag{10}$$

$$n = \frac{1}{mc^2}\left[\rho - \frac{3}{2} p\right]$$
$$= \frac{\left[-8\, v v' \, r + \left(v^2 + 9\right)\left(2\, v^2 + 1\right)\right] M}{\kappa\, mc^2\, r^3\, (1 + v^2)} + \frac{+4\, v v' \, r^2 - v^2 \left(7 + v^2\right) r}{\kappa\, mc^2\, r^3\, (1 + v^2)}. \tag{11}$$

The quantity $mc^2\, n$ in (11) follows directly from Eqs. (4) and (5), while n in (10) also follows from p in (4) with $A \propto \exp[\int (v^2/r) dr]$. Consistency requires that (10) and (11) yield the same expression for n.

4. Dark fluid hydrodynamics

We shall assume for v^2 the empiric dark halo rotation velocity law given by Persic and Salucci [30, 31]

$$v^2 = \frac{v_0^2\, x^2}{a^2 + x^2}, \qquad x \equiv \frac{r}{r_{\rm opt}} \tag{12}$$

where $r_{\rm opt}$ is the "optical radius" containing 83% of the galactic luminosity, whereas the empiric parameters a and v_0, respectively, the ratio of "halo core radius" to $r_{\rm opt}$ and the "terminal" rotation velocity, depend on the galactic luminosity. For spiral galaxies we have: $v_0^2 = v_{\rm opt}^2 (1 - \beta)(1 + a^2)$, where $v_{\rm opt} = v(r_{\rm opt})$ and the best fit to rotation curves is obtained for: $a = 1.5\, (L/L_*)^{1/5}$ and $\beta = 0.72 + 0.44 \log_{10}(L/L_*)$, where $L_* = 10^{10.4} L_\odot$. The range of these parameters for spiral galaxies is 125 km/sec $\leq v_0 \leq$ 250 km/sec and $0.6 \leq a \leq 2.3$.

Inserting (12) into (2) and (3) we obtain

$$A = \left[1 + x^2\right]^{v_0^2/2} \Rightarrow T = T_c \left[1 + x^2\right]^{-v_0^2/2}, \tag{13}$$

$$M = \frac{(v_0^2 - 2)(a^2 + x^2)^{2 - v_0^2}\, v_0^2\, x^3}{\left[a^2 + (1 + v_0^2) x^2\right]^{2/(1+v_0^2)}} \times r_{\rm opt} \int \frac{\left[a^2 + (1 + v_0^2) x^2\right]^{(1 - v_0^2)/(1+v_0^2)}\, x dx}{(a^2 + x^2)^{3 - v_0^2}}, \tag{14}$$

where $T_c = T(0)$ and we have set an integration constant to zero in order to comply with the consistency requirement that $v_0 = 0$ implies flat spacetime ($A = 1$, $M = 0$). Since the velocities of rotation curves are newtonian, $v_0 \ll c$ (typical values are $v_0/c \approx 0.5 \times 10^{-3}$), instead of evaluating (14) we will expand this quadrature around v_0/c (in order to keep the notation simple, we write v_0 instead of v_0/c). This yields

$$M = \frac{x^3\, r_{\rm opt}}{a^2 + x^2}\, v_0^2 \left[1 - \frac{5x^2 + 2a^2}{2(a^2 + x^2)} v_0^2 + \frac{12x^4 + 11a^2x^2 + 3a^4}{2(a^2 + x^2)^2}\, v_0^4 + \mathcal{O}(v_0^6)\right], \tag{15}$$

we obtain the expanded forms of ρ and p by inserting (12) and (14) into (4) and (5) and then expanding around v_0, leading to

$$\kappa \rho r_{\rm opt}^2 = \frac{2(3a^2 + x^2)}{(a^2 + x^2)^2} v_0^2 - \frac{5x^4 + 23a^2x^2 + 6a^4}{(a^2 + x^2)^3}\, v_0^4 + \mathcal{O}(v_0^6), \tag{16}$$

$$\kappa p r_{\rm opt}^2 = \frac{2a^2 + x^2}{(a^2 + x^2)^2}\, v_0^4 - \frac{2x^4 + 7a^2x^2 + 3a^4}{(a^2 + x^2)^3}\, v_0^6 + \mathcal{O}(v_0^8), \tag{17}$$

while the expanded form for T follows from (13)

$$T = T_c \left[1 - \frac{1}{2} \ln\left(1 + x^2\right), v_0^2 + \frac{1}{8} \ln^2\left(1 + x^2\right)\, v_0^4 - \mathcal{O}(v_0^6)\right]. \tag{18}$$

In order to compare n obtained from (10) and (11), we substitute (12) and (14) into (11) and expand around v_0, leading to

$$n r_{\rm opt}^2 = \frac{1}{\kappa\, m\, c^2}\left[\frac{2(3a^2 + x^2)}{(a^2 + x^2)^2} v_0^2 - \frac{18x^4 + 55a^2x^2 + 13a^4}{2(a^2 + x^2)^3}\, v_0^4 + \mathcal{O}(v_0^6)\right], \tag{19}$$

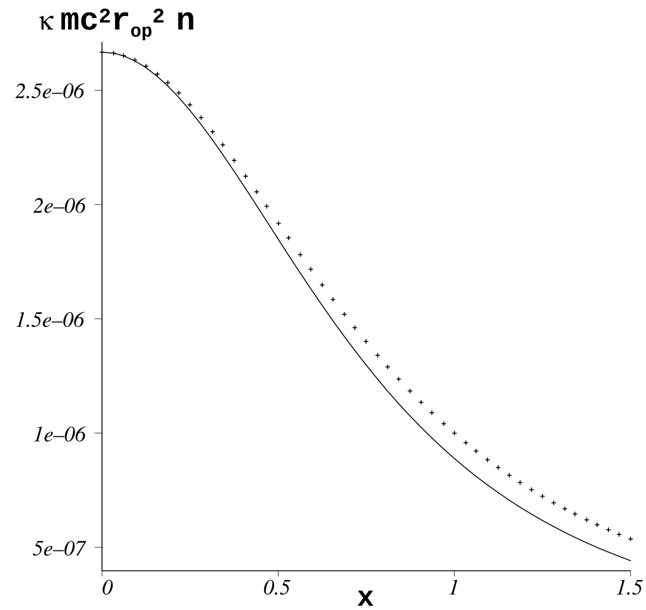

FIGURE 1. Comparison of n obtained from (19) and (20). This plot displays the adimensional quantity $\kappa mc^2 r_{\rm op}^2\, n$, as a function of x, obtained from truncating the right hand sides of (19) (solid curve) and (20) (dotted curve with crosses) up to fourth order in v_0/c and assuming the consistency condition (21). Since we are using an empiric law for observed galactic rotation curves [the URC given by (12)], the fact that these two expressions for n are so close to each other provides an empirical justification for the compatibility between MB distribution and these rotation curves.

while n in (10) follows by substituting (14) into (17), using T from (13) and then expanding around v_0. This yields

$$n r_{\rm opt}^2 = \frac{1}{\kappa\, k_{\scriptscriptstyle B} T_c}\left[\frac{2a^2+x^2}{(a^2+x^2)^2}\, v_0^4 + \frac{(2a^2+x^2)(a^2+x^2)\ln\left(1+x^2\right)}{2(a^2+x^2)^3} v_0^6 - \frac{2(a^2+2x^2)(3a^2+x^2)}{2(a^2+x^2)^3}\, v_0^6 + \mathcal{O}(v_0^8)\right]. \quad (20)$$

Since $v_0/c \ll 1$, a reasonable approximation is obtained if the leading terms of n from (19) and (20) coincide. By looking at these equations, it is evident that this consistency requirement implies

$$\frac{1}{2}\, m v_0^2 \;=\; \frac{3}{2}\, k_{\scriptscriptstyle B} T_c, \quad (21)$$

where v_0 denotes a velocity (cm/sec) and not the adimensional ratio v_0/c. Since higher order terms in v_0/c have a minor contribution, the two forms of n are approximately equal. This is shown in Fig. 1 displaying the adimensional quantity $\kappa\, mc^2 n\, r_{\rm opt}^2$ from (19) and (20) as functions of x for typical values $v_0/c = 0.0006$, $a = 1$ and eliminating T_c with (21). Equation (14) shows how "flattened" rotation curves, as obtained from the empiric form (12), lead to $M \propto r^3$ for $r \approx 0$ and $M \propto r$ for large r. Equations (15) to (21) represent a relativistic generalization of the "isothermal sphere" that follows as the newtonian limit of an ideal Maxwell-Boltzmann gas characterized by $\rho \approx mc^2 n, p \ll \rho$ and $T \approx T_c$. In fact, using newtonian hydrodynamics we would have obtained only the leading terms of Eqs. (15) to (21). It is still interesting to find out that the isothermal sphere can be obtained from General Relativity in the limit $v_0/c \ll 1$ by demanding that rotation curves have a form like (12). The total mass of the galactic halo, usualy given as M evaluated at the radius $r = r_{200}$ (the radius at which ρ is 200 times the mean cosmic density). Assuming this density to be $\approx 10^{-29}$ gm/cm^3 together with typical values $v_0 = 200$ km/sec and $a = 1$ yields $r_{200} \approx 150$ kpc. Evaluating M at this values yields about $10^{17} M_\odot$, while M evaluated at a typical "optical radius" $r = 15$ kpc leads to about $10^{12}\, M_\odot$, an order of magnitude larger than the galactic mass due to visible matter.

5. Discussion

So far we have found a reasonable approximation for galactic dark matter to be described by a self gravitating Maxwell-Boltzmann gas, under the assumption of the empiric rotation velocity law (12). The following consistency relations emerge from Eqs. (19), (20) and (21)

$$n_c \approx \frac{3\, v_0^2}{4\pi G\, m\, a^2\, r_{\rm opt}^2}, \qquad T_c \approx \frac{m\, v_0^2}{3\, k_{\scriptscriptstyle B}} \quad (22)$$

hence, bearing in mind that $n \le n_c$ and $T \approx T_c$, the condition (6) for the validity of the MB distribution together with (22) yields the condition

$$m \gg \left[\frac{3^{5/2}\, \hbar^3}{4\pi G\, a^2\, r_{\rm opt}^2\, v_0}\right]^{1/4}, \quad (23)$$

a criteria of aplicability of the MB distribution that is entirely given in terms of m, the fundamental constants G, $\hbar$ and the empiric parameters v_0 and $a\, r_{\rm opt}$ (the "terminal" rotation velocity and the "core radius") [29]. For dark matter dominated galaxies (spiral and low surface brightness (LSB)) [30] these parameters have a small variation range: $r_{\rm opt} \approx 15$ kpc, $0.6 \le a \le 2.3$ and 125 km/sec $\le v_0 \le$ 300 km/sec, the constraint (23) does provide a tight estimate of the minimal value for the mass of the particles under the assumption that these particles form a self gravitating ideal dark gas complying with MB statistics. As shown in Fig. 2, this minimal value lies between 30 and 60 eV, thus implying that appropriate particle candidates must have a much larger mass than this figure [29]. This minimal bound excludes, for instance, light mass thermal particles such as the electron neutrino ($m_{\nu_e} < 2.2$ eV). The axion is also very light ($m_A \approx 10^{-5}$ eV) but it is not a thermal relique and so we cannot study it under the present framework. The currently accepted estimations of cosmological bounds on the sum of

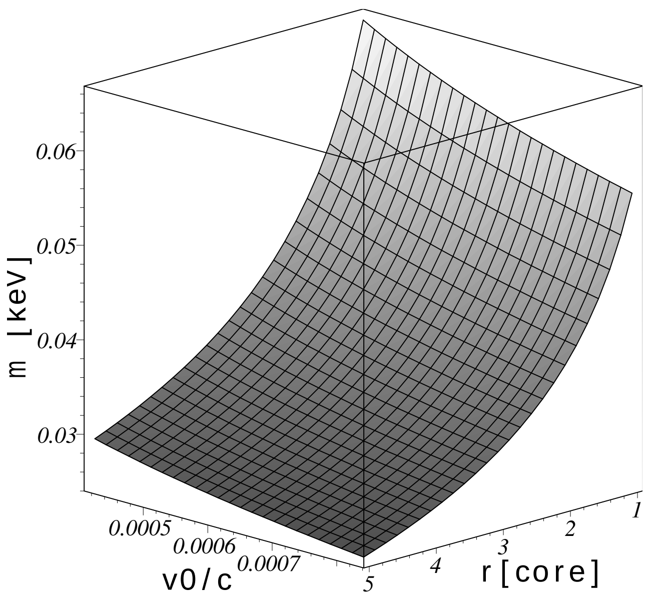

FIGURE 2. Minimal mass for which the Maxwell-Boltzmann distribution is applicable. This graph displays m (in keV's) as a function of v_0/c and $b = a\,x$, respectively, the terminal velocity and 'halo core radius' associated with the URC given in (12). Assuming typical ranges for spiral galaxies: 125 km/sec $\leq v_0 \leq 250$ km/sec and 1 kpc $\leq b \leq 5$ kpc, we obtain masses in the range of 30 eV $\leq m \leq 60$ eV that follow from the right hand side of the relation (23), providing the criterion for applicability of the Maxwell-Bolzmann distribution. Dark matter particle candidates complying with an MB distribution must have much larger mass than the plotted values 30 eV $\leq m \leq 60$ eV.

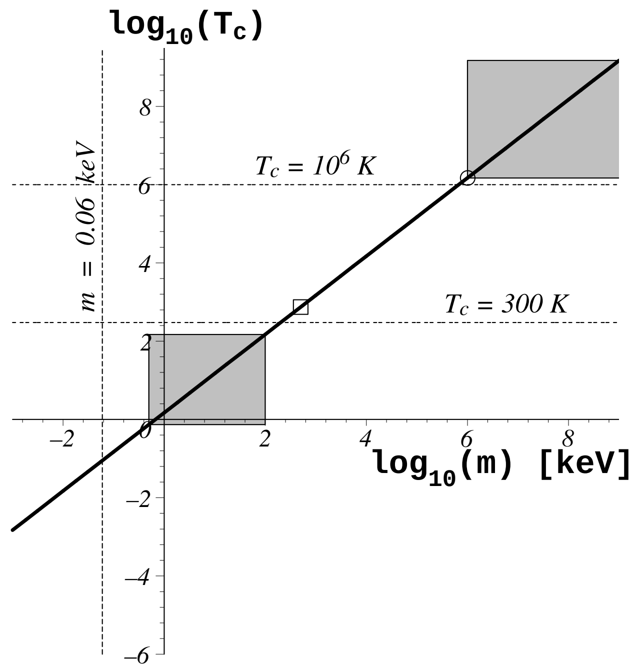

FIGURE 3. Relation between particle mass and central temperature. This graph displays the relation between $\log_{10}(T_c)$ (in K) and $\log_{10}(m)$ (in keV's) that follows from Eq. (24) for a terminal velocity $v_0 = 200$ km/sec. Almost identical plots are obtained for other velocities in the observed range 125 km/sec $\leq v_0 \leq$ 250 km/sec. The circle and box symbols respectively denote the proton and electron mass yielding central temperatures of the order $T_c \approx 10^6$, 10^3 K. The central temperature for light particles in the range 0.5 keV $\leq m \leq 100$ keV is less than 300 K (rectangle in the left), while for massive sypersymmetric particles in the range 1 GeV $\leq m \leq$ 100 GeV, we have T_c as large as 10^9 K (rectangle on the right). However, such high temperatures cannot rule out these weakly interactive particles as components of the dark matter MB gas.

masses for the three active neutrino species is about 24 eV, a value that would apparently rule out all neutrino flavours. However, recent estimations of these cosmological bounds have raised this sum to about 1 keV [42], hence more massive neutrinos could also be accomodated as dark matter particle candidates. Estimates of masses of various particle candidates are displayed in Table I.

Since $T \approx T_c$, the consistency condition (21) provides the following constraint on the temperature and particles mass of the dark gas

$$\frac{m}{T_c} = \frac{3\,k_B}{v_0^2} \approx 0.4 \times 10^3\,\frac{\mathrm{eV}}{\mathrm{K}}, \tag{24}$$

where we have taken $v_0 = 300$ km/sec. Considering in (24) the minimal mass range that follows from (23), we would obtain gas temperatures consistent with the assumed typical temperatures of relic gases: $T_c \approx 2$ to 4 K. However, since we have no way of infering a value for the temperature of the ideal dark gas, we have no clear cut criterion for the estimate of a maximal bound for this mass. If we assume that the ideal dark gas is made of electrons or barions, so that $m = m_p$ or $m = m_e$, then condition (23) for applicability of the MB distribution is certainly satisfied and (24) implies a temperature of the order of $T_c \approx 10^3$ K for electrons and $T_c \approx 10^6$ K for barions. Obviuosly, barions or electrons at such a high temperatures would radiate and certainly not remain unobservably "dark". However, as long as the interaction is weak and the particles are not charged, we cannot rule out any other particle candidate only on the basis of the gas temperature, even if this temperature is very high (see Fig. 3) as in the case of massive supersymmetric particles. As shown in Table I, a wide range of weakly interactive particles can be considered as possible main components of a MB dark gas, including popular supersymmetric particles (the neutralino), as well as hypothetical light particles predicted by current literature based on WDM models of structure formation [2, 3]. The main novelty of the present paper is the fact that it is based on a general relativistic hydrodynamics, as opposed to numerical simulations [7–9], newtonian or Kinetic Theory perturbative approaches (see Refs. 12–22).

Finaly, the fact that we have obtained a minimal mass on the range 30–60 eV, that seems to discriminate against very light thermal particles like the electron neutrino, coincides with the fact that these HDM particle candidates tend to be ruled out because of their inability to produce sufficient matter clustering [2, 3, 29]. In spite of these arguments, a self gravitating gas of this type of particles accounting for a galactic halo, would have to be modeled, either as a relativistic MB gas (very light particles can be relativistic even at low temperatures) and/or in terms of a distribution that takes into account Fermi-Dirac or Bose-Einstein statistics. These studies will be undertaken in future papers [29].

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