Unification of cosmological scalar fields

Abdel Pérez-Lorenzana,* Merced Montesinos,† and Tonatiuh Matos‡

Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N., Apdo. Post. 14-740, 07000, México, D.F., México

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We present a model where early inflation and late accelerating expansion of the Universe are driven by the real and imaginary parts of a single complex scalar field, which we identified as the inflaton and phantom field, respectively. This inflaton-phantom unification is protected by an internal $SO(1, 1)$ symmetry, with the two cosmological scalars appearing as the degrees of freedom of a sole fundamental representation. The unification symmetry allows to build successful potentials. We observe that our theory provides a matter-phantom duality, which transforms scalar matter cosmological solutions into phantom solutions and vice versa. We also suggest that a complete unification of all scalar fields of cosmological interest is yet possible under a similar footing.

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As cosmology entered the precision stage with the accurate measurements of the microwave cosmic background spectrum by the Cosmic Background Explorer (COBE) [1] and most recently by the Wilkinson Microwave Anisotropy Probe (WMAP) [2], and with the observations of distant type Ia supernovae [3–5] and galaxy cluster measurements [6], the necessity for dark components of the Universe seems to be unavoidable. Current data indicates that common (standard) matter barely constitutes about 3% of the critical density of the Universe, whereas about 27% corresponds to dark matter (DM). The remaining 70% is a form of dark energy (DE), which is responsible for the current accelerating expansion of the Universe.

Understanding the origin of dark components of the Universe has been one of the leading motivations for a large number of theoretical works in last years. Apart from the cosmological constant, one of the favorite candidates for DE are scalar fields, for which acceleration is easy to achieve by choosing an appropriate potential energy and tuning model parameters, as in quintessence and tachyonic scalar models (for examples see [7,8]). However, for all such models the equation of state $p = \omega\rho$ leads to $\omega > -1$. In contrast, observations, including recent results of SNLS [5], not only constrain ω to be close to $\omega = -1$, but also seem to allow and even favor the parameter region where $\omega < -1$ (see also [9]), indicating an apparent exotic source for DE, which violates the weak energy condition $\rho > 0$, $\rho + p > 0$ [10]. A simple way to realize this scenario is to introduce a scalar field φ, called phantom, for which the kinetic term comes with the "wrong" sign, i.e. $\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$, which gives the pressure $p = -\frac{1}{2}\dot{\varphi}^2 - V$ and the energy density $\rho = -\frac{1}{2}\dot{\varphi}^2 + V$, leading to $\rho + p = -\dot{\varphi}^2 < 0$.

An increasing number of studies has been advocated to analyze phantom cosmology (for references see for example [11–13]). Few ideas exist, however, about the possible origin of the phantom, many of them as exotic as the nature of the field itself. They range from string motivated models [14] to higher order theories of gravity and supergravity [15] and nonminimally coupled scalar field theories [16].

The aim of this paper is to show, first, that despite the sign of its kinetic term, the phantom can actually be understood as the imaginary part of a complex scalar field, ϕ. Free ϕ satisfies standard equations of motion and thus our theory would not have to relay on more exotic physics. Second, that the extra degree of freedom which appears as the real part of ϕ has a standard kinetic term, and so it could in principle play the role of any other cosmological scalar field. The conclusion is striking. One can unify in a simple way the phantom with some other field of cosmological use, and there is a natural candidate for the latter: the inflaton, which drove another stage of cosmological acceleration at the early Universe. Thus, in this scenario, DE would be just a remnant of the very early stages of cosmological evolution, with the phantom as the other face of a more fundamental field (see Refs. [17,18] for similar ideas with quintessence). One can also speculate about phantom and DM unification on this same theoretical ground, but we will leave such speculations for a later discussion [19].

In many areas of physics, however, true unification is usually a very profound concept that implies the existence of protecting symmetries that interrelate the dynamical degrees of freedom of the theory. Such is the case, for instance, of electrodynamics where gauge invariance arises, the same later became the fundamental link for the electroweak unification in the standard model of particle physics. Remarkably, the theory we are about to develop also fulfills this concept. It poses an $SO(1, 1)$ symmetry with the inflaton and phantom belonging to a fundamental two-dimensional representation. Once promoted to a fundamental level in the theory, the symmetry allows us to build successful potentials to account for inflation and late time acceleration with the so given degrees of freedom. The symmetry also implies a duality

*aplorenz@fis.cinvestav.mx
†merced@fis.cinvestav.mx
‡tmatos@fis.cinvestav.mx

among matter and phantom cosmological solutions. Furthermore, the theory has an immediate extension to include other scalars of cosmological interest. Those are the central points of the present paper.

To elaborate our theory, let us start by considering a single complex scalar field $\phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$. As fundamental symmetries can usually be read out of the kinetic terms, we will for the moment switch off any possible potential, and, contrary to standard lore, we write a complex kinetic term for ϕ, such that the real Lagrangian is given in the noncanonical form

$$\mathcal{L} = \tfrac{1}{2}[\partial_\mu \phi \partial^\mu \phi + \partial_\mu \phi^* \partial^\mu \phi^*]; \tag{1}$$

where the contraction of space-time indices with the background metric is to be understood. As it should be clear, despite its unusual form, the Lagrangian describes the correct equations of motion for the free fields ϕ and ϕ^*. Nevertheless, when written in terms of real and imaginary parts of ϕ, the above Lagrangian acquires the remarkable expression

$$\mathcal{L} = \tfrac{1}{2}[\partial_\mu \varphi_1 \partial^\mu \varphi_1 - \partial_\mu \varphi_2 \partial^\mu \varphi_2]; \tag{2}$$

where not only the two real component fields are decoupled, but also the imaginary part has the opposite sign in the kinetic term with respect to φ_1. Thus, φ_2 can indeed be identified as a phantom field, while the standard scalar field φ_1 still allows for a variety of interpretations. We would like to identify φ_1 as the inflaton, thus achieving a first step towards phantom-inflaton unification.

It is fair to mention that there already exist some models in the literature where phantom and an ordinary scalar field are treated together on a cosmological frame. By construction, it is clear that all such models are described in principle by the same kinetic Lagrangian term given in Eq. (2). Such are the cases of the so-called quintom models [20], and the hessence models [21]. Nevertheless, both models were constructed *ad hoc* to address a single problem, the nature of DE as produced by a complex system with a phantom component. Clearly, having the same action term is just an accidental overlapping. The actual physics a model can account for depends not only on the basic ingredients, but also on the extent to which they can be identified as specific degrees of freedom of a real system. In this sense, our model is different than any previously presented model. We intend to guide the construction of our cosmological model by the fundamental principles of simplicity and the use of symmetries. And so, as we will show, it is the identification of φ_1 with the inflaton and the identification of the right symmetry for the model that shall provide us the simplest, meaningful, and minimal realization of our cosmological unification idea by connecting both stages of accelerating expansion at a very fundamental level.

Next, we look for the symmetries of the above kinetic terms. First, notice that the Lagrangian in Eq. (1) can be put in the same functional form of Eq. (2), by the simple reparametrization $\phi^* \to i\phi^*$, which resembles Wick transformation that connects Euclidean to Minkowski $1 + 1$ dimensional spaces, where one changes $t \to it$ on the Euclidean metric $ds_E^2 = dx^2 + dt^2$ to get $ds^2 = dx^2 - dt^2$. As a matter of fact, the analogy has a deeper meaning which becomes transparent if we define the two-dimensional vectorlike array

$$\Phi_E = \begin{pmatrix} \phi \\ \phi^* \end{pmatrix}, \tag{3}$$

in terms of which Eq. (1) is simply written as the Euclidean metric contraction

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \Phi_E^T \cdot \partial^\mu \Phi_E \tag{4}$$

on field space (aside from derivatives), where T stands for the transpose. Similarly, the Lagrangian in Eq. (2) can be expressed as

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \Phi^T \cdot \sigma_3 \cdot \partial^\mu \Phi = \tfrac{1}{2}\eta_{ij}\partial_\mu \Phi^i \partial^\mu \Phi^j, \tag{5}$$

where we have now written the real field components as

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \tag{6}$$

and we have recognized the $1 + 1$ dimensional Minkowski metric as $\eta = \sigma_3 = \mathrm{diag}(1, -1)$.

The conclusion is straightforward. The simple Lagrangian theory we have just presented for inflaton and phantom unification has an internal guarding symmetry. That corresponds to the symmetry transformations on field space that preserve the $1 + 1$ Euclidean (or Minkowski) metric. For the Euclidean contraction on Eq. (4) we have the symmetry group $O(2, \mathbb{C})$, consisting of all two by two orthogonal complex matrices. $O(2, \mathbb{C})$ is a dimension one, nonconnected, complex Lie group, which is isomorphic to the indefinite orthogonal group $O(1, 1, \mathbb{C})$, whose transformations preserve the Minkowski metric contraction in Eq. (5). The identity component of $O(2, \mathbb{C})$, i.e. the subgroup of symmetry transformations connected to the identity, is $SO(2, \mathbb{C})$ which is isomorphic to $SO(1, 1, \mathbb{C}) \subset O(1, 1, \mathbb{C})$. The algebra of the former has a single generator over $\mathbb{C}$, that one identifies as $i\sigma_2$, whereas the generator of $SO(1, 1, \mathbb{C})$ is σ_1. $SO(1, 1, \mathbb{C})$ contains two Lorentz-like subgroups $SO(1, 1)$, which by using the exponential mapping are expressed as the complex rotations $g_\alpha = \exp(i\alpha\sigma_1)$ and the standard (real) Lorentz boosts $h_\alpha = \exp(\alpha\sigma_1)$; for α real. Φ corresponds to the fundamental (two-dimensional complex) representation. It is easy to see that the Wick transformation $\phi^* \to i\phi^*$ on Φ_E gives

$$\Phi_E \to \begin{pmatrix} \phi \\ i\phi^* \end{pmatrix} = e^{i(\pi/4)\sigma_1}\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \tag{7}$$

which shows the equivalence of both prescriptions.

Given any two arbitrary doublet representations, Φ and Ψ, there is only one invariant bilinear form under $O(1, 1, \mathbb{C})$ transformations, $\Phi^T \sigma_3 \Psi = \eta^{ij} \Phi_i \Psi_j$. The reduced group $SO(1, 1, \mathbb{C})$, however, has the extra invariant $\Phi^T i\sigma_2 \Psi = \epsilon^{ij} \Phi_i \Psi_j$, where ϵ^{ij} is the usual skewsymmetric tensor with $\epsilon^{12} = 1$. Two extra invariants exist if we restrict to any of the $SO(1, 1)$ subgroups. g_α complex rotations have also the invariants $\Phi^\dagger \sigma_1 \Psi = \Phi_1^* \Psi_2 + \Phi_2^* \Psi_1$; and $\Phi^\dagger \Psi = \delta^{ij} \Phi_i^* \Psi_j$. On the other hand, $\Phi^\dagger i\sigma_2 \Psi$; and $\Phi^\dagger \sigma_3 \Psi$ are invariant under the real boost transformations. However, notice that with the real component field representation given in Eq. (6), the last invariants are not independent from those of $SO(1, 1, \mathbb{C})$.

We will choose the $SO(1, 1)$ subgroup given by g_α transformations as the fundamental symmetry of our theory. This has the clear advantage that one can directly work with the two real components by using Φ as given in Eq. (6), thus keeping our initial identification for the phantom and inflaton candidate fields. Therefore, our theory explicitly breaks the $O(1, 1, \mathbb{C})$ isometry group down to an $SO(1, 1)$ residual symmetry through the potential terms. Notice that this is unlike hessence models in Ref. [21] where the other $SO(1, 1)$ subgroup was used.

Clearly, since $\Phi^T i\sigma_2 \Phi = 0$, we may only use the other three associated invariants to build the potential of our unified theory, which we may rewrite as

$$\Phi^T \sigma_3 \Phi = \varphi_1^2 - \varphi_2^2; \tag{8}$$

$$\Phi^\dagger \Phi = \varphi_1^2 + \varphi_2^2; \tag{9}$$

$$\Phi^\dagger \sigma_1 \Phi = 2\varphi_1 \varphi_2. \tag{10}$$

It is worth noticing that the first two expressions above allow us to write quite general and independent potentials for phantom and inflaton fields as far as they depend quadratically on such fields: $U(\varphi_1^2) = U(\Phi^T \sigma_3 \Phi + \Phi^\dagger \Phi)$ and $V(\varphi_2^2) = V(\Phi^\dagger \Phi - \Phi^T \sigma_3 \Phi)$, respectively. In this scenario, phantom and inflaton have consistently decoupled dynamics, provided the coupling term $\Phi^\dagger \sigma_1 \Phi$ is not allowed. Remarkably, we can again make use of symmetry arguments to insure this feature by requiring the theory to be invariant under the parity transformation generated by the metric $\eta = \sigma_3$, under which $\Phi^\dagger \sigma_1 \Phi$ is a pseudoscalar. Other bilinears remain invariant. Thus, a natural and simple choice is to take a chaotic potential for the inflaton, $U(\varphi_1) = \frac{1}{2}\mu^2 \varphi_1^2$. For the phantom field, on the other hand, there is not a preferred potential yet, but we can include most working examples in the literature (see [11,12] for references). For instance, one can consider the toy phantom potential with a bell profile

$$V_0 \exp\left(-\frac{\alpha}{M_P^2}\varphi_2^2\right). \tag{11}$$

The dynamics with such potentials follows the general features of scalar phantom cosmology. Particularly, phantom obeys the equation of motion

$$\ddot{\varphi}_2 + 3H\dot{\varphi}_2 = \frac{\partial V}{\partial \varphi_2}, \tag{12}$$

which indicates that the expansion of the Universe acts as a damping force, as usual, but the phantom moves towards local maxima, as if the potential were inverted, due to the sign on the right-hand side of Eq. (12). Initial conditions are expected to be fixed at same time for both scalars. Early inflation would proceed as usual, followed by reheating, big bang nucleosynthesis, structure formation, and a late-time matter-dominated era. The phantom should survive all those epochs without affecting them. This can be arranged by fine tuning the potential parameters such that the phantom would remain frozen at some large value away from zero, until matter density ρ_m catches up with it at late time. Indeed, if initial phantom conditions can be arranged for almost zero initial kinetic energy, $T = -\dot{\varphi}_2^2/2$, and a very flat potential (small α regime), the phantom starts with an equation of state with $\omega = -1$, with $\rho_m \gg \rho$ during matter domination. As the Universe expands, matter density scales as $\rho_m \propto a^{-3}$, with a the metric scale function, whereas phantom density goes as $\rho \propto \exp[-6\int(1-\theta(a))da/a]$, for $\theta(a) = (1 + T/2V)^{-1}$. Flatness of the potential would imply that $\theta \approx 1$, well up to the coincidence era, where $\Omega_{\rm matter}$ equals $\Omega_{\rm phantom}$. Thereafter, phantom gets released, gaining kinetic energy and moving towards the top of the potential producing the $\omega < -1$ era, where we live. This would be part of a period of phantom oscillations around the maximum of the potential, where the phantom should finally settle, returning to $\omega = -1$ within finite time. In this epoch, the Hubble parameter gets contributions from both sources, $H^2 = (\rho + \rho_m)/3M_P^2$. θ becomes larger than 1, and phantom energy increases with the expansion whereas matter keeps diminishing. Thus, it is natural to expect that $\Omega_m < \Omega_{\rm DE}$ as observations indicate. Actual values can be accommodated, including supernova data, for some models (see for instance Ref. [11]).

Alternative unified scenarios where the two field components are not completely decoupled are also possible. Those are, to our point of view, much more interesting. Consider, for instance, the simple potential

$$U(\Phi) = \tfrac{1}{4}M^2(\Phi^T \sigma_3 \Phi + \Phi^\dagger \Phi) - \tfrac{1}{2}m^2 \Phi^\dagger \sigma_1 \Phi + V_0, \tag{13}$$

built out of the three $SO(1, 1)$ invariants. It can be written in terms of real component fields as

$$U = \frac{1}{2}M^2\left(\varphi_1 - \frac{m^2}{M^2}\varphi_2\right)^2 - \frac{1}{2}\frac{m^4}{M^2}\varphi_2^2 + V_0. \tag{14}$$

The potential is unbounded, but this should not be a matter of concern due to unusual dynamics of the phantom. Above potential has a saddle point at $\Phi = 0$, which is a local minimum for φ_1 but a local maximum for φ_2. This is the stable configuration point where the fields should finally settle down, no matter what the initial conditions are. This point is an attractor for the system.

For $M \gg m$, the potential is steeper along φ_1, with an almost flat direction along $\varphi_1 = \frac{m^2}{M^2}\varphi_2$. If initial conditions are such that φ_1 and φ_2 are about M_P, with no kinetic energy, we would be in a situation where the dynamics of the system is initially reduced to chaotic inflation, with φ_1 as the inflaton. φ_1 shall roll down the potential towards the local minimum at $\varphi_1 = \frac{m^2}{M^2}\varphi_2$, inflating and then reheating the Universe, whereas φ_2 remains frozen due to its small effective mass, $\mu = m^2/M \ll H \approx M$. It is clear that to produce the observed amount of density perturbations in the cosmic microwave background, we require $\delta\rho/\rho \approx M/M_P \approx 10^{-5}$, which fixes the scale M. After the period of reheating, the system settles at the local minimum, where the effective potential becomes $V = V_0(1 - \alpha\varphi_2^2/M_P^2)$, with $\alpha = \frac{1}{2}m^4 M_P^2/V_0 M^2$. This potential has the form of the very first terms in the expansion of the potential in Eq. (11), which is actually true for almost any bell-shaped potential. Thus, dynamics should follow similar paths. The system would remain at such a point for a long period until the condition $\rho < \rho_m$ is broken when the system is again released. Thereafter, the configuration shall move towards zero with a phantom dynamics, controlled by the rolling of φ_2, which undergoes damped oscillations around zero. Time evolution of ω would help to avoid future singularities caused by the instabilities associated with the violation of null energy conditions (see first references in [12]). A detailed numerical analysis (that will be presented elsewhere) should probe this potential and fix the free parameters to reproduce data. By comparing with the analysis in Ref. [11], it is not difficult to realize that the values $V_0 \approx 10^{-44}$ GeV^4 and $m \approx 5.5 \times 10^{-13}$ eV, are likely to provide a successful scenario, where the initial condition on φ_2 is chosen to give a positive H^2. The smallness of those parameters indicates the need for a large fine tuning, not protected by symmetry, as in the cosmological constant problem.

Reheating by inflaton decay can also occurs in a way that does not break the protecting symmetry of our theory. Consider a singlet ψ_0 and a doublet, $\Psi^T = (\psi_1, \psi_2)$ of fermions with the $SO(1,1)$ invariant couplings

$$\alpha\bar{\psi}_0(\Phi^T\sigma_3\Psi + \Phi^\dagger\Psi) + \beta\bar{\psi}_0(\Phi^T i\sigma_2\Psi + \Phi^\dagger\sigma_1\Psi), \quad (15)$$

which, written in terms of field components, reduce to $2\alpha\bar{\psi}_0\varphi_1\psi_1 + 2\beta\bar{\psi}_0\varphi_1\psi_2$. Thus, one gets the decay channels $\varphi_1 \to \psi_0\psi_1, \psi_0\psi_2$ for the inflaton, with no coupling among phantom and fermion fields. We then get the reheating temperature $T_r \approx 6 \times 10^{-3}\max\{|\alpha|, |\beta|\}M_P$.

General models based on our $SO(1,1)$ symmetry, regardless of the identification of φ_1, also have an interesting property that it is worth commenting on. Consider again our initial analogy with the $1+1$ metric. By performing the exchange $t \leftrightarrow x$ we get a conformally equivalent space-time theory, since $ds^2 \to -ds^2$. In field space, this corresponds to the duality transformation $\Phi \to \sigma_1\Phi$, which exchanges $\varphi_1 \leftrightarrow \varphi_2$. $SO(1,1,\mathbb{C})$ invariant forms are odd under this transformation, whereas the other two $SO(1,1)$ invariants are even. Thus, one can use this duality to transform matter into phantom cosmological solutions, and vice versa, just by rewriting all terms in an invariant way. A similar duality was noticed in Refs. [13], but no connection to the underlying symmetry was made.

Last, but not least, our theory can straightforwardly be extended to provide the possibility of a completely unified treatment of all cosmological scalars. Briefly, by considering the kinetic terms for n scalar fields and a phantom, we can write them as the $n+1$ metric form $\frac{1}{2}\eta_{ij}\partial_\mu\Phi^i\partial^\mu\Phi^j$, with the obvious $O(n,1,\mathbb{C})$ symmetry, and Φ representation. A model for hybrid inflation, DM and DE unification would then have the suggestive symmetry $O(3,1,\mathbb{C})$, which contains the subgroup $SO(3,1)$. We will address these issues in a forthcoming paper.

Summarizing, we have shown that the phantom field that may cause current cosmic acceleration can be unified with the inflaton field that drove early expansion of the Universe, as the imaginary and real parts, respectively, of a more fundamental complex scalar field. The theory that describes the unification is protected by an internal $SO(1,1)$ symmetry. Therefore, one can achieve unification in a true sense, with the inflaton-phantom system belonging to a fundamental representation of an internal symmetry group. The symmetry allows us to write adequate invariant potentials with enough freedom to get a successful description of both inflation and DE from the same footing. To our knowledge, this is the very first time that the concept of unification via this nontrivial symmetry is realized for the cosmological setup.

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