Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **416,** 87–93 (2011) doi:10.1111/j.1365-2966.2011.19012.x

Structure formation with scalar-field dark matter: the fluid approach

A. Suárez⋆ and T. Matos⋆

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, 07000 México DF, México

Accepted 2011 May 4. Received 2011 April 28; in original form 2011 February 9

ABSTRACT

The properties of nearby galaxies that can be observed in great detail suggest that a theory other than cold dark matter (CDM) would describe better the mechanism by which matter is rapidly gathered into large-scale structures such as galaxies and groups of galaxies. In this work we develop and simulate a hydrodynamical approach to the early formation of structure in the Universe. This approach is based on the fact that dark matter is in the form of some kind of scalar field (SF) with a potential of form $\mu^2\Phi^2/2 + \lambda\Phi^4/4$. We expect that the fluctuations coming from the SF will give us some information about the matter distribution that we currently observe.

Key words: cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

We begin this work by remembering the framework of the standard cosmological model: a homogeneous and isotropic Universe, the evolution of which is best described by Friedmann's equations coming from general relativity and the main ingredients of which can be described by fluids with characteristics very similar to those we see in our Universe. Of course, the Universe is not exactly homogeneous and isotropic, but this standard model does give us a framework within which we can study the evolution of structures like observed galaxies or clusters of galaxies from small fluctuations in the density of the early Universe. In this model, 4 per cent of the mass in the Universe is in baryons, 22 per cent is non-baryonic dark matter and the rest is in some form of cosmological constant. Another idea that has been around just slightly under a hundred years, and on which many of the cosmological models are based, is that of a homogeneous and isotropic Universe, although it has always been clear that this homogeneity and isotropy are only found up to a certain level. Now we know that anisotropies are very important and can grow as large as the large-scale structure we see today.

Nowadays the most accepted model in cosmology that explains the evolution of the Universe is known as ΛCDM, because it has matched some observations with outstanding success, for example the fact that the cosmic microwave background can be explained in great detail and provides a framework within which one can understand the large-scale isotropy of the Universe and important characteristics of the origin, nature and evolution of the density fluctuations believed to give rise to galaxies and other cosmic structure. However, certain problems remain at galactic scales, such as the cusp profile of central densities in galactic haloes, over 500 substructures predicted by numerical simulations yet not found in observations, etc. For examples, see Moore et al. (1999), Clowe (2006) and Penny et al. (2009).

In the big bang model, gravity plays an essential role: it collects the dark matter in concentrated regions called 'dark matter haloes'. In large dark matter haloes, the baryons are believed to be so dense that they radiate enough energy to collapse into galaxies and stars. The most massive haloes, hosts for the brightest galaxies, are formed in the regions with the highest local mass density. Less massive haloes, hosts for the less bright galaxies, appear in regions with low local densities, i.e. regions where the local density is not well defined (Peebles & Nusser 2010). These situations appear to be the same as in our extragalactic neighbourhood, but there are still problems.

Observations point to a better understanding of the theory, beginning with the less occupied space called the 'local void', which contains just a few galaxies that are larger than expected. This problem would be solved if structure grew faster than in the standard theory, therefore filling the local void and giving rise to more matter in the surroundings (Peebles & Nusser 2010).

Another problem arises for so-called 'pure disc galaxies', which do not appear in numerical simulations of structure formation in the standard theory, because it is believed that their formation, which is relatively slow, began in thick stellar bulges. Again this problem would be solved if there were early formation of structure.

The incorporation of a new kind of dark matter, different from that proposed by the ΛCDM model, into the big bang theory offers the possibility of resolving some of these issues.

Recent works have introduced a dynamic scalar field with a certain potential $V(\Phi)$ as a candidate for dark matter, see for example Briscese (2011), although there is not yet an agreement for the correct form of the potential of the field. Lee & Koh (1996), and independently Matos & Guzmán (2000), suggested bosonic dark matter (SFDM) as a model for galactic haloes. Another interesting work in this direction was carried out by Matos & Ureña (2000) and

⋆E-mail: asuarez@fis.cinvestav.mx; tmatos@fis.cinvestav.mx

independently by Sahni & Wang (2000), where they used a potential of the cosh form to explain the core density problem for disc galaxy haloes in the ΛCDM model. Matos & Guzmán (2000) presented a model for the dark matter in spiral galaxies, in which they supposed that dark matter is a scalar field endowed with a scalar potential.

Several recent works have also suggested that SFDM can be composed of spin-0 bosons that give rise to Bose–Einstein condensates (BECs), which at the same time can make up the galaxies we are observing in our Universe (Lundgren et al. 2010). Hu, Barkana & Gruzinov (2000) proposed that dark matter is composed of ultra/light scalar particles, initially in the form of a BEC. In their work Woo & Chiueh (2009) used a bosonic dark matter model to explain structure formation via high-resolution simulations. Finally, Ureña (2010) reviewed the key properties that may arise from the bosonic nature of SFDM models.

The main objective of this work, in difference from others, is to introduce SFDM and assume that dark matter itself is a scalar field that involves an auto-interacting potential of the form $V(\Phi) = \mu^2\Phi^2/2 + \lambda\Phi^4/4$, where $\mu_\Phi \sim 10^{-22}$ eV is the mass of the scalar field (Lee & Koh 1996; Matos & Ureña 2001; Hu et al. 2000). With the mass $\mu_\Phi \sim 10^{-22}$ eV and only one free parameter when λ is taken equal to zero, the SFDM model fits the following important features.

(i) The cosmological evolution of the density parameters of all components of the Universe (Matos, Vázquez-González & Magaña 2009).

(ii) The rotation curves of galaxies (Boehmer & Harko 2007) and the central density profile of LSB galaxies (Bernal, Matos & Núñez 2008).

(iii) With this mass, the critical mass of collapse for a real scalar field is just $10^{12}\,\mathrm{M}_\odot$, i.e. the one observed in galactic haloes (Alcubierre et al. 2002).

(iv) The central density profile of the dark matter is flat (Bernal et al. 2008).

(v) The scalar field has a natural cut-off; thus the substructure in clusters of galaxies is avoided naturally. With a scalar field mass of $\mu_\Phi \sim 10^{-22}$ eV, the amount of substructure is compatible with that observed (Matos & Ureña 2001).

In this paper we show that SFDM predicts galaxy formation earlier than the cold dark matter (CDM) model, because BECs form at a critical temperature $T_\mathrm{c} \gg$ TeV. Therefore, if SFDM is correct, this would imply that we have to observe large galaxies at high redshifts. In order to do this, we study the density fluctuations of the scalar field from a hydrodynamical point of view, which will give us some information about the energy density of dark matter haloes necessary to obtain the observational results of large-scale structure. Here we will give some tools that might be necessary for study of the early formation of structure.

In Section 2 we analyse the analytical evolution of a scalar field (SF), then in Section 3 we treat the SF as a hydrodynamical fluid in order to study its evolution in terms of density contrast. In section 4 we compare our results with those obtained by the CDM model for the density contrast in the radiation-dominated era just before recombination, and finally we give our conclusions.

2 THE BACKGROUND

In this section we perform a transformation in order to solve the Friedmann equations analytically with the approximation $H \ll \mu_\Phi$. The scalar field (SF) that we deal with depends only on time, $\Phi = \Phi_0(t)$, and of course the background is purely time-dependent as well.

We use the Friedmann–Lemaıtre–Robertson–Walker (FLRW) metric with scalefactor $a(t)$. The background Universe is composed only of SFDM (Φ_0) endowed with a scalar potential. We begin by recalling the basic background equations. From the energy–momentum tensor $\boldsymbol{T}$ for a scalar field, the scalar energy density T_0^0 and the scalar pressure T_j^i are given by

$$T_0^0 = -\rho_{\Phi_0} = -\left(\frac{1}{2}\dot{\Phi}_0^2 + V\right), \tag{1}$$

$$T_j^i = p_{\Phi_0} = \left(\frac{1}{2}\dot{\Phi}_0^2 - V\right)\delta^i_j, \tag{2}$$

where the dots stand for the derivative with respect to the cosmological time and $\delta^i{}_j$ is Kronecker's delta. Thus, the equation of state (EoS) for the scalar field is $p_{\Phi_0} = \omega_{\Phi_0}\rho_{\Phi_0}$ with

$$\omega_{\Phi_0} = \frac{\frac{1}{2}\dot{\Phi}_0^2 - V}{\frac{1}{2}\dot{\Phi}_0^2 + V}. \tag{3}$$

Notice that background scalar quantities have the subscript 0. Now the following dimensionless variables are defined:

$$x \equiv \frac{\kappa}{\sqrt{6}}\frac{\dot{\Phi}_0}{H}, \quad u \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{V}}{H},$$

where $\kappa^2 \equiv 8\pi G$ and $H \equiv \dot{a}/a$ is the Hubble parameter. Here we take the scalar potential as $V = m^2\,\Phi^2/2\hbar^2 + \lambda\Phi^4/4$, where, $\mu = mc/\hbar$ and m is the mass given in kg, and from now on we will use units where $c = 1$. For an ultralight boson particle we then have $\mu_\Phi \sim 10^{-22}$ eV.

With these variables, the density parameter Ω_Φ for the background 0 can be written as

$$\Omega_{\Phi_0} = x^2 + u^2. \tag{4}$$

In addition, we may write the EoS of the scalar field as

$$\omega_{\Phi_0} = \frac{x^2 - u^2}{\Omega_{\Phi_0}}. \tag{5}$$

Since ω_{Φ_0} is a function of time, if its time average tends to zero this would imply that Φ^2 dark matter is able to mimic the EoS for CDM (see Matos, Magaña & Suárez 2010; Matos et al. 2009).

Now we express the SF, Φ_0, in terms of the new variables S and $\hat{\rho}_0$, where S is constant in the background and $\hat{\rho}_0$ is the energy density of the fluid, also in the background. Our background field is therefore proposed to be

$$\Phi_0 = \left(\psi_0 \mathrm{e}^{-\mathrm{i}mt/\hbar} + \psi_0^* \mathrm{e}^{\mathrm{i}mt/\hbar}\right), \tag{6}$$

where

$$\psi_0(t) = \sqrt{\hat{\rho}_0(t)}\,\mathrm{e}^{\mathrm{i}S/\hbar}, \tag{7}$$

and with this our background SF can be finally expressed as

$$\Phi_0 = 2\sqrt{\hat{\rho}_0}\cos(S - mt/\hbar). \tag{8}$$

With this we obtain

$$\dot{\Phi}_0^2 = \hat{\rho}_0\left[\frac{\dot{\hat{\rho}}_0}{\hat{\rho}_0}\cos(S - mt/\hbar) - 2(\dot{S} - m/\hbar)\sin(S - mt/\hbar)\right]^2. \tag{9}$$

To simplify, observe that the uncertanty relation implies that $m\Delta t \sim \hbar$, and for the background in the non-relativistic case the relation $\dot{S}/m \sim 0$ is satisfied. Notice also that for the background we have the density varying as $(\ln \hat{\rho}_0)\dot{} = -3H$, but we also have $H \sim 10^{-33}\,\mathrm{eV} \ll \mu_\Phi \sim 10^{-22}\,\mathrm{eV}$ so, with these considerations at hand for the background, in (9) we have

$$\dot{\Phi}_0^2 = 4\frac{m^2}{\hbar^2}\hat{\rho}_0 \sin^2(S - mt/\hbar). \tag{10}$$

Finally, substituting this last equation and equation (8) into (1) when taking $\lambda = 0$, we obtain

$$\rho_{\Phi_0} = 2\frac{m^2}{\hbar^2}\hat{\rho}_0 \left[\sin^2(S - mt/\hbar) + \cos^2(S - mt/\hbar)\right] = 2\frac{m^2}{\hbar^2}\hat{\rho}_0. \tag{11}$$

Comparing this result with (4), we see that identity $\Omega_{\Phi_0} = 2m^2\hat{\rho}_0/\hbar^2$ holds for the background, so, comparing with (11),

$$x = \sqrt{2\hat{\rho}_0}\frac{m}{\hbar}\sin(S - mt/\hbar), \tag{12}$$

$$u = \sqrt{2\hat{\rho}_0}\frac{m}{\hbar}\cos(S - mt/\hbar). \tag{13}$$

We plot the evolution of the energies (12) and (13) in Fig. 1, where for the evolution we used the e-folding number N defined as $N = \ln(a)$ and the fact that $a \sim t^n \to t \sim \mathrm{e}^{N/n}$. In terms of the two analytic results (12) and (13), Fig. 1 shows the kinetic and potential energies of the scalar field. Observe the excellent agreement with the numerical results in Matos et al. (2009) for the kinetic and potential energies of the background respectively.

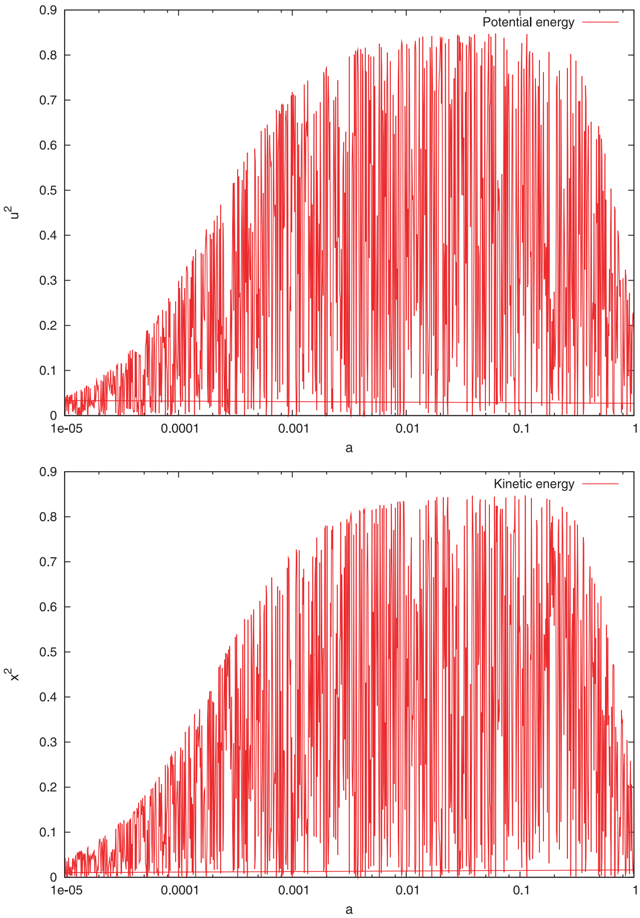

Figure 1. Analytical evolution of the potential (top panel) and kinetic (bottom panel) energies of scalar-field dark matter.

3 SCALAR-FIELD FLUCTUATIONS

If dark matter is some kind of elemental particle with mass μ, then there would be about $10^{68}\,\mu\,\mathrm{GeV}^{-1}$ particles to follow in a single galaxy.

Here we describe a model for the non-interacting matter such that (i) it can describe this matter more as a field than as particles and (ii) we find a function that only depends on the three spatial coordinates and time.

Nowadays it is known that our Universe is not exactly isotropic and spatially homogeneous, as the FLRW metric describes. There exist small deviations from this model; if we believe that these deviations are small enough then they can be treated by linear perturbation theory.

Then, if dark matter is composed of scalar particles with masses $\mu \ll 1\,\mathrm{eV}$, the occupation numbers in galactic haloes are so great that the dark matter behaves as a classical field that obeys the Klein–Gordon equation $(\Box^2 + m^2/\hbar^2)\Phi = 0$, where $\Box$ is the D'Alambertian and we have set $c = 1$.

By definition, a perturbation in any quantity is the difference between its value during some event in real space–time and its corresponding 'background' value. For the SF, for example, we have

$$\Phi = \Phi_0(t) + \delta\Phi(\boldsymbol{x}, t), \tag{14}$$

where the background is only time dependent while the perturbations also depend on the space coordinates (Dodelson 2003). Similar cases apply for the metric:

$$\begin{aligned} g_{00} &= -a^2(1 + 2\phi), \\ g_{0i} &= a^2 B_{,i}\,, \\ g_{ij} &= a^2[(1 - 2\psi)\delta_{ij} + 2E_{,ij}\,]. \end{aligned} \tag{15}$$

Here the scalefactor a depends on the conformal time, ψ is a perturbation associated with the curvature and E is associated with the expansion. We will work under the Newtonian gauge, which is defined when $B = E = 0$. An advantage of using this gauge is that here the metric tensor $g_{\mu\nu}$ is diagonal, and so the calculations become much easier. We will only work with scalar perturbations: vector and tensor perturbations are eliminated from the beginning, so that only scalar perturbations are taken into account. Another advantage in using this gauge is that ϕ will play the role of gravitational potential, which will help us to find a simpler physical interpretation, i.e. both potentials ϕ and ψ are then related. This metric has already been used in other works: Bardeen (1980), Ma & Bertschinger (1995) and Malik (2009).

For the perturbed Klein–Gordon, where we have used equation (14) and set $\dot{\phi} = 0$, we have

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} - \frac{1}{a^2}\hat{\nabla}^2\delta\Phi + V_{,\Phi\Phi}\,\delta\Phi + 2V_{,\Phi}\,\phi = 0. \tag{16}$$

The SF Φ has very hard oscillations from the beginning; these oscillations are transmitted to the fluctuations, which apparently seem to grow very fast and are too large. Nevertheless, this behaviour is not physical, because we see only the oscillations of the fields but cannot clearly see the evolution of density (Matos et al. 2010). In order to rid ourselves of these oscillations, in what follows we perform two transformations. The first one changes the perturbed Klein–Gordon equation into a kind of 'Schrödinger' equation and the second transforms this last equation into a hydrodynamical system, in which we can interpret the physical quantities more easily and the observable quantities become much clearer. Now we express the perturbed SF $\delta\Phi$ in terms of the field Ψ:

$$\delta\Phi = \Psi e^{-imt/\hbar} + \Psi^* e^{imt/\hbar}, \tag{17}$$

which oscillates with a frequency proportional to m and $\Psi = \Psi(\boldsymbol{x}, t)$, i.e. proportional to a wavefunction of an ensemble of particles in the condensate (Chaikin & Lubensky 1995). With this equation and the expression for the potential of the scalar field, (16) transforms into

$$-i\hbar\left(\dot{\Psi} + \frac{3}{2}H\Psi\right) + \frac{\hbar^2}{2m}(\Box\Psi + 9\lambda|\Psi|^2\Psi) + m\phi\Psi = 0, \tag{18}$$

where we have defined

$$\Box = \frac{d^2}{dt^2} + 3H\frac{d}{dt} - \frac{1}{a^2}\hat{\nabla}^2. \tag{19}$$

Notice that this last equation could represent a kind of 'Gross–Pitaevskii' equation in an expanding Universe. The only modification of equation (18) in comparison with the Schrödinger or Gross–Pitaevskii equations is the scalefactor a^{-1} associated with the comoving spatial gradient and the fact that the Laplacian $\hat{\nabla}^2 = \partial_{\boldsymbol{x}}^2$ transforms into the D' Alambertian $\Box$.

To explore the hydrodynamical nature of bosonic dark matter, we will use a modified fluid approach (Bohm 1952; Chiueh 1998). Then, to make the connection between the theory of the field and the condensate wavefunction, the field is proposed as

$$\Psi = \sqrt{\hat{\rho}}\, e^{iS}, \tag{20}$$

where Ψ is the condensate wavefunction with $\hat{\rho} = \hat{\rho}(\boldsymbol{x}, t)$ and $S = S(\boldsymbol{x}, t)$ (Ginzburg & Landau 1950). Here we have separated Ψ into a real phase S and a real amplitude $\sqrt{\hat{\rho}}$ and the condition $|\Psi|^2 = \Psi\Psi^* = \hat{\rho}$ is satisfied (Pitaevskii & Stringari 2003). From (20) we have

$$\begin{aligned}
&\dot{\hat{\rho}} + 3H\hat{\rho} - \frac{\hbar}{m}\hat{\rho}\Box S + \frac{\hbar}{a^2 m}\hat{\nabla}S\hat{\nabla}\hat{\rho} - \frac{\hbar}{m}\dot{\hat{\rho}}\dot{S} = 0,\\
&\hbar\dot{S}/m + \omega\hat{\rho} + \phi + \frac{\hbar^2}{2m^2}\left(\frac{\hat{\Box}\sqrt{\hat{\rho}}}{\sqrt{\hat{\rho}}}\right) + \frac{\hbar^2}{2a^2}\left[\hat{\nabla}(S/m)\right]^2\\
&\quad - \frac{\hbar^2}{2}(\dot{S}/m)^2 = 0.
\end{aligned} \tag{21}$$

Now, taking the gradient of (21) then dividing by a and using the definition

$$\boldsymbol{v} \equiv \frac{\hbar}{ma}\hat{\nabla}S, \tag{22}$$

we have

$$\begin{aligned}
&\dot{\hat{\rho}} + 3H\hat{\rho} - \frac{\hbar}{m}\hat{\rho}\Box S + \frac{1}{a}\boldsymbol{v}\nabla\hat{\rho} - \frac{\hbar}{m}\dot{\hat{\rho}}\dot{S} = 0,\\
&\dot{\boldsymbol{v}} + H\boldsymbol{v} + \frac{1}{2a\hat{\rho}}\nabla p + \frac{1}{a}\nabla\phi + \frac{\hbar^2}{2m^2 a}\nabla\left(\frac{\Box\sqrt{\hat{\rho}}}{\sqrt{\hat{\rho}}}\right)\\
&\quad + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \hbar(\dot{\boldsymbol{v}} + H\boldsymbol{v})(\dot{S}/m) = 0,
\end{aligned} \tag{23}$$

where in (21) $\omega = 9\hbar^2\lambda/2m^2$ and in (23) we have defined $p = \omega\hat{\rho}^2$.

It is worth noting that up to this point this last set of equations do not involve any approximations with respect to equation (18) and can be used in both linear and non-linear regimes.

Now, neglecting squared terms, second-order time derivatives and products of time derivatives in this last set of equations, we obtain

$$\frac{\partial\hat{\rho}}{\partial t} + \nabla\cdot(\hat{\rho}\boldsymbol{v}) + 3H\hat{\rho} = 0, \tag{24}$$

$$\begin{aligned}
&\frac{\partial\boldsymbol{v}}{\partial t} + H\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \frac{\hbar^2}{2m^2}\nabla\left(\frac{1}{2\hat{\rho}}\nabla^2\hat{\rho}\right) + \omega\nabla\hat{\rho}\\
&\quad + \nabla\phi = 0,
\end{aligned} \tag{25}$$

$$\nabla^2\phi = 4\pi G\hat{\rho}, \tag{26}$$

where the equation for the gravitational field is given by Poisson's equation (26). In these equations we have introduced $\boldsymbol{r} = a(t)\boldsymbol{x}$, such that $1/a\hat{\nabla} = \nabla = \partial_r$ (Matos 2003).

Equation (22) shows the proportionality between the gradient of the phase and the velocity of the fluid. Note that $\boldsymbol{v}$ can represent the velocity field for the fluid and $\hat{\rho}$ will be the particle density number within the fluid. Also, there exists an extra term of third order for the partial derivatives in the waves amplitude, which varies as the gradient of

$$\frac{\hbar^2}{2m^2}\frac{\Box\sqrt{\hat{\rho}}}{\sqrt{\hat{\rho}}}.$$

This term would result in a sort of 'quantum pressure' that would act against gravity. We keep the condition that ϕ represents the gravitational field. These two sets of equations (24) and (25) would be analogous to Euler's equations of classical 'fluids', see Godréche & Manneville (1998), with the main difference that there exists a 'quantum part', which we call Q, given by

$$Q = \frac{\hbar^2}{2m^2}\frac{\Box\sqrt{\hat{\rho}}}{\sqrt{\hat{\rho}}},$$

which can describe a force or a sort of negative quantum pressure.

For equation (24) we have that $\hat{\rho}$ represents the mass density or particle number density of the fluid, where all the particles have the same mass. Finally, these equations describe the dynamics of a great number of non-interacting identical particles that manifest themselves in the form of a fluid; also, equation (18) can describe a great number of non-interacting but self-interacting identical particles similar to a Bose gas, when the probability density is interpreted as the density number.

Now, these hydrodynamical equations are a set of complicated non-linear differential equations. To solve them we will restrict ourselves to the vicinity of total equilibrium.

For this, let $\hat{\rho}_0$ be the mass density of the fluid in equilibrium. The average velocity $\boldsymbol{v}_0$ is taken as zero in equilibrium, so we only have $\boldsymbol{v}\,(\boldsymbol{x}, t)$ out of equilibrium. Then, the matter in the Universe is considered as a hydrodynamical fluid inside a Universe in expansion. This system will then evolve in this Universe and later on they will collapse because of their gravitational attraction.

Then from (24) for the mass density of a fluid in equilibrium, we have

$$\frac{\partial\hat{\rho}_0}{\partial t} + 3H\hat{\rho}_0 = 0, \tag{27}$$

with solutions of the form

$$\hat{\rho}_0 = \frac{\rho_{0i}}{a^3}, \tag{28}$$

where, as we know, in general if we have an equation of state of the form $\hat{p} = \omega\hat{\rho}$ and consider CDM or dust as dark matter such that $\hat{p} = 0$ then it holds that $\hat{\rho} \propto a^{-3}$. Thus when the scalefactor was small the densities were necessarily bigger. Now, the particle number density is inversely proportional to the volume and must be proportional to a^{-3}, therefore the matter energy density will also be proportional to a^{-3}, a result that is consistent with our expression (28).

Now, for a system out of equilibrium we have

$$\frac{\partial \delta\hat{\rho}}{\partial t} + 3H\delta\hat{\rho} + \hat{\rho}_0 \nabla \cdot \delta \boldsymbol{v} = 0,$$
$$\frac{\partial \delta \boldsymbol{v}}{\partial t} + H\delta\boldsymbol{v} - \frac{\hbar^2}{2m^2}\nabla\left(\frac{1}{2}\nabla^2 \frac{\delta\hat{\rho}}{\hat{\rho}_0}\right) + \omega\nabla\delta\hat{\rho} + \nabla\delta\phi = 0,$$
$$\nabla^2\delta\phi = 4\pi G\delta\hat{\rho}, \tag{29}$$

equations that are valid in a Universe in expansion. In order to solve system (29) we look for solutions in the form of plane waves; for this a convenient ansatz is

$$\delta\hat{\rho} = \hat{\rho}_1(t)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}/a),$$
$$\delta\boldsymbol{v} = \boldsymbol{v}_1(t)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}/a),$$
$$\delta\phi = \phi_1(t)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}/a),$$

where $\boldsymbol{x}$ is the position vector and $\boldsymbol{k}$ is a real wavevector corresponding to a wavelength λ. If we substitute the above ansatz in the set of equations (29), we then have

$$\frac{\mathrm{d}\hat{\rho}_1}{\mathrm{d}t} + 3H\hat{\rho}_1 + \mathrm{i}\frac{\hat{\rho}_0}{a}\boldsymbol{k}\cdot\boldsymbol{v}_1 = 0, \tag{30}$$

$$\frac{\mathrm{d}\boldsymbol{v}_1}{\mathrm{d}t} + H\boldsymbol{v}_1 + \mathrm{i}\frac{\hat{\rho}_1}{a}\left(\frac{v_\mathrm{q}^2}{\hat{\rho}_0} - 4\pi G\frac{a^2}{k^2} + \omega\right)\boldsymbol{k} = 0, \tag{31}$$

$$\phi_1 + 4\pi G\frac{a^2}{k^2}\hat{\rho}_1 = 0, \tag{32}$$

where we have defined the velocity

$$v_\mathrm{q}^2 = \frac{\hbar^2 k^2}{4a^2m^2}. \tag{33}$$

To solve the system, it is convenient to rotate the coordinate system so that the propagation of the waves will be along the direction of one of the axes. For this we know that the velocity vector can be divided into longitudinal (parallel to $\boldsymbol{k}$) and transverse (perpendicular to $\boldsymbol{k}$) parts, so we have $\boldsymbol{v}_1 = \lambda\boldsymbol{k} + \boldsymbol{v}_2$, where $\boldsymbol{v}_2$ is the vector perpendicular to the wave propagation vector $\boldsymbol{k}\cdot\boldsymbol{v}_2 = 0$. In terms of $\boldsymbol{v}_2$ for equations (30)–(32), we have

$$\frac{\mathrm{d}\hat{\rho}_1}{\mathrm{d}t} + 3H\hat{\rho}_1 + \mathrm{i}\frac{\hat{\rho}_0}{a}k^2\lambda = 0, \tag{34}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} + H\lambda + \frac{\mathrm{i}}{a}\left(\frac{v_\mathrm{q}^2}{\hat{\rho}_0} - 4\pi G\frac{a^2}{k^2} + \omega\right)\hat{\rho}_1 = 0, \tag{35}$$

in addition to an equation for $\boldsymbol{v}_2$, $\mathrm{d}\boldsymbol{v}_2/\mathrm{d}t + H\boldsymbol{v}_2 = 0$, with solutions $\boldsymbol{v}_2 = C/a$ with C a constant of integration, i.e. perpendicular modes to the wavevector are eliminated with the expansion of the Universe. Now, if we use the result (28), then equation (34) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\hat{\rho}_1}{\hat{\rho}_0}\right) = -\frac{\mathrm{i}k^2\lambda}{a}. \tag{36}$$

System (34)–(35) can be treated as in the case of a Universe with no expansion, so combining the two equations and with the aid of (36) we obtain

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}\delta}{\mathrm{d}t} + \left[\left(v_\mathrm{q}^2 + \omega\hat{\rho}_0\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right]\delta = 0, \tag{37}$$

where $\delta = \hat{\rho}_1/\hat{\rho}_0 = \rho_1/\rho_0$ is defined as the density contrast. This will be a fundamental equation for an understanding of the evolution of primordial fluctuations.

4 RESULTS

First we will give a brief summary of the results for the ΛCDM model; this will enable us then to make a direct comparison with our results.

For CDM, the equation for the evolution of the density contrast is given by

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}\delta}{\mathrm{d}t} + \left(c_\mathrm{s}^2\frac{k^2}{a^2} - 4\pi G\hat{\rho}_0\right)\delta = 0, \tag{38}$$

where c_s is defined as the sound velocity (which in our case it is not). Now let us analyse equation (38) at the beginning of the the matter-dominated era, a time just after the epoch of equality and just before recombination (when the radiation has cooled down and the photons do not interact with the electrons any more); for a relativistic treatment see Gorini et al. (2008). In this era, $a \geq a_\mathrm{eq}$, practically all the interesting fluctuation modes are well within the horizon and the evolution of the perturbations can be well described within the Newtonian analysis. At this time, matter behaves like dust with zero pressure. We therefore have $a \sim t^{2/3}$, $c_\mathrm{s}^2k^2/a^2 \approx 0$ and $\hat{\rho}_0 \sim t^{-2}$; therefore $H = (2/3)1/t$. For equation (38) we have

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + \frac{4}{3}\frac{1}{t}\frac{\mathrm{d}\delta}{\mathrm{d}t} - \frac{2}{3}\frac{1}{t^2}\delta = 0. \tag{39}$$

The solutions to this equation are of the form

$$\delta(t) \to t^{2/3}C_1 + \frac{C_2}{t}, \tag{40}$$

where C_1 and C_2 are integration constants. From this solution we can see that we have modes that will disappear as time goes by and modes that grow proportionally to the expansion of the Universe. This is an important result, because then the density contrast will grow proportionally to the expansion of the Universe when this is dominated by matter. Thus, these fluctuations can perhaps grow and give life to galaxies, clusters of galaxies and all the large-scale structure we see currently.

Now let us see what happens to the SFDM at this epoch ($a \geq a_\mathrm{eq}$). The evolution of the perturbations in this case will be given by equation (37).

In general, we have that in equation (37) the term v_q is very small throughout the evolution of the perturbations ($v_\mathrm{q} \leq 10^{-3}\,\mathrm{m\,s^{-1}}$ for small k), so it really does not have a significant contribution to the evolution.

When the condition $\lambda = 0$ is taken, we have a BEC that may or may not be stable. If stability exists, the results of SFDM are consistent with those obtained from CDM (in this case both equations (37) and (38) are almost equal). The condition of stability for the BEC in the SFDM case will come from the study of λ together with Q:

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}\delta}{\mathrm{d}t} + \left(v_\mathrm{q}^2\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0. \tag{41}$$

As we can see in Fig. 2, the perturbations used for the ΛCDM model grow in a similar way to those in the SFDM model when $\lambda = 0$. In this case both perturbations can give birth to structures quite similar in size, and this will happen with all fluctuations as long as k is kept small.

When $\lambda \neq 0$ the results are quite different, so when discussing the evolution of the density perturbations there are two different cases.

(i) In the case of $\lambda > 0$ the amplitude of the density contrast tends to decrease as λ moves further away from zero, until the amplitude

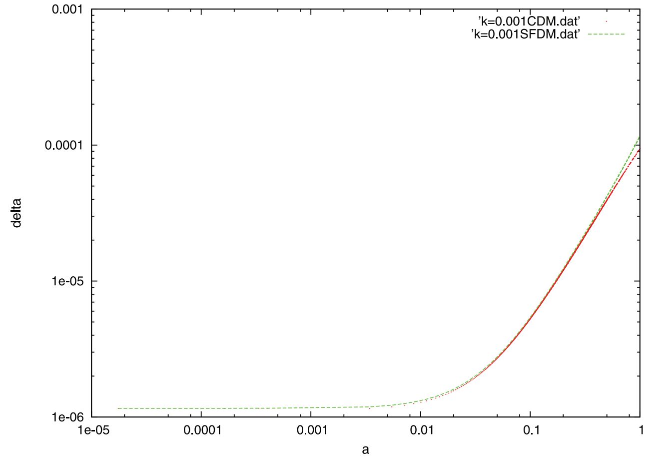

Figure 2. Evolution of perturbations for the CDM model (dots) and SFDM model (lines) for $k = 1 \times 10^{-3}\,h\,\mathrm{Mpc}^{-1}$. Notice how after the epoch of equality ($a_{\mathrm{eq}} \sim 10^{-4}$) the evolution of both perturbations is nearly identical, $a = 1$ today. In this case we have taken $\lambda = 0$.

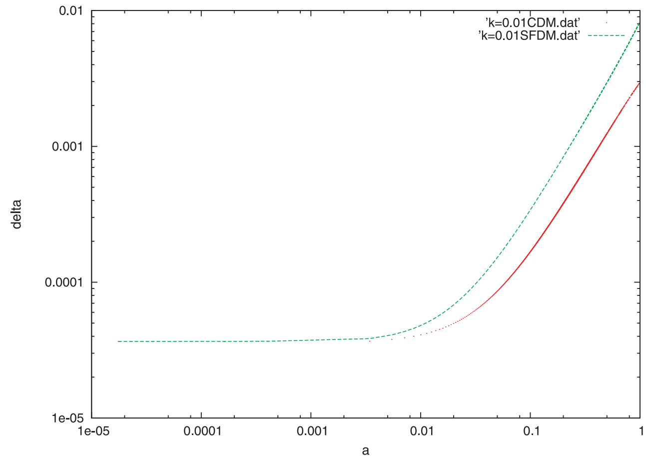

Figure 3. Evolution of perturbations for the CDM model (dots) and SFDM model (lines) for $k = 1 \times 10^{-2}\,h\,\mathrm{Mpc}^{-1}$ and $\lambda \neq 0$ and negative. Notice how after the epoch of equality ($a_{\mathrm{eq}} \sim 10^{-4}$) the evolution of both perturbations is now different from that in Fig. 2, $a = 1$ today. In this case we can clearly see that the SFDM fluctuations grow more quickly than those for the CDM model.

of the density takes negative values (around $\lambda \sim 10^8$), telling us that this kind of fluctuation cannot grow in time and hence will not form a BEC.

(ii) On the other hand, if $\lambda < 0$ the fluctuations for the density contrast always grow despite their size. This result means that either the fluctuations grow and form a stable BEC or the density grows, because it is collapsing into a single point, and our BEC might be unstable. The study of the stability of such fluctuations needs then to be made with non-linear perturbation theory.

These results are shown in Fig. 3. In both Figs 2 and 3 the initial condition for δ varies as $\delta \sim 1 \times 10^{-5}$, in accordance with the data obtained from *WMAP*.

If the resulting fluctuations are stable then, because they are large in size, this means that they can only give birth to large structures. These fluctuations can then help in the formation of large clusters or other large-scale structure in the Universe during its early stages (around $a \geq a_{\mathrm{eq}}$). As this kind of SFDM can only interact with radiation in a gravitational form, it is not limited by its interaction with radiation, and dark matter haloes can then create potential wells that will collapse early in time, giving enough time for structures to form. Then, if DM is some kind of SFDM, luminous matter will follow the DM potential, giving birth to large-scale structure.

5 CONCLUSIONS

New observational instruments and telescopes have, to date, perceived objects as far as $z = 8.6$ (Lehnert et al. 2010). Cosmic background radiation can bring us information from $z = 1000$ to $z = 2000$. However, we cannot see anything from the intermediate region. We now know of a possible galaxy that might be found at a distance of $z = 10.56$, but this has yet to be confirmed.

As seen earlier, as expected for the CDM model we obtained that for the matter-dominated era the low-k modes grow. When CDM decouples from radiation at a time just before recombination, it grows in a milder way than it does in the matter-dominated era (Fig. 2).

Although in general a scalar field is not a fluid, it can be treated as if it behaves like one. The evolution of its density can be an appropriate consideration for the purpose of structure formation, because locations with a high density of dark matter can support the formation of galactic structure.

In this work we have assumed that there is only one component to the mass density, and that this component is given by the scalar-field dark matter. In this case equation (37) is valid for all subhorizon-sized perturbations in our non-relativistic species, so for subhorizon perturbations a Newtonian treatment working within the evolution of the perturbations suffices.

The SFDM has provided to be an alternative model for the dark matter nature of the Universe. We have shown that a scalar field with an ultralight mass of 10^{-22} eV simulates the behaviour of CDM in a Universe dominated by matter when $\lambda = 0$, because in general in a matter-dominated Universe for low k, v_{q} tends to be a very small quantity tending to zero, so from (37) we can see that in this era we will have the CDM profile given by (38), i.e. the SFDM density contrast profile is very similar to that of the ΛCDM model (Fig. 2). In contrast, for $\lambda \neq 0$ both models have different behaviour as we can see from Fig. 3, a result that shows that linear fluctuations in SFDM can grow in comparison with CDM, even at early times when large-scale modes (small k) have entered the horizon just after $a_{\mathrm{eq}} \sim 10^{-4}$ (when it has decoupled from radiation), so that the amplitudes of the density contrast start to grow faster than those for CDM around $a \sim 10^{-2}$. Here an important point is that although CDM can grow it does so in a hierarchical way, while from Fig. 3 we can see that SFDM can have larger fluctuations just before the ΛCDM model does, i.e. it may be that no hierarchical model of structure formation is needed for SFDM. It is expected that for non-linear fluctuations the behaviour will be exactly the same as soon as the scalar field condenses, at a very early epoch when the energy of the Universe is about $\sim$ TeV. These facts point to the crucial difference between both models.

As mentioned before, recent observations have taken us to very early epochs in the origin of the Universe, and have made us think that structure had already been formed, corresponding to $z \approx 7$. It is clear from Fig. 3 that at recombination $z \approx 1300$ there already existed defined perturbations in the energy density for the SFDM model, which could contribute to the early formation of structure. Then, if clusters could be formed as early as these z values, this would imply that $\Phi^2 + \lambda\Phi^4$ as a model for dark matter could give an explanation for the characteristic masses that are being observed, and therefore it could solve some of the problems present in the standard ΛCDM model.

Although the observational evidence seems to be in favour of some kind of cold dark matter, the last word has not been had on the matter. Astronomers hope to send satellites that will detect the finest details of cosmic background radiation, which will help us to obtain information about structure at the time of recombination, from which it will be possible to deduce the evolution of structure up till the present day.

ACKNOWLEDGMENTS

This work was partially supported by CONACyT México, Instituto Avanzado de Cosmologia (IAC) collaboration. A. Suárez is supported by a CONACYT scholarship.

REFERENCES

Alcubierre M., Guzmán F. S., Matos T., Núñez D., Ureña L. A., Wiederhold P., 2002, Class. Quant. Grav., 19, 5017
Bardeen J. M., 1980, Phys. Rev. D, 22, 1882
Bernal A., Matos T., Núñez D., 2008, Rev. Mex. Astron. Astrofís., 44, 149
Boehmer C. G., Harko T., 2007, J. Cosmol. Astro-Particle Phys., 706, 025
Bohm D., 1952, Phys. Rev., 85, 180
Briscese F., 2011, Phys. Lett. B, 696, 315
Chaikin P. M., Lubensky T. C., 1995, Principles of Condensed Matter Physics. Cambridge Univ. Press, Cambridge
Chiueh T., 1998, Phys. Rev. E, 57, 4150
Clowe D., Bradac M., Gonzalez A. H., Markevitch M., Randall S. W., Jones C., Zaritsky D., 2006, ApJ, 648, 2
Dodelson S., 2003, Modern Cosmology. Academic Press, New York
Ginzburg V. L., Landau L. D., 1950, Zhurnal Eksperimental noi i Teoreticheskoi Fiziki, 20, 1064
Godréche C., Manneville P., 1998, Hydrodynamics and Nonlinear Instabilities. Cambridge Univ. Press, Cambridge
Gorini V., Kamenshchik A. Y., Moschella U., Piattella O. F., Starobinsky A. A., 2008, J. Cosmol. Astro-Particle Phys., 802, 016
Hu W., Barkana R., Gruzinov A., 2000, Phys. Rev. Lett., 85, 1158
Lee J., Koh I., 1996, Phys. Rev. D, 53, 2236
Lehnert M. D. et al., 2010, Nat, 467, 940
Lundgren A. P., Bondarescu M., Bondarescu R., Balakrishna J., 2010, ApJ, 715, L35
Ma C.-P., Bertschinger E., 1995, ApJ, 455, 7
Malik K. A., 2009, Phys. Rep., 475, 1
Matos T., 2003, Rev. Mex. Astron. Astrofís., 49, 16
Matos T., Guzman F. S., 2000, Class. Quantum Grav., 17, L9
Matos T., Ureña L. A., 2000, Class. Quantum Grav., 17, L75
Matos T., Ureña L. A., 2001, Phys. Rev. D, 63, 063506
Matos T., Vázquez-González A., Magaña J., 2009, MNRAS, 393, 1359
Matos T., Magaña J., Suárez A., 2010, The Open Astron. J., 3, 94
Moore B., Ghigna S., Governato F., Lake G., Quinn T., Stadel J., Tozzi P., 1999, ApJ, 524, L19
Peebles P. J. E., Nusser A., 2010, Nat, 465, 565
Penny S. J., Conselice C. J., De Rijcke S., Held E. V., 2009, MNRAS, 393, 1054
Pitaevskii L., Stringari S., 2003, Bose-Einstein Condensation. Oxford Univ. Press, Oxford
Sahni V., Wang L., 2000, Phys. Rev. D, 62, 103527
Ureña L. A., 2010, in Macias A., Maceda M., eds, AIP Conf Proc. Vol. 1318, Recent Development in Gravitation and BEC's Phenomenology. Am. Inst. Phys., New York, p. 82
Woo T. P., Chiueh T., 2009, ApJ, 697, 850

This paper has been typeset from a TEX/LATEX file prepared by the author.