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RESEARCH ARTICLE

Flat rotation curves in scalar field galaxy halos

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Abstract We start with a model where the dark matter is of scalar field nature, which condensates and form the dark halos of galaxies. In this work we study Bose–Einstein condensates (BEC) where the scalar field particles are in many different states, and not only in the ground state, as in a realistic BEC. We find that this model is in better agreement with the rotation curves of galaxies than previous models with scalar field dark matter.

1 Introduction

If one accepts Einstein's General Relativity as the correct description of the gravity force in galaxies, one is led to conclude that the so-called *flat* rotation curves are gravitationally sustained by the existence of some type of non-baryonic matter. As observations seem to suggest, this *dark matter* makes up to 90% of the total material content in most galaxies [1,2]. Experiments aimed to detect dark matter particles have been unsuccessful in having any positive signal for their existence [3], and then flat rotation curves still stand as one of the most striking observation for the presence of dark matter in galaxies. More recently, new micro-lensing observations also support the separate existence of dark matter [4,5].

The common wisdom assumes that baryonic matter dominates the central region of galaxies, whereas dark matter takes control of the outer parts. A simple exercise using

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Newtonian gravity in a spherically symmetric galaxy halo, shows that the rotation velocity is given by $v(r) = \sqrt{GM(r)/r}$. If the rotational velocity is a constant at large radii, then the total mass at radius r should increase as $M(r) \propto r$.

Going further, the mass density corresponding to this mass profile is $\rho_{mass}(r) \propto 1/r^2$. This result is quite odd, because it corresponds to an unbounded mass configuration, and one would be lead to believe that a galaxy halo should extend at least to very large distances.

Another observational fact that should be taken into account is the existence of ripples in the rotation curves, and the great variety of curve profiles (see for instance [6]). Whether these facts point out to particular properties of the dark matter particles is still a matter of debate.

Our aim in this paper is to give first steps towards a different approach to the problem of rotation curves from the perspective of the scalar field nature of dark matter in galaxies. Although scalar fields and their role as dark matter in the universe have been thoroughly studied in the literature, only few authors have considered whether a scalar field galaxy halo can fit observed rotation curves.

Scalar field dark matter (SFDM) is the generic name for models in which the dark matter particle is of scalar nature. This idea has had many proponents, and its properties have been studied by many authors, see for instance [7–13] and references therein.

We believe that it is illustrative to repeat part of the summary given in [14] about earlier attempts to fit galactic rotation curves using a scalar field. A first proposal appeared in a couple of papers by Sang–Jin–Sin [15,16]. He took a massive scalar field and were able to fit some galactic observations taken into account also the contribution of baryons. A key point in this work was the use of the so-called excited configurations, configurations in which the radial profile of the scalar fields has nodes. From that, they determined that the mass of the scalar field should be of order of $\sim 10^{-24}$ eV.

Next proposal appeared in a paper by Schunck [17]. It is shown in there that a massless complex scalar field can be used a dark matter model in galaxies and to fit rotation curves. In this model the internal frequency of the field plays the role of an adjustable parameter, and the radial profile of the scalar field also has nodes.

However, as pointed out in [14], none of the above proposals can be realistic. Any excited configuration is intrinsically unstable, as it collapses into a ground configuration under the smallest perturbation [18]. On the other side, a massless scalar field (whether real or complex) cannot form a gravitationally bound configuration [19].

More recently, Arbey et al. [20] worked on the rotation curves of galaxies using a massive scalar field and baryonic matter. As they were aware of the intrinsic instability of excited configurations, the only configurations they considered were ground (nodeless) configurations. Remarkably, their results pointed out to a scalar field mass of the order of $\sim 10^{-23}$ eV.

Our particular proposal is that a correct answer to the rotation curve riddle lays in between those briefly described above. In other words, excited states are appropriate to fit rotation curves but the only stable configuration is the ground (nodeless) one. The idea we want to discuss here is the following; we will suppose that the halo of a galaxy is made of a condensed scalar field, a Bose–Einstein condensate (BEC), where most of the particles are in the ground state, but there is a cloud of particles surrounding the BEC which are in different *excited* states. This model resembles a realistic BEC made in the laboratory.

Yet there is another possibility that goes beyond the model we will discuss in this work. In principle, we should also include finite-temperature effects in the configuration of scalar object. As studied in [21], bosonic particles out of the condensate may imprint its particular signature on the rotation curves of galaxies. It is interesting to note that a flat profile can be naturally achieved in this case.

The plan for the rest of paper is as follows. In Sect. 2 we describe how self-gravitating scalar objects are constructed, and focus our discussion to the case of real scalar fields. In Sect. 3, we study the properties of mixed scalar configurations by making use of some semi-analytical approximations. Finally, we conclude in Sect. 4.

2 Self-gravitating scalar objects

In this section, we shall discuss the theory behind the formation of self-gravitating objects made of real scalar fields. Dark matter is described by the classical evolution of a real scalar field Φ endowed with a quadratic scalar field potential of the form $V(\Phi) = (m_\Phi^2/2)\Phi^2$, where m_Φ is the mass parameter. The scalar field Lagrangian is $\mathcal{L}_\Phi = -(1/2)\partial_\mu \Phi \partial^\mu \Phi - (m_\Phi^2/2)\Phi^2$.

For simplicity, we shall consider a metric with spherical symmetry,

$$ds^2 = \alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{1}$$

and the equation of motion of the scalar field is the Klein–Gordon (KG) equation

$$\left(\Box - m^2\right)\Phi = 0, \tag{2}$$

where $\Box$ is the covariant d'Alembertian operator. The most general spherically symmetric solution of (2) is of the form

$$\Phi(t,r,\theta,\varphi) = R_{nl}(t,r)Y_{lm}(\theta,\varphi). \tag{3}$$

Function $R(t,r)$ is a solution to the radial part of the KG (2). It is well known that a globally regular and stable solution of $R(t,r)$ should be fully time-dependent, such a solution is dubbed an oscillaton[1] [22–24].

Using the formalism of second quantization, the scalar field is considered a quantum operator,

$$\hat{\Phi} = \sum_{nlm} R_{nl}(t,r)\left[\hat{b}_{nlm}Y_{lm}(\theta,\varphi) + \hat{b}^\dagger_{nlm}Y^*_{lm}(\theta,\varphi)\right]. \tag{4}$$

[1] This is unlike the case of a complex scalar field for which separation of variables can be used to further split the time and radial dependent parts of the scalar field [13,25,29].

The expansion coefficients in (4) are interpreted as creation $\hat{b}^{\dagger}_{nlm}$ and annihilation $\hat{b}_{nlm}$ quantum operators for particles with angular momentum $\hbar l$, azimuthal angular momentum $\hbar m$ and total mass M_{nl} [25]. As usual, these quantum operators satisfy

$$\left[\hat{b}_{nlm}, \hat{b}^{\dagger}_{n'l'm'}\right] = \delta_{nn'}\delta_{ll'}\delta_{mm'}, \tag{5a}$$

$$\left[\hat{b}_{nlm}, \hat{b}_{n'l'm'}\right] = \left[\hat{b}^{\dagger}_{nlm}, \hat{b}^{\dagger}_{n'l'm'}\right] = 0. \tag{5b}$$

Using these relationships for the scalar-field quantum operator, one can construct orthonormal states of the form

$$|N_{100}, N_{200}, N_{211}, N_{210}, \ldots\rangle, \tag{6}$$

where $|N_{100}, 0, 0, \ldots\rangle$ is the ground state.

We can now calculate the expectation value of the energy-momentum tensor for the scalar field. Even though it depends non-linearly in $\hat{\Phi}$, the orthonormality of the quantum states assures its expectation value is given as a superposition of the expectation values of the energy-momentum for each state,

$$\langle T_{\mu\nu}\rangle = \sum_{nlm} c_{nlm}\langle T_{\mu\nu}\rangle_{nlm}, \tag{7}$$

where c_{nml} are convenient normalization coefficients [25].

Therefore, in the case more than one state is populated, the source on the r.h.s. of the Einstein equations is the equivalent to the *superposition* of many uncoupled real scalar fields, each one represented by one of the radial functions $R(t, r)_{nl}$ that appear in expansion (4). This also means that an equal number of KG equations has to be solved simultaneously, one KG equation for each radial function.

We arrive to the conclusion that the (semi-classical) configuration that arises when the ground state and some of the excited states are populated is equivalent to that made of many real scalar fields. This type of objects have been studied before and dubbed *multi-scalar stars* [26].

However, the kind of configurations studied in [27] are closer to our idea, and are in fact the source of inspiration for the present paper. It was investigated there the profile formed by a scalar field in the presence of a central gravitational field. The result was a configuration similar to the excited states of a scalar field object, namely, the field's profile shows nodes.

These results can be interpreted in another way. The central gravitational field could be provided by a (nodeless) ground state, and the field "outside" can be supposed to be in some excited states. Interestingly enough, we have evidence for the stability of such configurations [28].

3 Toy model for a mixed state

Our main objective in this section is to show some of the main properties of what we shall call a *mixed state*, i.e., a configuration in which different excited states of the scalar object are populated. For simplicity let us start supposing that only excited states with zero angular momentum ($\ell = 0$) are populated. That is, we shall work on the mixed state $|N_{100}, N_{200}, N_{300}, N_{400}, \ldots\rangle$.

Taking into account that gravity in galaxies is weak, we will consider the weak field limit of the coupled Einstein–Klein–Gordon (EKG) equations, which is the so-called Schroedinger–Poisson (SP) system [18,25,29]. The Newtonian expansion of the relativistic scalar field is [18]

$$\sqrt{8\pi G}\, R_n(t, r) = \psi_n(t, r) e^{-imt} + \text{c.c.} \tag{8}$$

The Klein–Gordon equation then becomes the Schroedinger equation for the wave function ψ. For the *stationary* mixed state we chose, we can assume that $\psi_n = \psi_n(r) e^{-iE_n t}$, and then the SP system reads

$$\nabla^2 \psi_n = (U + E_n)\, \psi_n, \tag{9a}$$

$$\nabla^2 U = \sum_n |\psi_n|^2, \tag{9b}$$

where ∇^2 is the Laplacian operator in spherical coordinates, and E_n are the energy eigenstates of the system.

If only one state were to appear in (9), we would recover the well known solutions of the SP system for a single state(see [18] and references therein for a detailed study of the SP system). However, if many states are populated, they are gravitationally mixed through (9b), where $U(r)$ is the Newtonian gravitational potential.

A complete solution for a mixed state is a complicated matter that requires the use of numerical techniques [30–34]. For the present work, we will present the simplest approximation to the complete solution of (9), namely, we will represent a mixed state by the *naive superposition* of individual states. We have already mentioned that such solution is not realistic because of the presence of gravity. However, the approximated solution will help us to illustrate the basic features that may be present in a full numerical solution.

In that respect, Fig. 1 speaks by itself. Main figure shows the resulting rotation curve corresponding to the complete (and numerical) solution of (9) for a mixed stated with two wave functions; ψ_1 is nodeless, whereas ψ_2 has one node only. (9) were solved using a shooting procedure to adjust the energy eigenvalues E_1, E_2 until appropriate boundary conditions for an isolated object were satisfied. The wave functions were given the same central value $\psi_1(0) = \psi_2(0) = 1$.

The numerical solution is then compared to the resulting rotation curve arising from the naive superposition of two single states, corresponding to the (nodeless) ground state and the first excited one. Obviously, there is no exact matching between the two curves; but we observe that the naive superposition illustrates well the qualitative properties of the (true) mixed state.

On the other hand, the inset shows the rotation curve arising from the naive superposition of the first four single states. This is *not* a solution of (9), but we believe its qualitative features illustrate the properties of a more general mixed state.

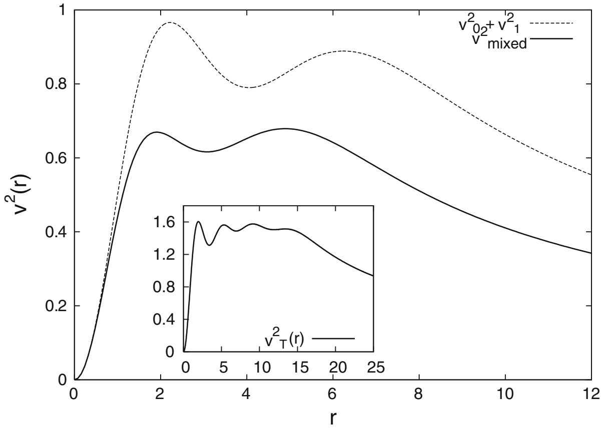

Fig. 1 *Main* the rotation curve $v^2(r)$ for the (true) mixed state $|N_{100}, N_{200}, 0, \ldots\rangle$; also shown for comparison is the rotation curve corresponding to the naive superposition $|N_{100}, 0\rangle + |0, N_{200}\rangle$. Notice that both curves are qualitatevely similar. *Inset* rotation curve for the naive superposition of four single states; see text for details. In both cases, the flatness of the rotation curves and the Keplerian tail at large distances (signature of the finite size of the scalar field galaxy halo) are manifest. Radii and velocity values are given in arbitrary units

As anticipated, the flatness of the rotation curve provided by the mixed state is flat in certain region, and presents the typical Keplerian tail at large distances indicating that we are dealing with a bound object of finite size. We also notice that the ripples of the rotation curve are a consequence of the nodes and ripples of the mass profiles of excited states, and idea anticipated in [15,16]. Moreover, the first peak is approximately set by the ground state, whereas the size of the object is fixed by the most excited state.

There is another particular feature we would like to mention. One expects any realistic model to have but a few fundamental parameters to explain the rotation curves in galaxies, and some other free parameters whose values will depend upon the particular properties of any single galaxy. In previous works in which the scalar field galaxy halo is composed of a single state, the only fundamental parameter was the scalar field mass m_Φ, and the only free parameter was the field's central value $\psi(0)$ [20]. The latter quantity is directly proportional to the number of scalar particles in the given state [19,18].

The introduction of mixed states provides us of more free parameters to fit realistic rotation curves. The extra parameters are the occupation numbers (the number of particles) of the mixed state, namely N_{100}, N_{200}, N_{210}, etc., which are respectively proportional to the $\psi_n(0)$ values. These values would be determined by the local conditions a scalar halo could be subjected to during its formation. Moreover, the (approximated) example shown in Fig. 1 is made only of excited states with zero angular momentum. It is yet to be investigated what effect a non-zero angular momentum excited state will have on the rotation curves.[2]

[2] We speak of the angular momentum related to the scalar field Φ, as given by the general solution (4), even though the metric is still spherically symmetric.

A more detailed study of mixed states will be necessary, as one should investigate the stability of mixed states with a full time evolution of the equations of equations of motion. This is work under progress we expect to publish elsewhere [28].

4 Final remarks

It has been shown many times that SFDM can behave as cold dark matter at cosmological scales, for early references see [35,36]. More recently, it has become clear that SFDM predicts a cut-off scale for the formation of structure. This cut-off could manifest itself in the regular (no cuspy) energy density profile in the central parts of a scalar field galaxy halo, and in the small number of dwarf galaxies observed around large galaxies [8,7].

As we mentioned in the introduction, the properties at galactic scales of SFDM have also been thoroughly studied, but the results were not conclusive. We hope that the ideas presented in this paper can open new vistas for the study of scalar field galaxy halos. Future work seems promising and SFDM model appears as a competitive and cheap one; the simplest model has only one fundamental parameter: m_Φ, the mass of the scalar field.

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