

INFLATION FROM IIB SUPERSTRINGS WITH FLUXES

ERANDY RAMÍREZ^{*,†} and TONATIUH MATOS^{*,‡}

*Departamento de Física,
Centro de Investigación y de Estudios Avanzados del IPN,
A.P. 14-740, 07000 México D.F., Mexico*

[†]erandy@fis.cinvestav.mx

[‡]tmatos@fis.cinvestav.mx

ISRAEL QUIROS

*Departamento de Física, Universidad Central de Las Villas,
54830 Santa Clara, Cuba
israel@uclv.edu.cu*

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We study the conditions needed to have an early epoch of inflationary expansion with a potential coming from type IIB superstring theory with fluxes involving two modulus fields. The phenomenology of this potential is different from the usual hybrid inflation scenario and we analyze the possibility that the system of field equations undergoes a period of inflation in three different regimes with the dynamics modified by a Randall–Sundrum term in the Friedmann equation. We find that the system can produce inflation and, due to the modification of the dynamics, a period of accelerated contraction can follow or precede this inflationary stage, depending on the sign of one of the parameters of the potential. We discuss the viability of this model in a cosmological context.

Keywords: Inflationary cosmology; string theory phenomenology.

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1. Introduction

The last 50 years have been among the most fruitful ones in the field of physics; the standard model of particles (SM) and the standard model of cosmology (SMC), essentially developed in this period, are now able to explain a great number of observations in laboratories and cosmological observatories as never before. In only 50 years there have been great steps in the understanding of the origin and development of the universe. Nevertheless, many questions are still open, e.g. the SMC contains two periods, inflation and structure formation. For the understanding of the

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structure formation epoch we need to postulate the existence of two kinds of substances: dark matter and dark energy. Without them it is impossible to explain the formation of galaxies and clusters of galaxies, or the observed accelerated expansion of the universe. On the other hand, there has been postulated a period of inflation in order to give an explanation for several observations, such as the homogeneity of the universe, the close value of the density of the universe to the critical density or the formation of the seeds which formed the galaxies. However, there is not a theory that unifies these two periods; essentially they are disconnected from each other.

In this work we study the possibility that superstring theory could account for the unification between inflation and the structure formation using a specific example. Recently Frey and Mazumdar¹ were able to compactify the IIB superstrings including 32 fluxes. Earlier works discussing string flux compactifications are,^{2–8} and works addressing the issue that the inflaton carries the SM gauge charges as opposed to an arbitrary gauge singlet in the context of string theory can be found in Refs. 9 and 10. In the context of the type IIB supergravity theory on the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold with self-dual three-form fluxes, it has been shown that after compactification, the effective dilaton–axion potential is given by^{1,a}

$$V_{\text{dil}} = \frac{M_P^4}{4(8\pi)^3} h^2 e^{-2\Sigma_i \sigma_i} \left[e^{-\Phi^{(0)}} \cosh(\Phi - \Phi^{(0)}) + \frac{1}{2} e^\Phi (C - C^{(0)})^2 - e^{-\Phi^{(0)}} \right], \quad (1)$$

where $h^2 = \frac{1}{6} h_{mnp} h_{qrs} \delta^{mq} \delta^{nr} \delta^{ps}$. Here h_{mnp} are the NS–NS integral fluxes, the superscript (0) in the fields stands for the fields in the vacuum configuration, and σ_i ($i = 1, 2, 3$) are the overall size of each factor \mathbf{T}^2 of the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold. This potential contains two main scalar fields (modulus fields), the dilaton Φ and the axion C . In Ref. 11 the dilaton was interpreted as dark matter and under certain conditions the model reproduces the observed universe, i.e. the structure formation. In this work we investigate if the same theory could give an inflationary period in order to obtain a unified picture between these two epochs. In other words, we want to find out whether it is possible that the same low energy Lagrangian of IIB superstrings with the scalar and axion potential (1) can give an acceptable inflationary period. To handle the fluxes, we work in the brane representation of space–time, working with an RS-II modification in the equations. Thus, due to the presence of the fluxes during this period, these models have the phenomenology of the Randall–Sundrum models^{16,17,1} and we have chosen to work with the RS-II one, but the same type of analysis could be done for the other model. An important point to mention before discussing the equations is that we are holding the σ_i fixed, not considering at the moment a potential to stabilize them. In particular, the modification of the dynamics into a Randall–Sundrum one is a consequence of this approximation.^b

^aWe thank Andrew Frey for pointing out a mistake in the previous version of the potential. This was due to a typographical error in Ref. 1 and it does not affect our work.

^bThanks to Andrew Frey for clarifying this point.

This paper is organized as follows. In Sec. 2 we introduce an appropriate parametrization of the potential (1) in order to give unities to physical quantities. As a first insight into studying the cosmology of the low energy Lagrangian, we make an approximation to the potential and give the field equations, leaving the analysis of the complete system to Sec. 4. In Sec. 3 we find the solutions to the equations in different regimes and conditions, and the results they give. In Sec. 4 the dynamical evolution of the full system is performed for large values of the field, and in Sec. 5 we discuss some conclusions and perspectives.

2. The Potential

In order to study the cosmology of this model, it is convenient to define the following quantities:

$$\begin{aligned} \lambda\sqrt{\kappa}\phi &= \Phi - \Phi^{(0)}, & V_0 &= \frac{M_P^4}{4(8\pi)^3} h^2 e^{-2\Sigma_i\sigma_i} e^{-\Phi^{(0)}}, \\ C - C^{(0)} &= \sqrt{\kappa}\psi, & \psi_0 &= e^{\Phi^{(0)}}, \end{aligned} \tag{2}$$

where λ is the string coupling $\lambda = e^{\langle\Phi\rangle}$ and $\lambda\sqrt{2\kappa}$ is the reduced Planck mass $M_P/\sqrt{8\pi}$. With these new variables, the dilaton potential is transformed into

$$V_{\text{dil}} = V_0(\cosh(\lambda\sqrt{\kappa}\phi) - 1) + \frac{1}{2} V_0 e^{\lambda\sqrt{\kappa}\phi} \psi_0^2 \kappa\psi^2 = V. \tag{3}$$

In what follows we want to study the behavior of this potential at early times when the scalar field takes large values, and study the conditions that the parameters λ and V_0 need to meet in order to have inflation. Thus, expressing the cosh function in terms of exponentials and taking the limit ϕ big, we arrive at the following expression for the potential:

$$V(\phi, \psi) = \frac{1}{2} V_0 e^{\lambda\sqrt{\kappa}\phi} (1 + \kappa\psi_0^2\psi^2) - V_0. \tag{4}$$

From the approximation, there remains a term V_0 which acts during the inflationary period like a “cosmological constant” and which we will address as a free parameter of the model. In contrast with the usual hybrid inflation scenario,¹² there is no critical value for which this potential exhibits a phase transition triggering the end of inflation (if any such process occurs). The potential follows an exponential behavior in the ϕ field that prevents it from staying at a fixed value from the start, i.e. it cannot relax at $\phi = 0$ or at any other different value (apart from infinity). A plot of the potential illustrates this behavior; see Fig. 1. Then we will assume in this section and the next that there is some mechanism by which the ψ field rolls down to its minimum at $\psi = 0$, oscillating around it at the very early stages of evolution, and that any processes such as inflation take place afterward. So any information concerning such a process is erased by the expansion led by the ϕ field

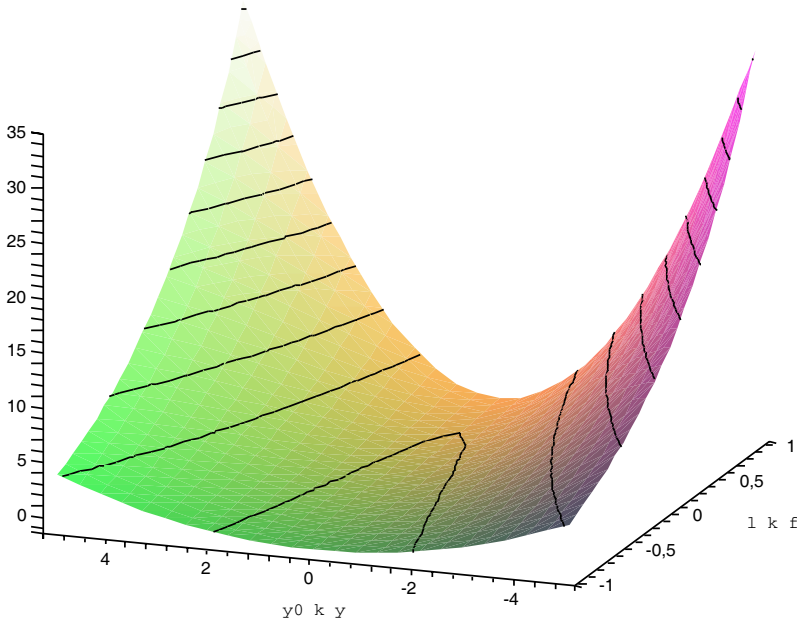


Fig. 1. Potential.

if inflation is to happen. Thus, we can work with the following expression for the potential:

$$V(\phi) = \frac{1}{2}V_0 e^{\lambda\sqrt{\kappa}\phi} - V_0. \tag{5}$$

We will follow closely the analysis done by Copeland *et al.*¹³ and Mendes and Liddle¹⁴ in order to obtain the conditions for this potential to undergo inflation in the cases where the scalar field or the V_0 term dominates the dynamics, as well as in the intermediate stage.

Our calculations are performed in the high energy regime within the slow-roll approximation, since a potential slow-roll formalism has already been provided for this scenario.¹⁹

2.1. Field equations

In the presence of branes, in the RS-II scenario, the Friedmann equation for this cosmology changes from its usual expression to¹⁷

$$H^2 = \frac{\kappa}{3}\rho \left(1 + \frac{\rho}{\rho_0} \right), \tag{6}$$

with $H \equiv \dot{a}/a$, where a is the scale factor of the universe, the dot means a derivative with respect to time, and ρ_0 is the brane tension. The total density ρ and the

equations of motion for the fields in the standard cosmology case are deduced in Ref. 18:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 e^{\lambda\sqrt{\kappa}\phi} + V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi}. \tag{7}$$

In our case,

$$V_\phi = \frac{1}{2}V_0 e^{\lambda\sqrt{\kappa}\phi} - V_0, \quad V_\psi = \frac{1}{2}V_0 \kappa \psi_0^2 \psi^2, \tag{8}$$

which in the slow-roll approximation can be written as

$$H^2 \simeq \frac{\kappa}{3}(V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi}) \left[1 + \frac{(V_\phi + V_\psi e^{\lambda\sqrt{\kappa}\phi})}{\rho_0} \right]. \tag{9}$$

The equations of motion for both fields are given considering only the presence of both scalar fields with no radiation fluid; they are¹⁸

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_\phi}{\partial \phi} &= \lambda\sqrt{\kappa} e^{\lambda\sqrt{\kappa}\phi} \left(\frac{1}{2}\dot{\psi}^2 - V_\psi \right), \\ \ddot{\psi} + 3H\dot{\psi} + \frac{\partial V_\psi}{\partial \psi} &= -\lambda\sqrt{\kappa}\dot{\phi}\dot{\psi}, \end{aligned} \tag{10}$$

which in the slow-roll approximation read

$$\begin{aligned} 3H\dot{\phi} + \frac{\partial V_\phi}{\partial \phi} &\simeq -\lambda\sqrt{\kappa} e^{\lambda\sqrt{\kappa}\phi} V_\psi, \\ 3H\dot{\psi} + \frac{\partial V_\psi}{\partial \psi} &\simeq 0, \end{aligned} \tag{11}$$

since the right-hand side of the second equation in (10) can be taken as a kinetic term to the square.

A consequence of considering the ψ field in the value corresponding to the minimum of its potential ($\psi = 0$) is that the system of equations does not have source terms and couplings of the fields. This allows one to do an inflationary analysis equivalent to the usual one in terms of the potential slow-roll parameters for the RS-II model. But this restriction lacks the information about how the interaction of the two fields affects the dynamics. The results presented in these sections are therefore a particular case of a more general analysis presented in Sec. 4.

3. Inflationary Phenomenology of the Model

The analysis that follows will be confined to the high energy limit of this model, which simplifies calculations further. Consequently, we have the following system of equations within the slow-roll approximation and the high energy limit:

$$H \simeq \sqrt{\frac{\kappa}{3\rho_0}} V, \quad 3H\dot{\phi} + \frac{\partial V_\phi}{\partial \phi} \simeq 0, \tag{12}$$

which can be solved analytically. In this work we take the convention $\dot{\phi} < 0$, and hence the ϕ field is a decreasing function of time. The solutions to the field equations will be given in Subsec. 3.6.

The expressions for the potential slow-roll parameters having the inflation field confined to the brane in the intermediate regime and high energy limit of this cosmology were deduced by Maartens *et al.*¹⁹ For the high energy limit they are

$$\epsilon \simeq \frac{1}{\kappa} \left(\frac{V'}{V} \right)^2 \frac{\rho_0}{V}, \quad \eta \simeq \frac{1}{\kappa} \left(\frac{V''}{V} \right) \frac{\rho_0}{V}, \tag{13}$$

where the primes indicate derivatives with respect to the ϕ field. The slow-roll approximation is satisfied as long as the slow-roll parameters defined previously accomplish the following conditions:

$$\epsilon \ll 1, \quad |\eta| \ll 1. \tag{14}$$

The number of e-foldings of inflation in terms of the potential for this model is given in our notation by¹⁹

$$N \simeq -\frac{\kappa}{\rho_0} \int_{\phi_N}^{\phi_e} \frac{V^2}{V'} d\phi, \tag{15}$$

where ϕ_N represents the value of the field ϕ at N e-foldings of expansion before the end of inflation and ϕ_e is the value at the end of inflation.

For the potential (5) we have

$$\epsilon = \frac{\rho_0 \lambda^2}{4V_0} \frac{e^{2\lambda\sqrt{\kappa}\phi}}{\left(\frac{e^{\lambda\sqrt{\kappa}\phi}}{2} - 1 \right)^3}, \tag{16}$$

$$\eta = \frac{\rho_0 \lambda^2}{2V_0} \frac{e^{\lambda\sqrt{\kappa}\phi}}{\left(\frac{e^{\lambda\sqrt{\kappa}\phi}}{2} - 1 \right)^2}, \tag{17}$$

$$N \simeq \frac{2V_0}{\lambda^2 \rho_0} \left[-\frac{1}{4} \left(e^{\lambda\sqrt{\kappa}\phi_e} - e^{\lambda\sqrt{\kappa}\phi_N} \right) + \left(e^{-\lambda\sqrt{\kappa}\phi_e} - e^{-\lambda\sqrt{\kappa}\phi_N} \right) + \lambda\sqrt{\kappa}(\phi_e - \phi_N) \right]. \tag{18}$$

Since in our model there is no value of ϕ that may lead ψ to a global minimum, the only way in which inflation can finish is by the violation of the slow-roll approximation with ϵ exceeding unity.

The value of ϕ at which ϵ becomes equal to unity is

$$\sqrt{\kappa}\phi_e = \frac{1}{\lambda} \ln \left\{ \left[\frac{2\lambda^2 \rho_0}{3V_0} \right] \left[\frac{4^{1/3} B^{1/3}}{2\lambda^2 \rho_0} + \frac{(6V_0 + \lambda^2 \rho_0) 4^{2/3}}{2B^{1/3}} + \frac{3V_0 + \lambda^2 \rho_0}{\lambda^2 \rho_0} \right] \right\}, \tag{19}$$

with

$$B = \lambda^2 \rho_0 \left[27V_0^2 + 18V_0 \lambda^2 \rho_0 + 2\lambda^4 \rho_0^2 + (3V_0)^{3/2} \sqrt{4\lambda^2 \rho_0 + 27V_0} \right]. \tag{20}$$

We can rearrange the previous expression so that

$$\sqrt{\kappa}\phi_e = \frac{1}{\lambda} \ln \left\{ \left[\frac{2\lambda^2\rho_0}{3V_0} \right] \left[1 + \frac{3V_0}{\lambda^2\rho_0} + \frac{2^{2/3}B^{1/3}}{2\lambda^2\rho_0} + \frac{(6V_0 + \lambda^2\rho_0)2^{1/3}}{B^{1/3}} \right] \right\}. \tag{21}$$

Thus, if

$$\frac{3V_0}{\lambda^2\rho_0} + \frac{B^{1/3}}{2^{1/3}\lambda^2\rho_0} + \frac{2^{1/3}(6V_0 + \lambda^2\rho_0)}{B^{1/3}} \ll 1 \tag{22}$$

the ϕ field dominates the potential (5) and we will have an exponential potential, which in the standard cosmology case corresponds to power law inflation, but not for the RS-II modification.

The bound found before depends on the choice of values for λ and V_0 . As we will see in Subsec. 3.2, λ is fixed once the value of the brane tension is given. So actually Eq. (22) depends only on the choice of V_0 . We use the computing package Mathematica to find a value of V_0 that satisfies the condition (18). This happens if

$$\lambda^2\rho_0 < 0 \Rightarrow -\frac{4\lambda^2\rho_0}{27} \leq V_0 < -\frac{\rho_0\lambda^2}{6}, \tag{23}$$

$$\lambda^2\rho_0 > 0 \Rightarrow V_0 < -\frac{1}{6}\rho_0\lambda^2. \tag{24}$$

From these the second case satisfies having a brane tension with no incompatibilities with nucleosynthesis.^{14,22}

3.1. *Density perturbations*

The field responsible for inflation produces perturbations which can be of three types: scalar, vector and tensor. Vector perturbations decay in an expanding universe and tensor perturbations do not lead to gravitational instabilities that may result in structure formation. The adiabatic scalar or density perturbations can produce these types of instabilities through the vacuum fluctuations of the field driving the inflationary expansion. So they are usually thought to be the seed of the large scale structures of the universe. One of the quantities that determined the spectrum of the density perturbations is δ_H , which gives the density contrast at the horizon crossing (if evaluated at that scale). For the RS-II modification and in our notation, this quantity is^{14,20}

$$\delta(k)_H^2 \simeq \frac{\kappa^3}{75\pi^2} \frac{V^3}{V'^2} \frac{V^3}{\rho_0^3}, \tag{25}$$

evaluated at the moment of the horizon crossing when the scale $k = aH$. The slow-roll approximation guarantees that δ_H is nearly independent of the scale when scales of cosmological interest are crossing the horizon. The reason is that the field is almost constant in time satisfying the new COBE constraint updated to $\delta_H = 1.9 \times 10^{-5}$.²³ Here we take 60 e-foldings before the end of inflation to find the scales of cosmological interest.

We use Eq. (18), provided that we know the value of ϕ_e given by Eq. (19), to find the value ϕ_N corresponding to $N = 60$, i.e. 60 e-foldings before the end of inflation, and evaluate δ_H as

$$\delta_H^2 \simeq \frac{4\kappa^2}{75\pi^2} \frac{V_0^4}{\lambda^2 \rho_0^3} e^{-2\lambda\sqrt{\kappa}\phi_{60}} \left(\frac{e^{\lambda\sqrt{\kappa}\phi_{60}}}{2} - 1 \right)^6. \tag{26}$$

The results are given in Table 2, in Subsec. 3.7.

3.2. *Field-dominated region*

Considering the case where the ψ field plays no role and ϕ governs the dynamics of the expansion alone, from Eq. (5) we have a potential of the exponential type resembling that of power law inflation.²¹ The first term of Eq. (5) dominates, giving an exponential expansion, but not to power law inflation because the dynamics of RS-II changes this condition. The slow-roll parameters are given by

$$\epsilon = \eta \simeq \frac{2\rho_0}{V_0} \frac{\lambda^2}{e^{\lambda\sqrt{\kappa}\phi}}, \tag{27}$$

in contrast with the standard cosmology, where they are not only the same but constant. The fact that the dynamics is modified due to the Randall–Sundrum cosmology allows the existence of a value of ϕ that finishes inflation, since, as we just saw, the potential slow-roll parameters are equal but not constant, showing a dependence on ϕ and therefore an evolution.

We see that in this regime, the value of ϕ_e corresponding to the end of inflation is given by $\epsilon \simeq 1$. The \simeq is used because we are in the potential slow-roll approximation, not the Hubble one:¹⁵

$$\sqrt{\kappa}\phi_e \simeq \frac{1}{\lambda} \ln \left(\frac{2\lambda^2\rho_0}{V_0} \right). \tag{28}$$

Inserting this value in the expression for the number of e-foldings (15) for this regime and evaluating, we find that ϕ_N , with $N = 60$, is

$$\sqrt{\kappa}\phi_{60} \simeq \frac{1}{\lambda} \ln \left(122 \frac{\lambda^2\rho_0}{V_0} \right), \tag{29}$$

and we can evaluate the density contrast Eq. (25) for this regime:

$$\delta_H^2 \simeq \frac{61^4 \kappa^2 \lambda^6 \rho_0}{75\pi^2}. \tag{30}$$

We can see from this equation that given the value of δ_H from observations, it is possible to completely constrain λ as

$$\lambda^6 \simeq \frac{75\pi^2 \delta_H^2}{61^4 \kappa^2 \rho_0}. \tag{31}$$

We have a dimensionless number that fixes one of the parameters of the potential and can be contrasted with the value predicted by this supergravity model when interpreted as dark matter.¹¹

If we substitute the last equation into Eq. (22), we get in principle a set of values for the constant V_0 that satisfy the field domination condition.

3.3. Vacuum energy-dominated regime

We consider now the regime in which the second term of Eq. (5) dominates the dynamics. In this case the slow-roll parameters (13) are:

$$\epsilon \simeq -\frac{\rho_0 \lambda^2 e^{2\lambda\sqrt{\kappa}\phi}}{4V_0}, \tag{32}$$

$$\eta \simeq \frac{\rho_0 \lambda^2 e^{\lambda\sqrt{\kappa}\phi}}{2V_0}. \tag{33}$$

We find ourselves here with the fact that ϵ is negative, i.e. with a period of deflation.²⁴

It is necessary to point out that this is a consequence of the modification of the dynamics. The definition of ϵ for the Randall–Sundrum II cosmology in the high energy limit is not positive definite as in the standard cosmology case. Thus, a stage of accelerated contraction for this regime on the potential is only a result of the modification in the field equations.

Following the evolution of the dynamics with the potential (5), from a region where ϕ dominates to a stage in which the energy V_0 drives the behavior of the expansion, we observe a primordial inflationary expansion that erases all information concerning any process that the ψ field is undergoing under the influence of the potential (4). The intermediate regime, in which the two terms in Eq. (5) are of the same order, produces further expansion. Finally, the field ϕ reaches a value on the potential that commences a stage of accelerated contraction. This value is obtained when the denominator in Eq. (16) changes sign,

$$\sqrt{\kappa}\phi_d = \frac{\ln 2}{\lambda}, \tag{34}$$

corresponding to the value of the vacuum-dominated regime. Such a process takes place when the field ϕ takes on values below $\sqrt{\kappa}\phi_d$. Substituting the value of ϕ_d in the potential (5), we have $V(\phi_d) = 0$. Thus, the balance of the terms in Eq. (5) and the sign of V_0 determine the place where deflation starts as the point where the potential crosses the ϕ axis.

In principle, there would not be a physical reason that could prevent this deflationary stage from stopping. But the argument mentioned before, concerning the modification of the dynamics, applies again. We observe that ϵ has a dependence on the field and therefore undergoes an evolution accordingly. The condition for

ending deflation is $\epsilon = -1$, in opposition to inflation. In consequence, we can also find from the first of the equations (32) a value of ϕ corresponding to this;

$$\epsilon = -1 \Rightarrow \sqrt{\kappa}\phi_e = \frac{1}{2\lambda} \ln \left(\frac{4V_0}{\rho_0\lambda^2} \right), \tag{35}$$

where, this time, the subscript e , indicates the end of deflation. This value, as we can see, depends on V_0 and λ .

The choice of the parameter V_0 will be given in Subsec. 3.7, where we give different values according to the conditions we find in the following. We will see whether or not an inflationary stage takes place under the value of λ found in the previous analysis.

3.4. *Intermediate regime*

The intermediate regime corresponds to the region where the two terms in Eq. (5) are of the same order. To obtain a bound on the values of V_0 that satisfy the COBE constraint (26), we need to solve numerically Eqs. (18) and (19). Since both terms in Eq. (5) are important, this means that the exponential is of $\mathcal{O}(1)$, and thus we can expand it in a Taylor series as $\exp(\lambda\sqrt{\kappa}\phi) \simeq 1 + \lambda\sqrt{\kappa}\phi$ and arrive at a value of ϕ_f equal to

$$\phi_e = \frac{(4B)^{1/3}}{a\lambda V_0} + \left(2 + \frac{\lambda^2\rho_0}{3V_0} \right) \frac{\lambda\rho_0 4^{2/3}}{\kappa B^{1/3}} + \frac{2\lambda\rho_0}{3\sqrt{\kappa}V_0} + \frac{1}{\lambda\sqrt{\kappa}}, \tag{36}$$

where B is now given by

$$B = \frac{\rho_0\lambda^2}{\kappa^3} \left(18\rho_0\lambda^2V_0 + 27V_0^2 + 2\rho_0^2\lambda^4 + \sqrt{(3V_0)^3(4\rho_0\lambda^2 + 27V_0)} \right). \tag{37}$$

In order to have a real scalar field, we find two bounds for the values that V_0 can take on following from the roots in the previous expressions:

$$V_0 < -\frac{1}{6}\rho_0\lambda^2, \quad V_0 > -\frac{4}{27}\rho_0\lambda^2. \tag{38}$$

The first bound coincides with the value given by Eq. (24) needed to have field domination. It is an upper bound for the allowed values of V_0 in the expression (19). On the other hand, we see from Eqs. (19) and (20) that positive values of V_0 also satisfy the condition that there exists a real value of ϕ_e in the intermediate regime. But there is another condition coming from Eq. (20) in which we see that V_0 cannot be 0. So we have an interval for allowed values of V_0 as

$$V_0 < -\frac{1}{6}\lambda^2\rho_0, \quad V_0 > 0. \tag{39}$$

If $V_0 > 0$, Eq. (32) is negative and we have the period of deflation mentioned before. But $V_0 < -1/6\lambda^2\rho_0$ means that even in the region of vacuum domination ϵ can be positive and we return to the usual picture of inflation. However, as we have chosen a value of V_0 below this bound, Eq. (27) becomes negative, and so now we have a period of deflation translated to the epoch of field domination.

We can then choose values for V_0 below $-1/6\lambda^2\rho_0$ being in a period of deflation for the intermediate regime [Eq. (16) is negative] where Eqs. (20) and (19) still have real values because the approximation made in this section to find the upper limits of V_0 is a lower bound on Eqs. (19) and (20). This is in fact redundant, since we have also said that the two terms in the potential (5) are of the same order, and so the expansion is valid for the general case.

Once the choice of V_0 is made, we can solve numerically to find a value for ϕ_N in Eq. (18); then it is introduced into Eq. (25) and can be accepted or rejected, depending on whether or not it fulfills the left-hand side. This value depends on ρ_0 , the brane tension, and we present the results for different values of it considering that in order to have no incompatibilities with nucleosynthesis the brane tension must satisfy¹⁴ $\rho_0 \geq 2 \text{ MeV}^4$, the authors take the number 1 MeV^4 , the difference arises due to the change of notation. We also check that the choice of ρ_0 satisfies the COBE constraint.

The results are shown in Table 2.

3.5. Vacuum-dominated region revisited

Following the argument in the preceding subsection, it would be possible to continue with the usual analysis to find the value of ϕ that finishes inflation, in the vacuum-dominated region, provided that V_0 is negative, and calculate the number of e-foldings and then use Eq. (25) in the region of vacuum domination to find a constraint on V_0 . We find from Eq. (32) that

$$\sqrt{\kappa}\phi_e = \frac{1}{2\lambda} \ln \left(-\frac{4V_0}{\lambda^2\rho_0} \right), \tag{40}$$

and from Eq. (15) that

$$\sqrt{\kappa}\phi_{60} \simeq -\frac{1}{\lambda} \ln \left[\left(-\frac{\lambda^2\rho_0}{4V_0} \right)^{1/2} - \frac{30\rho_0\lambda^2}{2V_0} \right]. \tag{41}$$

And, finally, from (25)

$$(-V_0)^{3/2} - 60\rho_0^{1/2}\lambda V_0 = \frac{\sqrt{75}\pi\delta_H\rho_0}{\kappa}. \tag{42}$$

This equation can be solved numerically to give a value of V_0 that is in accordance with the COBE constraint for the perturbations. As we shall see later, this process will not be applied to the case of vacuum domination, since we find ϵ to be a decreasing function of time, and thus for this case inflation never ends. Consequently, it is not possible to find a value for the parameter V_0 satisfying this condition in the case of vacuum domination. So, in fact, the value of ϵ corresponding to 1 indicates in this case the place, where inflation starts to take place, as from there onward we will have $\epsilon < 1$.

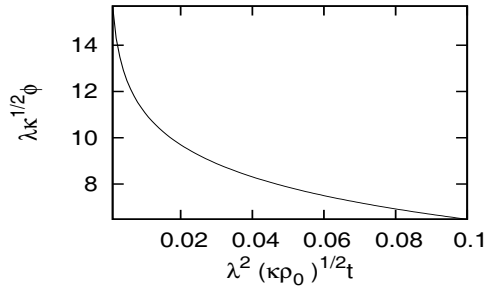


Fig. 2. The behavior for the scalar field during inflation.

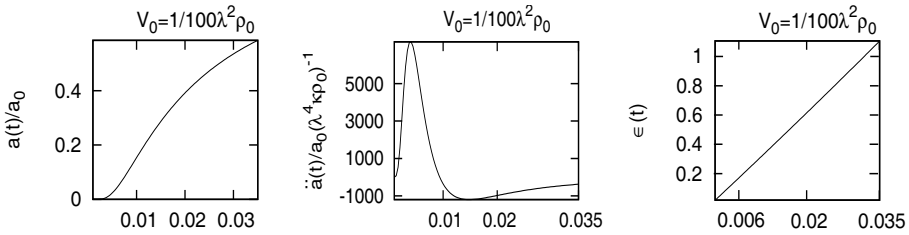


Fig. 3. The scale factor (left) shows an inflationary behavior but the acceleration factor (middle) grows and decreases in the same interval. On the right we plot the inflationary parameter ϵ .

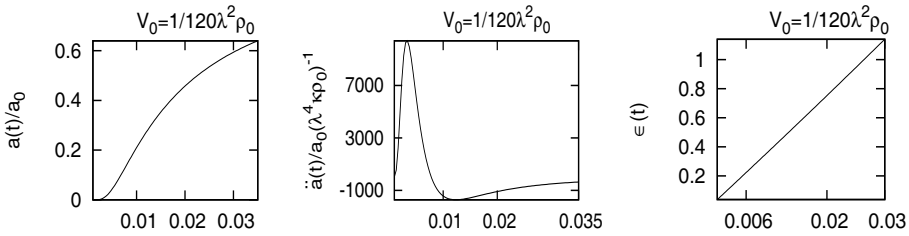


Fig. 4. In this case the scale factor (l.h.s.) increases more rapidly than in the previous case and the acceleration factor (middle) reaches higher numbers within the same interval. On the r.h.s., we again plot the ϵ parameter.

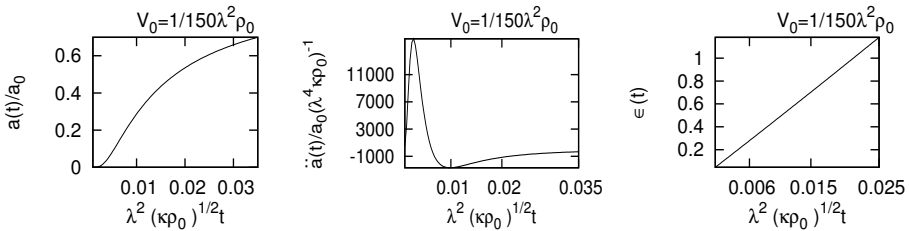


Fig. 5. Again in this case the scale factor (l.h.s.) increases more rapidly than in the first case and the acceleration factor (middle) reaches much higher numbers within the same interval. On the r.h.s., we again plot the ϵ parameter.

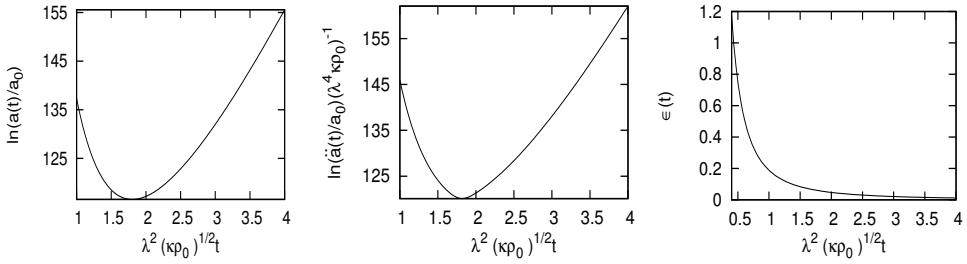


Fig. 6. The plot of the scale factor, the acceleration factor and ϵ for V_0 negative; here we use $V_0 = -56\lambda^2\rho_0$.

3.6. Field equations

In this subsection we solve the system of field equations. The numeric results are presented in the next subsection. Integrating Eqs. (12) for the potential (5) yields

$$a(t) = a_0 \exp \left[-\frac{V_0}{\lambda^4 t \sqrt{3\kappa\rho_0^3}} (\lambda^4 \kappa \rho_0 t^2 + 24e^{-2}) \right], \tag{43}$$

$$\sqrt{\kappa}\phi(t) = -\frac{2}{\lambda} + \frac{1}{\lambda} \ln \left(\frac{48}{\lambda^4 \kappa \rho_0 t^2} \right). \tag{44}$$

One can immediately see that the behavior of the field does not depend on the value of the parameter V_0 , but only on λ , whose value is given by Eq. (31). From this solution and its plot, one can check that indeed the field is a decreasing function of time, having the same behavior regardless of the regime it is in. The scale factor shows little dependence on the value of V_0 , as shown in the plots, following an increasing behavior for $V_0 > 0$. Figures 3–5 show that for $V_0 > 0$, ϵ is a growing positive function which corresponds to an inflationary stage, as expected from the analysis in the previous subsections. The acceleration factor is also shown, and one can observe that for the range used in the plots \ddot{a}/a changes sign before ϵ reaches 1 in the same interval.

We have plotted the scale and acceleration factors as well as ϵ in Fig. 6 for a negative value of V_0 . The scale and acceleration factors change the sign of their slope at $\lambda^2(\kappa\rho_0)^{1/2}t = 1.8$, which is the same value as that from Eq. (34), indicating the start of the vacuum-dominated regime. So one ends with a stage of inflation after deflation in the other two regimes. We find that indeed ϵ is a positive decreasing function of time. This means that although there is a period of inflation, after deflation it will never end and there is no meaning in calculating the value of the potential from Eq. (42) satisfying the COBE constraint since there is no value of the field corresponding to 60 e-folds before the end of inflation. The case of vacuum domination with a negative potential is not realistic for this model.

The bounds that the field needs to meet in order to have inflation for the intermediate regime are presented in Table 3. The value of the parameter V_0 in the region

Table 1. Results for the sign of the first slow-roll parameter ϵ according to the choice of V_0 for the three different regions of the potential (5).

Region	$V_0 > 0$	Dynamics	$V_0 < -\frac{1}{6}\lambda^2\rho_0$	Dynamics
ϕ -dominated	$\epsilon > 0, \phi_e$ real	inflation	$\epsilon < 0$	deflation
Intermediate	$\epsilon > 0, \phi_e$ real	inflation	$\epsilon < 0$	deflation
Vacuum-dominated	$\epsilon < 0$	deflation	$\epsilon > 0, \phi_e$ real	inflation

of vacuum domination remains unconstrained, and therefore it is not possible to give a bound on the value of the field for the onset of inflation.

3.7. Results

Before starting with the numeric results for the three regions just analyzed, we summarize what we have found so far. This is shown schematically in Table 1.

Despite the fact that $V_0 > 0$ gives a positive value for Eq. (27), it does not correspond to the region of field domination. So one has to employ the value of $V_0 > 0$ in Eqs. (16) and (19), not in (28), i.e. in the intermediate regime. We have checked that $V_0 > 0$ does not meet the condition (22) even for very small values of V_0 compared to unity. Instead, the smaller this parameter is, the closer to 2 Eq. (22) will be. So we are left with only two regions where we have inflation. The intermediate regime for $V_0 > 0$, and the region of vacuum domination for $V_0 < -1/6\lambda^2\rho_0^2$.

We solved numerically the corresponding equations in the intermediate regime and found the values shown in Table 2. For this, we have taken $\kappa \simeq 25/m_{\text{Pl}}^2$.¹⁴

Table 2 shows three values of the potential that are in good agreement with the value of the density contrast δ_H . The numbers that appear in the third column correspond to $1/100\lambda^2\rho_0$, $1/120\lambda^2\rho_0$ and $1/150\lambda^2\rho_0$, respectively. Bigger values in the denominators seem to lead the decimals in the density contrast closer to 1.9×10^{-5} . We keep these numbers as a good approximation to the ideal value of the potential.

Table 2. Results for V_0 in the intermediate regime, for three different values of ρ_0 .

$\rho_0 \times 10^6$ (eV ⁴)	$\lambda \times 10^{14}$	$V_0 \times 10^{34}$ (eV ⁴)	$\phi_e \times 10^{13}$ (eV)	$\phi_{60} \times 10^{13}$ (eV)	$\delta_H \times 10^{-5}$
2	8.2	1.4	1.5	2.7	1.9
		1.1	1.6	2.8	1.9
		0.9	1.6	2.8	1.9
4	7.4	2.2	1.7	3.1	1.9
		1.8	1.8	3.1	1.9
		1.4	1.9	3.2	1.9
6	6.9	2.8	1.9	3.3	1.9
		2.4	1.9	3.3	1.9
		1.9	1.9	3.4	1.9

Table 3. Bounds for the scalar field ϕ multiplied by the Planck mass, for V_0 positive in eV units.

$\rho_0 \times 10^6$ (eV ⁴)	$\lambda \times 10^{14}$	V_0 (eV ⁴)	$\phi > \times 10^{-15} \times m_{\text{Pl}}$ (eV)
2	8.2	$\frac{1}{100} \rho_0 \lambda^2$	1.3
		$\frac{1}{120} \rho_0 \lambda^2$	1.3
		$\frac{1}{150} \rho_0 \lambda^2$	1.4
4	7.4	$\frac{1}{100} \rho_0 \lambda^2$	1.4
		$\frac{1}{120} \rho_0 \lambda^2$	1.5
		$\frac{1}{150} \rho_0 \lambda^2$	1.5
6	6.9	$\frac{1}{100} \rho_0 \lambda^2$	1.5
		$\frac{1}{120} \rho_0 \lambda^2$	1.6
		$\frac{1}{150} \rho_0 \lambda^2$	1.7

4. The Two Field Analysis

In this section we perform the analysis of the dynamics using only the $\cosh \sim \exp$ approximation. The complete field equations now read

$$H^2 = \frac{\kappa}{3} \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\psi}^2 e^{\lambda\sqrt{\kappa}\phi} + V_\phi + e^{\lambda\sqrt{\kappa}\phi} V_\psi + \rho_\gamma e^{\alpha\sqrt{\kappa}\phi} + \rho_{V_0} \right), \tag{45a}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_\phi}{d\phi} = \lambda\sqrt{\kappa} e^{\lambda\sqrt{\kappa}\phi} \left(\frac{1}{2} \dot{\psi}^2 - V_\psi \right) \alpha\sqrt{\kappa} e^{\alpha\sqrt{\kappa}\phi} \rho_\gamma, \tag{45b}$$

$$\ddot{\psi} + 3H\dot{\psi} + \frac{dV_\psi}{d\psi} = -\lambda\sqrt{\kappa} \dot{\phi} \dot{\psi}, \tag{45c}$$

$$\dot{\rho}_\gamma + 3H\rho_\gamma = 0, \tag{45d}$$

$$\dot{\rho}_{V_0} = 0, \tag{45e}$$

where we have set

$$\begin{aligned} V_\phi &= \frac{1}{2} V_0 e^{\kappa\lambda\phi}, \\ V_\psi &= \frac{1}{2} V_0 \kappa^2 \psi_0^2 \psi^2, \\ \rho_{V_0} &= V_0. \end{aligned} \tag{46}$$

A dot stands for a derivative with respect to the cosmological time and H is the Hubble parameter $H = \dot{a}/a$. The slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2}$ can be obtained by differentiating Eq. (45a) with respect to t . We arrive at

$$\dot{H} = -\frac{\kappa^2}{2} (\dot{\phi}^2 + \dot{\psi}^2 + \gamma\rho_\gamma). \tag{47}$$

We perform the following definitions for the dynamical variables:

$$x = \frac{\kappa}{\sqrt{6}} \frac{\dot{\phi}}{H}, \quad u = \frac{\kappa}{\sqrt{6}} \frac{\sqrt{V_\phi}}{H}, \tag{48a}$$

$$A = \frac{\kappa}{\sqrt{6}} \frac{\dot{\psi}}{H} e^{\frac{1}{2}\lambda\kappa\phi}, \quad w = \frac{\kappa}{\sqrt{6}} \frac{\sqrt{V_\psi}}{H} e^{\frac{1}{2}\lambda\kappa\phi}, \tag{48b}$$

$$y = \frac{\kappa}{\sqrt{6}} \frac{\sqrt{\rho\gamma}}{H} e^{\frac{1}{2}\alpha\kappa\phi}, \quad l = \frac{\kappa}{\sqrt{6}} \frac{\sqrt{V_0}}{H}, \tag{48c}$$

$$s = \kappa\psi_0 \frac{\sqrt{V_0}}{H}. \tag{48d}$$

With these variables the field equations reduce to

$$x' = -3x + \frac{\lambda}{2}u^2 - \sqrt{\frac{3}{2}}(\lambda(w^2 - A^2) + \alpha y^2) + \frac{3}{2}\Pi x, \tag{49a}$$

$$A' = -3A - ws - \sqrt{\frac{3}{2}}\lambda Ax + \frac{3}{2}\Pi A, \tag{49b}$$

$$u' = \frac{\lambda}{2}ux + \frac{3}{2}\Pi u, \tag{49c}$$

$$w' = As + \lambda\sqrt{\frac{3}{2}}wx + \frac{3}{2}\Pi w, \tag{49d}$$

$$y' = \frac{3}{2}\left(\Pi - \gamma + \alpha\sqrt{\frac{2}{3}}x\right)y, \tag{49e}$$

$$l' = \frac{3}{2}\Pi l, \tag{49f}$$

$$s' = \frac{3}{2}\Pi s, \tag{49g}$$

where now the prime stands for the derivative with respect to the number of e-foldings $N = \ln(a)$. The quantity Π is related to the slow-roll parameter ϵ by $\Pi = \frac{2}{3}\epsilon$ and is defined as

$$\Pi = 2x^2 + 2A^2 + \gamma y^2. \tag{50}$$

The Friedman equation (45a) becomes a constriction for the variables such that

$$F = x^2 + A^2 + u^2 + w^2 + y^2 + l^2 = 1. \tag{51}$$

In order to see the dynamical evolution, we analyze the stability of the system (49) using the theorem of Hartman and Grobman. We first perform a small perturbation $\delta\vec{x}$ of the system around of the minima $\vec{x} = \vec{x}_c + \delta\vec{x}$, in order to analyze the stability of these points. In doing so, the perturbation fulfills the relation

$$\delta\vec{x}' = \mathcal{M}\delta\vec{x}. \tag{52}$$

The phase space we are interested in is defined by the vector $\vec{x} = (x, u, A, w, l, s)$. In this case the matrix \mathcal{M} is given by

$$\begin{bmatrix} -3A_0^2 + 3 - 9x_0^2 & 0 & -\sqrt{6}(\sqrt{6}x_0 + \lambda)A_0 & \sqrt{6}\lambda w_0 & 0 & 0 \\ -\frac{1}{2}u_0(\lambda + 12x_0) & -3x_0^2 - \frac{1}{2}\lambda x_0 & -6A_0u_0 & 0 & 0 & 0 \\ \sqrt{\frac{3}{2}}(-2\sqrt{6}x_0 + \lambda)A_0 & -3A_0^2 & \sqrt{\frac{3}{2}}\lambda x_0 + 3 & 0 & 0 & 0 \\ -\sqrt{\frac{3}{2}}(2\sqrt{6}x_0 + \lambda)w_0 & 0 & -9A_0^2 - 3x_0^2 & -\sqrt{\frac{3}{2}}\lambda x_0 & 0 & -A_0 \\ -6x_0l_0 & 0 & -s_0 - 6A_0w_0 & -3x_0^2 - 3A_0^2 & -3x_0^2 - 3A_0^2 & 0 \\ -6x_0s_0 & 0 & -6A_0l_0 & 0 & 0 & -3x_0^2 - 3A_0^2 \end{bmatrix}$$

The critical points $\vec{x}_c = (x_0, u_0, A_0, w_0, l_0, s_0)$ which fulfill the Friedman constriction (51) are then

- $(0, 0, 0, 0, l, s) \rightarrow V_0$ dominance;
- $(\pm 1, 0, 0, 0, 0, 0) \rightarrow$ the dilaton’s kinetic part dominance;
- $\frac{1}{\sqrt{6}}(-\lambda, 0, 0, \sqrt{6 - \lambda^2}, 0, 0) \rightarrow$ the dilaton’s kinetic part plus the axion’s potential dominance;
- $\frac{1}{2}\left(-\frac{\sqrt{6}}{\lambda}, 0, \frac{\sqrt{2\lambda^2 - 6}}{\lambda}, \sqrt{2}, 0, 0\right) \rightarrow$ the dilaton’s kinetic part plus the axion’s dominance;
- $\frac{1}{6}\left(-\lambda, \frac{1}{\sqrt{2}}, 0, 0, 0, 0\right) \rightarrow$ the dilaton’s dominance, with $\lambda = \pm\sqrt{\frac{71}{2}}$;
- $\frac{1}{5}\left(\frac{6(1 - \sqrt{6})}{\lambda}, \sqrt{5(6 - \sqrt{6})}, \frac{\sqrt{36(2\sqrt{6} - 7) - 5\lambda^2(1 - \sqrt{6})}}{\lambda}, 0, 0, 0\right) \rightarrow$ the dilaton plus the axion’s kinetic part dominance.

In order to obtain the stability of these points, we substitute them into the matrix (52) and obtain their eigenvalues. We arrive at:

- V_0 dominance; the eigenvalue of \mathcal{M} is

$$(-3, -3, 0, 0, 0, 0).$$

This is an attracting wall.

- The dilaton’s kinetic part dominance; the eigenvalue of \mathcal{M} is

$$\left(3 + \sqrt{\frac{3}{2}}\lambda, -\sqrt{\frac{3}{2}}\lambda, 3 + \frac{\lambda}{2}, 3, 3, 6\right).$$

This point is a repelling focus for $-3\sqrt{\frac{3}{2}} < \lambda < 0$, but it is a saddle point otherwise.

- The dilaton’s kinetic part plus the axion’s potential dominance; the eigenvalue of \mathcal{M} is

$$\left(\frac{1}{2}\lambda^2 - 3, \lambda^2, \frac{1}{2}\lambda^2, \frac{1}{2}\lambda^2, \lambda^2 - 3, \frac{1}{2}\lambda^2\left(1 - \frac{\sqrt{6}}{12}\right)\right).$$

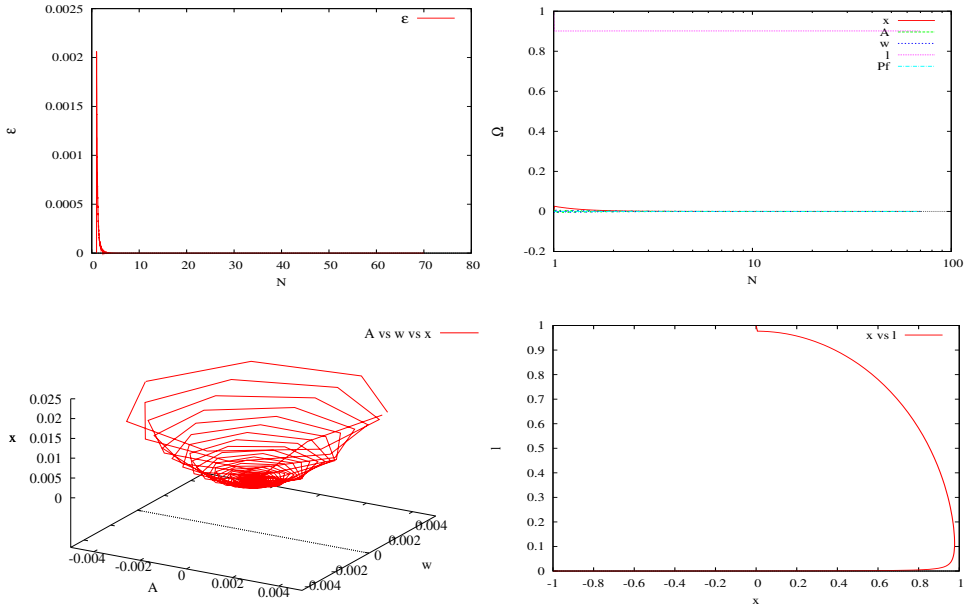


Fig. 7. We plot ϵ (left upper panel), the evolution of the fields (right upper panel), and the phase spaces (A, w, x) (left lower panel) and (x, l) (right lower panel) for the initial conditions $(0, 0, 0, 0, 1, 100)$. These initial conditions correspond to the V_0 dominance; observe how the (A, w, x) fields oscillate around $(0, 0, 0)$ and $l = 1$ dominates the evolution. The slow-roll parameter ϵ is always very small.

This is a repelling focus for $\lambda > \pm\sqrt{6}$, but the critical point then becomes imaginary. If $\lambda \neq 0$ but $-\sqrt{6} < \lambda < \sqrt{6}$ this is a saddle point, and it is an attractor wall if $\lambda = 0$, then the point becomes an axion’s potential dominance.

- The dilaton’s kinetic part plus the axion’s dominance; the eigenvalue of \mathcal{M} is

$$\left(-\frac{3}{4} - \frac{1}{4}\sqrt{153 - 48\lambda^2}, -\frac{3}{4} + \frac{1}{4}\sqrt{153 - 48\lambda^2}, -\frac{\sqrt{6}}{4} + \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 3\right).$$

This is a saddle point.

- The dilaton’s dominance; the eigenvalue of \mathcal{M} is

$$\left(\frac{1}{24}(1 - 71\sqrt{6}), \frac{71}{24}(-1 + \sqrt{6}), \frac{71}{24}, \frac{71}{24}, \frac{47}{8}, 0\right).$$

This is a saddle point.

- The dilaton’s plus the axion’s kinetic part dominance; this is a saddle point.

From these results we can see that only the constant part of the dilaton V_0 can dominate the evolution of the system. In all the other cases the system contains only saddle or source points. In Fig. 7 we see that the slow-roll parameter ϵ remains small during the evolution; we infer that during this time the system undergoes an inflationary period of evolution. In this figure the system starts with initial conditions where the constant V_0 dominates the evolution, but this point on the

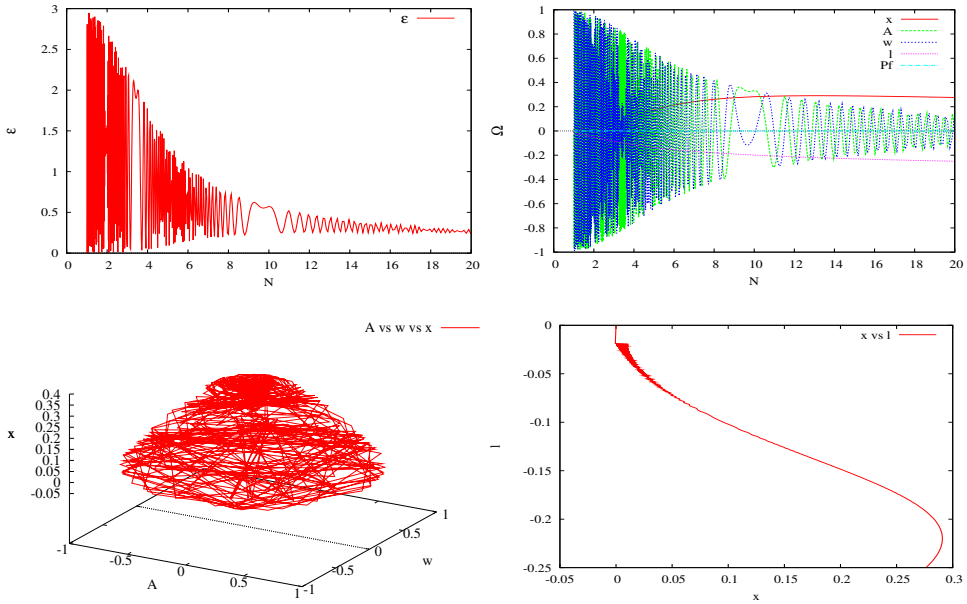


Fig. 8. We plot ϵ (left upper panel), the evolution of the fields (right upper panel), the phase space (A, w, x) (left lower panel) and the phase space (x, l) (right lower panel) for the initial conditions $(0, 0.01, 0, 0.99, 0, 100)$. This initial condition corresponds to a saddle point with $\lambda = 1$. The fields oscillate, but ϵ goes oscillating to a fixed point less than $1/2$. Here none of the fields dominate the evolution; there is a dynamical equilibrium of all of them.

phase space is stable, and thus the system evolves into an eternal period of inflation. We plot the evolution of each field and the phase spaces (A, w, x) and (x, l) to see the evolution of the fields in this period. In Fig. 8 which represents the cases where the axion’s kinetic part and the dilaton’s potential part dominate the evolution, we see that the slow-roll parameter ϵ is always small; we could expect an inflationary period of evolution from this initial condition. Figure 9 corresponds to the dilaton’s kinetic part plus the axion’s potential dominance with $\lambda = 1$. Here the slow-roll parameter ϵ oscillates very hard; we cannot expect an inflationary period with these initial conditions.

5. Conclusions

In this work we have seen the conditions that the parameters of the potential (5) have to fulfill in order to have early universe inflation. Addressing the question of whether the potential as quoted in Ref. 11 can give a unified picture between inflation and structure formation, we have done the first part of the analysis, approximating the potential to be only a function of the dilaton field assuming the axion at the minimum and using the slow-roll approximation. We found that the value of the parameter λ can be fixed in the region of field domination and there can be two possibilities for the sign of the parameter V_0 . Each of them determines

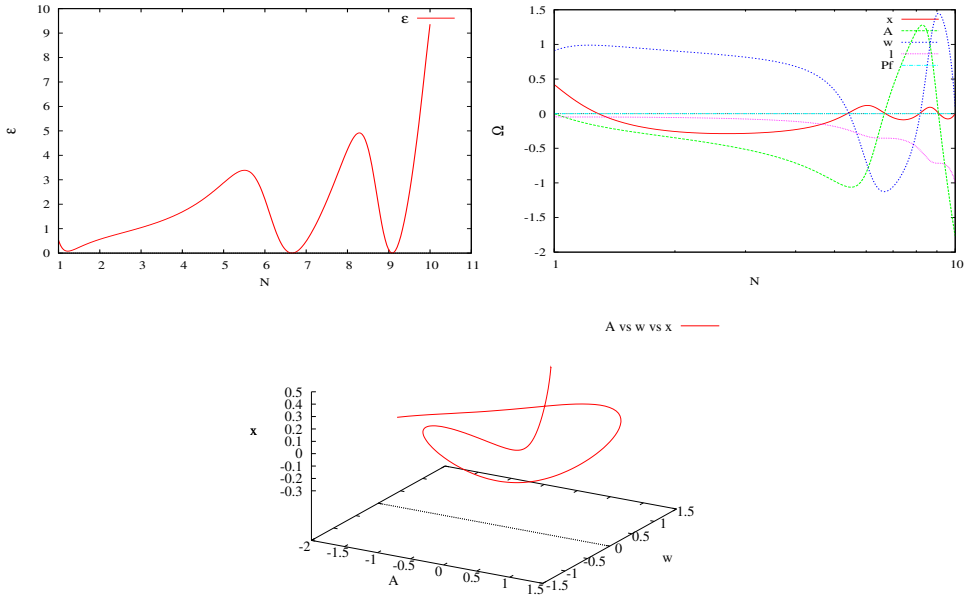


Fig. 9. We plot ϵ (left upper panel), the evolution of the fields (right upper panel), the phase space (A, w, x) (left lower panel) and the phase space (x, l) (right lower panel) for the initial conditions given by the dilaton’s kinetic part plus the axion’s potential dominance with $\lambda = 1$

different dynamics in the evolution of the field equations. A positive sign leads to a period of inflation followed by one of deflation, whereas the opposite sign implies the contrary. In the first case, the values of V_0 we have found to be viable in meeting the COBE constraint, are not in agreement with those found in Ref. 11 by several orders of magnitude. The second case, corresponding to vacuum domination, has proven that it is unrealistic to have a viable model of inflation since this process does not end and we do not have other mechanisms to finish it as in the usual hybrid inflation scenario.

As mentioned before, the previous analysis was made under the assumption that the axion played no significant role during inflation. In Sec. 4, the study of the evolution with the full dynamics was addressed; the results of this section indicated more possibilities to take into account for the analysis as a natural consequence of the inclusion of the axion. The only parameter of the potential that can dominate the evolution of the system is V_0 , which leads to an eternal period of inflation and, as a consequence, has to be discarded as a viable model of inflation. This keeps a resemblance to the result found previously in Subsecs. 3.6 and 3.7.

These results seem to be generic in superstring theory, implying that if we would like to relate the modulus fields to the inflaton, dark energy or dark matter, the model could fit observations either during the inflationary epoch or during the structure formation, but the challenge is to derive a model which fits our observing universe during the whole history of the universe. Otherwise, superstring theory has

to give alternative candidates for these fields and explain why we do not see the modulus fields in our observations. Another possibility to explore is to consider a potential capable of stabilizing the σ_i fields in Eq. (1) instead of considering them as constants. In this case the dynamics should correspond to the standard cosmology, but with more complicated field equations.

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