5D Model for the Magnetosphere of Static Bodies

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Abstract: Starting from a five-dimensional gauge model invariant under an U(1)-group, we present a new class of exact solutions which represent magnetic monopoles, dipoles, quadripoles etc., whose gravitational potential possess a Schwarzschild-like behavior.

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There exist a discrepancy between general relativity and particle physics in how they interpret interactions between matter. In gauge theory and particle physics interactions are understood as interchanging of virtual particles. For instance a proton interacts with an electron because they interchange a virtual photon. For general relativity interaction is understood in a very different way. Two mass bodys interact because they modify their space-time. Of course virtual particles are used in weak, strong and electromagnetic interactions where gauge theory has had a enormous success, while geometry is used only in gravitational interactions. General relativity was inspired in that "the idea of a thing (the space-time continium) that acts, but on which one can not act, is contrary to the line of scientific thought" which was formulated by Einstein [1] for justifying the effect of matter on the space-time, this effect is then interpreted as originator of the gravitational interaction. Following this idea, although these interactions are quite different, there is no reason for supposing that they shout be interpreted in a different way. One could adopt the philosophy of gauge theory postulating the existence of an interactions particle and call it the graviton, like in many modern text books of gauge theory, or adopt the geometrization of gauge theory. The first way has some problems because of the no-linearity of the Einstein's field equations and is explaned in modern text books. The second way is based in the minimal couplings principle which defines a covariant derivative over the space-time. The field equations are then derived from a Lagrangian invariant under a Lie group. All field equations are second-order differential equations for the gauge potentials. In this letter we deal with the geometrized U(1)-gauge theory in a curved space-time. The main goal of this work is to give a model for the magnetic field of celestial bodys based in a five-dimensional (5D) geuge theory.

Let us start from a 5D-pseudo-Riemannian space P over which is acting a U(1) group of motion. We interpret $B^4 = P/U(1)$ as the spacetime [2]. P can be viewed as a principal fibre bundle with typical fibre U(1) and projection $\Pi : P \rightharpoonup B^4$. Let $\{\hat{e}_A\}$ $A = 1,...,5$ a base of the tangent bundle TP of P, and $\{\hat{\omega}^A\}$ be its dual. The projection Π defines the vertical subspace V of P, such that $d\Pi(v) = 0$ iff $v\epsilon TV$, and the horizontal subspace, such that $P = H \bigoplus V$ [3]. Of course, because P is 5D and B^4 is 4D, V must be 1-dimensional. Let $\{\hat{e}\}$ be a base of TV and $\{\hat{e}_\alpha\}, \alpha = 1,...,4$ a base of TH, we can write $\{\hat{e}_\alpha, \hat{e}\} = \{\hat{e}_A\}$, with dual $\{\hat{\omega}^\alpha, \hat{\omega}\} = \{\hat{\omega}^A\}$. We interpret the action of U(1) over P as the existence of electromagnetic interactions on P.

A metric on P can be defined as $\hat{g} = \eta_{AB}\hat{\omega}^A \otimes \hat{\omega}^B = \eta_{\mu\nu}\hat{\omega}^\mu \otimes \hat{\omega}^\nu + I^2\hat{\omega} \otimes \hat{\omega}$. For many calculations it is convenient to work in an open set $U \subset B^4$. If ϕ is a trivialization, then $\phi : P \rightharpoonup U \times S^1$, where S^1 is the cyrcle. If we know the trivialization ϕ we can work in $U \times S^1$ and map all the physical quantities back into P through ϕ . It can be shown that the projection of the base $\{\hat{e}_A\}$ into $U \times S^1$ is $d\phi(\{\hat{e}_\alpha, \hat{e}\}) = \{e_\alpha - A_\alpha \frac{\partial}{\partial y}, \frac{\partial}{\partial y}\}$ [4] where the $\{e_\alpha\}$ are defined by $d\Pi(\hat{e}_\alpha) = e_\alpha$ and $\{\frac{\partial}{\partial y}\}$ is a base of S^1. Of course $\{e_\alpha - A_\alpha \frac{\partial}{\partial y}, \frac{\partial}{\partial y}\}$ is a base of the tanget bundle T $U \times S^1$ of $U \times S^1$, whose dual is $\{\omega^\alpha, dy + A_\alpha\omega^\alpha\}$. In terms of this base the metric $\bar{g}$ such that $\phi^*\bar{g} = \hat{g}$ can be written as

$$\bar{g} = \eta_{\mu\nu}\omega^\mu \otimes \omega^\nu + I^2(A_\alpha\omega^\alpha + dy)(A_\beta\omega^\beta + dy) \tag{1}$$

where $g = \eta_{\mu\nu}\omega^\mu \otimes \omega^\nu$ is the metric defined in B^4 . To see the meaning of the quantities A_α , let $\hat{A}$ be a one-form of connection on $P, \hat{A} = \hat{\omega}$, and s a cross section over $U, s : U \rightharpoonup P$. It is easy to see that $s^*(\hat{\omega}) = A_\alpha\omega^\alpha$ [4], i.e. A_α are the components of the projection of the one-form of connection into U. From the action of $U(1)$ over P it follows the existence of an isometry on P and thus of a Killing-vector field X [5]. We can choose local coordinates on $U \times S^1$ such that $X = \frac{\partial}{\partial x^5}$, then the metric components of (1) do not depend on $x^5 = y$. The metric components are now the gauge fields where coordinate transformations (choosing of a local chart on P) are just gauge transformations. In the Kaluza-Klein theory metric (1) is postulated [6], so as the functional dependence of the metric components. In this formulation we only suppose the fibre bundle structure of the whole space and interpret the action of the $U(1)$ group as electromagnetic interactions. The second part of the formulation is to write the field equations. We follow here the Kaluza-Klein ideas. Field equations are second order differential equations for the gauge potentials in general relativity and in Yang-Mills theory. For the same reasons as in general relativity it is natural to postulate that the field equations are derived from the Lagrangian [5]

$$I_5 = \frac{1}{\kappa_0}\int \sqrt{-\bar{g}_5}\bar{R}d^5x \tag{2}$$

where κ_0 is constant, $\bar{g}_5$ is the determinant of the metric components of $\bar{g}$ and $\bar{R}$ is the curvature scalar of $U \times S^1$. From Kaluza-Klein theory we know that action (2) is a theory of gravity coupled with electromagnetism and a scalar field. In this theory, gravity, electromagnetism and the scalar field have the same origin.

A great amount of celestial motions are stationary. Let us suppose this symmetry in the metric (1). If we do so we can define five potentials in covariant manner. Let Y be the Killing vector associated with stationarity, then $\Psi^A = (f, \epsilon, \psi, \chi, \kappa)$ defined as [7]

$$I^2 = \kappa^{4/3} = X_A X^A \qquad f = -IY^AY_A + I^{-1}(X^AY_A)^2 \qquad \psi = -I^{-2}X_AY^A$$

$$\epsilon_{,A} = \epsilon_{ABCDE}X^BY^CX^{D;E} \qquad \chi_{,A} = \epsilon_{ABCDE}X^BY^CY^{D;E} \tag{3}$$

are the gravitational, rotational, electrostatic, magnetostatic and scalar potentials respectively. They are analogous to the Ernst potentials. If we suppose in addition to stationary, axisymmetry the field equations derived from (2) in terms of the potentials Ψ^A can be cast into the form [7]

$$(\rho\Psi^A_{,z})_{,\bar{z}} + (\rho\Psi^A_{,\bar{z}})_{,z} + 2\rho\Gamma^A_{BC}\Psi^B_{,z}\Psi^C_{,\bar{z}} = 0 \tag{4}$$

where $z = \rho + i\zeta$, and Γ^A_{BC} are the Christoffel symbolds of the metric

$$dS^2 = G_{AB}d\Psi^A d\Psi^B$$

$$= \frac{1}{2f^2}[df^2 + (d\epsilon - \psi d\chi)^2] + \frac{1}{2f}[\kappa^2 d\psi^2 + \frac{1}{\kappa^2}d\chi^2] + \frac{2}{3}\frac{d\kappa^2}{\kappa^2} \tag{5}$$

For solving equations (4) we make the following ansatz [8]. Suppose that $\Psi^A = \Psi^A(\lambda, \tau)$ where λ and τ fulfill the Laplace equation

$$(\rho\lambda_{,z})_{,\bar{z}} + (\rho\lambda_{,\bar{z}})_{,z} = 0 \qquad (\rho\tau_{,z})_{,\bar{z}} + (\rho\tau_{,\bar{z}})_{,z} = 0 \tag{6}$$

the equation (4) transforms to

$$\Psi^A_{,\lambda\lambda} + \Gamma^A_{BC}\Psi^B_{,\lambda}\Psi^C_{,\lambda} = 0$$

$$\Psi^A_{,\tau\tau} + \Gamma^A_{BC}\Psi^B_{,\tau}\Psi^C_{,\tau} = 0 \tag{7}$$

Here we deal with three solutions of equation (7) (the complete set of solutions is given in [9])

a) $\chi = \dfrac{a_1e^{q\tau} + a_2e^{-q\tau}}{g_{22}} \qquad g_{22} = be^{q\tau} + ce^{-q\tau} \qquad b + c = \dfrac{1}{I_0}$

b) $\chi = \dfrac{a_1\tau + a_2}{g_{22}} \qquad g_{22} = b\tau + \dfrac{1}{I_0}$

$$c) \quad \chi = \frac{a_1 e^{iq\tau} + \bar{a}_1 e^{-iq\tau}}{g_{22}} \qquad g_{22} = b e^{iq\tau} + \bar{b} e^{-iq\tau} \qquad b + \bar{b} = \frac{1}{I_0} \tag{8}$$

where a_1, a_2, q, b, c, I_0, *and* Λ are constants restricted to $bcq^2 = I_0\delta \neq 0$, and

$$f = \frac{e^{\Lambda\lambda}}{\sqrt{I_0 g_{22}}} \quad , \quad I^2 = \frac{I_0 e^{-\frac{2}{3}\Lambda\lambda}}{g_{22}} \quad , \quad \psi = \epsilon = 0$$

for each case. The space-time metric can be writen in Boyer-Lindquist local coordinates $z = \sqrt{r^2 + 2mr\, sin\theta} + i(r-m)cos\theta$ where $X = \frac{\partial}{\partial y}$, $Y = \frac{\partial}{\partial t}$ and $Z = \frac{\partial}{\partial \varphi}$ as

$$\bar{g} = \frac{1}{If} e^{2k}[1 - \frac{2m}{r} + \frac{m^2 sin^2\theta}{r^2}][\frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\theta^2]$$

$$+ \frac{1}{If}(1 - \frac{2m}{r}) r^2 \sin^2\theta d\varphi^2 - \frac{f}{I} dt^2 + I^2(A_3 d\varphi + dx^5)^2 \tag{9}$$

The function k and A_3 are related with the potentials Ψ^A through

$$k_{,z} = \rho \left[\frac{(f_{,z})^2}{2f^2} + \frac{1}{2f}\frac{(\chi_{,z})^2}{\kappa^2} + \frac{2}{3}\frac{(\kappa_{,z})^2}{\kappa^2} \right]$$

$$A_{3,z} = \frac{\rho}{fI^3}\chi_{,z}. \tag{10}$$

Observe that $\chi = \chi(\tau)$ while f and κ depend on both parameters λ and τ . It is easy to see that

$$A_{3,r} = sin\theta\, \tau_{,\theta}$$

$$A_{3,\theta} = -(r^2 - 2mr) sin\theta\, \tau_{,r} \tag{11}$$

depends only on τ and is allways integrable because τ fulfills the Laplace equation (6)

$$((r^2 - 2mr)\tau_{,r})_{,r} + \frac{1}{sin\theta}(sin\theta\tau_{,\theta})_{,\theta} = 0 \tag{12}$$

Three interesting solutions of (12) into (11) are [10]

$$i) \quad \tau = \tau_0 ln(1 - \frac{2m}{r}) \qquad A_3 = 2\tau_0 m(1 - cos\theta)$$

$$ii) \quad \tau = m\tau_0(r-m)/\Delta \qquad A_3 = \tau_0 m^3 sin^2\theta cos\theta/\Delta$$

$$\Delta = (r-m)^2 - m^2 cos^2\theta \qquad + 2\tau_0 m(1 - cos\theta)$$

$$iii) \quad \tau = \tau_0 m^2 cos\theta/\Delta \qquad A_3 = m^2\tau_0(r-m)sin^2\theta/\Delta \tag{13}$$

For $r >> m$ the electromagnetic potential $A = A_\mu dx^\mu = A_3 d\varphi$ behavies for i) like a magnetic monopole, for ii) like a magnetic quadripole plus a magnetic monopole and for iii) like a magnetic dipole. If we set

$$\lambda = \lambda_0 ln(1 - \frac{2m}{r})$$

in (9) and choose λ_0 conveniently, this metric represents the exterior space-time of a field with magnetic four potential $A = A_3 d\varphi$ and a Schwarzschild-like gravitational potential with metric

$$\bar{g} = \frac{1}{I}\left\{ \sqrt{I_0 g_{22}} e^{\tau_0' \tau_d} \left(\frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\theta^2 \right) \right.$$

$$\left. + \sqrt{I_0 g_{22}}\, r^2 sin^2\theta d\varphi^2 - \frac{\left(1 - \frac{2m}{r}\right)}{\sqrt{I_0 g_{22}}} dt^2 \right\} + I^2(A_3 d\varphi + dx^5)^2 \tag{14}$$

where τ_0' is a constant and $\tau_{d,z} = \rho(\tau_{,z})^2$.

Observe that for $\tau = 0$ the expression between the brakets $\{...\}$ is just the Schwarzschild metric. Metric (14) is asymptotically flat for $r >> m$ and flat for $m = 0$ for any combination of i), ii) and iii) in (13) with a), b) and c) in (8). In all cases I^2 is a function which is very big for $r \sim 2m$ and goes very fast to I_0 for $r >> 2m$. That means that the scalar potential becomes important only near of the λ-singularity $r = 2m$ and desapears far a way from it. Nevertheless we suspect that the singularity $r = 2m$ is not esential. Any solution of (12) for λ and τ and any linear combination of them will be an exact solution of the field equations with metric (8)-(9).

To end we remark that in this theory the electromagnetic and the gravitational potentials are the projection of a unic 5D-potential into the four dimensional spacetime, therefore it is possible that metric (9) could be used as model for magnetized celestial bodys.

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References

[1] Einstein A. The Meaning of Relativity. Princeton Univesity Press (1953).

[2] Cho, Y. M. J. Math. Phys.16, (1975), 2029

[3] Choquet-Brouhat, Y.; De Witt-Morette and Dikard-Bleik, M. Analysis Manifolds and Physics. North Holand (1977)

[4] Matos, T. and Nieto, A. In "Proceedings of the III Taller de Particulas y Campos" Morelia, Mich. Ed. by J. L. Rodriguez et al. Rev. Mex. Fis. complemento (1992).

[5] Schmutzer, E. In "Proceedings GR9", ed. E. Schmutzer, Cambridge University Press (1982), 39.

[6] Duff, M.; Nilsson B. and Pope C. V.Phys. Rep. 130 (1986) 1.Bailin, D. and Love, A. Rep. Prog. Phys. 50 (1987) 1087.

[7] Neugebauer, G. Habilitationsschrift Jena-Universität (Dr. in Sci. Thesis) (1969).

[8] Kramer, D. and Neugebauer, G. Ann. Phys. (Leipzig), 24 (1969) 62.

[9] Matos, T. To be published.

[10] Becerril, R. and Matos, T. Phys. Rev. *D46* (1992) 1540.

PLASMA PHYSICS IN THE EARLY UNIVERSE

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Abstract

Evidence that plasma physics is important in the early Universe is presented: I first give a short summary of the evidence that in the past there occurred a Big Bang and there existed a high temperature plasma of temperature $\gtrsim$ 1 MeV and particle density $\gtrsim$ 4 x 10^{31} cm^{-3}; I then discuss some properties of: 2) plasmons (quantized plasma waves) in the early Universe; 3) charged bubbles created in the quark-baryon phase transition; 4) creation of intense zero frequency electromagnetic waves in the early Universe; and 5) the mass change of particles due to the dense plasma of the early Universe.

I. Evidence of a Big Bang In The Past And A High Temperature Plasma Of Temperature T > 1 MeV And Particle Density $\gtrsim$ 4 x 10^{31} cm^{-3}

The evidence is:

1. Stars and galaxies are moving away from one another (i.e. the "Hubble flow") as if there occured a large explosion $\sim$ 10 - 20 billion years ago;

2. A $\sim$ 2.75 ^{o}K microwave background exists, remnant of a hot dense Big Bang plasma;

3. The observed abundances of the light elements such as D, ^{3}He, ^{4}He and ^{7}Li cannot be produced in stars and can be produced in a Big Bang that occurred $\sim$ 10 - 20 billion years ago after the first few seconds;

4. The age of globular clusters of stars from stellar evolution is $\sim$ 10 - 20 billion years