DIVISION DE GRAVITACION Y FISICA MATEMATICA
SOCIEDAD MEXICANA DE FISICA

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Dear Dr. Tonatiuh Matos,
Hugo Villegas-Brena

I am pleased to inform you that your paper

"Post-Post-Newtonian Limit of a Dilatonic Gravity Model"

has been accepted for publication in the Book **"Recent Developments in Gravitation and Mathematical Physics".** It appeared this year published by Since Network Publishing, in CD Rom, ISBN 3-9805735-0-8.

Sincerely Yours,

Darío Núñez

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By the editorial committee

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Post-Post-Newtonian Limit of a Dilatonic Gravity Model

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Abstract

We study a solution of the field equations for dilatonic gravity and obtain its post-post-newtonian limit. It turns out that terms to this and higher orders in the expansion may become important in strong gravitational fields, even though the post-newtonian limit coincides with that of General Relativity. This suggests that strong gravitational fields can only be studied by exact solutions of the field equations.

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1 Main Text

In recent years there has been an increasing interest in tensor-scalar theories of gravity, as they arise naturally in several unification models. However, for the slow motion, weak field limit, the PPN formalism [1] has proved to be a very powerful tool to restrict theories that are phisically viable. On the other hand, with binary pulsar observations the tests of the strong field regime have begun, but again, the measurements have further limited the possible alternatives to General Relativity [2]. Here we will suggest, by means of an example, that the PPN framework may be incomplete, and therefore, that exact solutions to the field equations may be the only way to describe certain aspects of physical systems in strong fields.

We shall be concerned with the action:

$$S = \int d^4x\sqrt{-g}(R - 2(\nabla\phi)^2 - e^{-2\alpha\phi}F^2), \tag{1}$$

where R is the scalar curvature, ϕ the dilaton, $F_{\mu\nu}$ the Maxwell tensor and α the (non-minimal) coupling parameter. For $\alpha = 0, \sqrt{3}, 1$ we obtain back, respectively, Einstein-Maxwell, Kaluza-Klein and Low-Energy Superstring theories, but we shall consider the general case with arbitrary α. It can be easily checked that the metric [3]:

$$ds^2 = -(1 - \frac{2m}{r})dt^2 + e^{2k_s}\frac{dr^2}{1-\frac{2m}{r}} + r^2(e^{2k_s}d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

with

$$e^{2k_s} = \begin{cases} \left(1 + \frac{m^2\sin^2\theta}{r^2(1-\frac{2m}{r})}\right)^{-1/\alpha^2} & \text{for } \alpha \neq 0, \\ 1 & \text{for } \alpha = 0 \text{ (Schwarzschild solution)}, \end{cases}$$

and

$$\Phi = \frac{1}{2\alpha}\ln\left(1 - \frac{2m}{r}\right),$$

is an axisymmetric static (indeed, quasi spherically symmetric) solution of the field equations for the action (1) in Boyer-Lindquist coordinates. The above expression for e^{2k_s}, which is the contribution from the dilaton to the metric, can be rewritten as:

$$e^{2k_s} = \left(1 + \frac{D^2\alpha^2\sin^2\theta}{r^2(1-\frac{2m}{r})}\right)^{-1/\alpha^2},$$

Here D is the dilatonic charge, defined by [4]:

$$D = \lim_{r\to\infty}\oint_S d^2S^\mu\nabla_\mu\phi = \frac{m}{\alpha}. \tag{3}$$

Now, let us obtain the series expansion of the metric (2) in powers of r, in order to be able to compare it with the PPN expansions. To do so, we perform a transformation to a "pseudo-isotropic" coordinate system by means of:

$$r = R(1 + \frac{m}{2R})^2,$$

thus obtaining:

$$\begin{aligned} ds^2 &= e^{2k_s}(1+\frac{m}{2R})^2(dx^2+dy^2+dz^2) + \frac{(1-e^{2k_s})(1+\frac{m}{2R})^2}{x^2+y^2}(xdy-ydx)^2 \\ &\quad - \frac{(1-\frac{m}{2R})^2}{(1+\frac{m}{2R})^2}dt^2, \end{aligned} \tag{4}$$

where (x, y, z) are related to (R, θ, ϕ) through the usual cartesian to spherical coordinate transformation. Note that, for large r, $R \sim r$. We have then the following expansions:

$$\begin{aligned} g_{tt} &= -(1 - \frac{2m}{R} + \frac{2m^2}{R^2} - \frac{3m^3}{2R^3} + \cdots) \\ g_{xx} &= 1 + \frac{2m}{R} + \frac{1}{R^2}\left[\frac{3m^2}{2} - D^2\frac{(x^2+y^2)}{R^2} + \frac{y^2}{R^2}\right] + \cdots \\ g_{yy} &= 1 + \frac{2m}{R} + \frac{1}{R^2}\left[\frac{3m^2}{2} - D^2\frac{(x^2+y^2)}{R^2} + \frac{x^2}{R^2}\right] + \cdots \\ g_{zz} &= 1 + \frac{2m}{R} + \frac{1}{R^2}\left[\frac{3m^2}{2} - D^2\frac{(x^2+y^2)}{R^2}\right] + \cdots \\ g_{xy} &= -\frac{1}{R^2}\left(\frac{xy}{R^2}D^2\right) + \cdots \end{aligned} \tag{5}$$

The post-newtonian limit corresponds to terms $O(1/R^2)$ in g_{tt} and $O(1/R)$ in g_{ij}, and it coincides with the same limit for General Relativity. It should not be surprising then if this metric passes the classical tests for the solar system, that is, for a weak field, slowly moving system of particles (for a study of this metric in the solar system see [5]), even when the scalar interaction is taken into account. Observe that the dilatonic charge D appears in the expansion only at orders of $O(1/R^2)$ in g_{ij}, which correspond to the post-post-newtonian expansion. Surprisingly, this charge cannot be eliminated from the metric for $\alpha \neq 0$ if the mass parameter $m \neq 0$. But, contrary to what happens in the PPN expansion with scalar field, here the interaction of the scalar field in the metric is weaker as the coupling parameter α gets larger. This is because D has a discontinuity for $\alpha = 0$, as can be seen from (3):

$$\lim_{\alpha \to 0} D = \infty \neq D\,|_{\alpha=0} = 0.$$

For the post-post-newtonian limit, corresponding to terms $O(1/R^4)$ in g_{tt} and $O(1/R^2)$ in g_{ij}, α appears in the metric terms through the value of D. Again contrary to the PPN expansion of scalar theories of gravity, the predictions of this theory will be closer to those of General Relativity if the value of α is large, that is, if the coupling with the scalar field is big. Of course, the special case $\alpha = 0$ should be treated just as the Schwarzschild solution.

Scalar theories of gravity have been studied using either spherically symmetric exact solutions, or the PPN formalism. This analysis shows that maybe it was too premature to discard some of these theories using only the former methods, and thus, assuming that the metric (2) could represent an actual physical system, it should be possible to describe effects that would be otherwise overlooked using the PPN formalism, at least for certain values of α. Finally, it remains to find out whether this metric actually passes the standard tests for the solar system, and whether its predictions for strong fields agree with binary pulsar observations.

References

[1] C.M. Will, "*Theory and Experiment in Gravitational Physics*", Cambridge University Press (1981).

[2] T. Damour and G. Esposito-Farèse, *Phys. Rev.* **D54**, 1474 (1996).

[3] T. Matos, D. Nuñez and H. Quevedo, *Phys. Rev.* **D51**, R310 (1995).

[4] D. Garfinkle, G. T. Horowitz and A. Strominger, *Phys. Rev.* **D43**, 3140 (1991).

[5] T. Matos, *Can Magnetic Fields of Astrophysical Objects be Fundamental?*. To be Published. T. Matos and C. Mora, *Class. Quantum Grav.* **14**, in Press.