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Tully-Fisher Relation and the Background space-time of a Galaxy

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It is presented a geometrical point of view about the conspiracy of a luminous disc and a dark halo in spiral galaxies. It is shown for a particular model of dark matter, that the evolution of spirals could be determined by a parameter of the space-time background which stablishes a kind of Tully-Fisher relation.

It is well known that spiral galaxies (SGs) are made of two components: luminous and dark matter. These have to act toghether in order to explain galaxy formation and the flatness of rotation curves for instance. In the standard interpretation, a fluctuation of the cosmological dark matter should create a potential into which baryonic matter falls just to form these objects. About the flatness of the rotation curves it is possible to say something:[1] according to Newton's law

$$\frac{GM_{dark}(r)}{r^2} = v_c^2 \Rightarrow M_{dark}(r) = \frac{v_c^2 r}{G} \tag{1}$$

where $M_{dark}(r)$ is the mass profile of the matter dominating where the curves are flat in terms of the radial coordinate, i.e. the dark matter and v_c is the velocity of stars and gas in this region. According to (1)

$$4\pi G \int \rho_{dark}(r) r^2 dr = v_c^2 r \Rightarrow \rho(r) = \frac{v_c^2}{4\pi G r^2} \tag{2}$$

which means that the distribution $\rho \sim 1/r^2$ when v_c is independent of r. Note that no information about the nature of dark particles is assumed. In the other hand, the luminous matter is succesfully modeled as an infinitely thin exponential disc $\rho_L \sim e^{-r/r_0}$, for which it is assumed that surface luminosity is proportional to the integrated mass distribution. Such disc predicts a keplerian behavior of rotation curves, i.e. v_c decays as $1/\sqrt{r}$, and therefore a combination of luminous and dark matter is required to reproduce the observed rotation curves, the first for the central part of the galaxy (which according to observations is newtonian) and the second for the exterior sector of the galaxy: the dark matter dominated region (DMDR).

Let $< v_C >$ the averaged nearly radii independent velocity of stars and gas, we mean, the velocity of particles in the DMDR. Observations have shown that the following relation is valid for spiral galaxies

$$< v_C > = 220 \left(\frac{L}{L_*}\right)^{0.22} Kms^{-1} \tag{3}$$

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being L_* the caracteristic luminosity for spirals. Equation (3) is known as the Tully-Fisher relation, and is valid for any meassured galaxy within a 5% of error.

It is worth to investigate if the Tully-Fisher relation could be a property of the space-time background of a galaxy. Let us assume that the space-time of a galaxy is axially symmetric and static, for which we use the line element

$$ds^2 = \frac{1}{f}[e^{2k}(d\rho^2 + d\zeta^2) + W^2 d\phi^2] - f dt^2, \tag{4}$$

where f, W and k depend only on ρ and ζ, and we consider that the galactic disc is contained in the equatorial plane. Such space-time would be the one produced by the presence of the dark matter, thus luminous particles behave as test particles on the backgrond given by (4) beacuse such component represents about the 5% of the whole mass of a galaxy. It has been shown that a solution possesing the property $f \sim W^l$ is the required condition to have flat rotation curves,[2] therefore it must be understood that (4) is valid in the DMDR. We shall take the example ($l = 1$) as was found when the dark matter is assumed to be a scalar field,[3] in order to ilustrate that the background space-time indeed determines the Tully-Fissher relation. Thus we start with $f = f_0 W = f_0(\rho^2 + \zeta^2) = r^2 + b^2$ in boyer lindquist coordinates for $\theta = \pi/2$, then the geodesic equations read[3]

$$\frac{\partial^2 R}{\partial \tau^2} - R\left(\frac{\partial \phi}{\partial \tau}\right)^2 + f_0^2 c^2 R \left(\frac{\partial t}{\partial \tau}\right)^2 = 0, \quad \frac{\partial \phi}{\partial \tau} = \frac{B}{R^2 f_0}, \quad \frac{\partial t}{\partial \tau} = \frac{A}{R^2 f_0} \tag{5}$$

where τ is the proper time of the test particle and $R = \int ds = \sqrt{(r^2 + b^2)/f_0}$ is the proper distance of the test particle at the equator from the galactic center, being B and A the angular momentum and energy per unit of mass of a test particle respectively. Observe that for a circular trajectory, the first of equations (5) reduces to

$$\dot{\phi} = f_0 c = \frac{B}{A} \tag{6}$$

after using the other two equations, where the dot means derivative with respect to t. We can estimate the constant A using the invariance of the metric. At the equator it is obtained

$$ds^2 = -\left(f_0 W - \frac{v^2}{c^2}\right) c^2 dt^2 = -\left(f_0(r^2 + b^2) - \frac{v^2}{c^2}\right) c^2 dt^2 = -c^2 d\tau^2 \tag{7}$$

with $v^2 = g_{ij} v^i v^j$, $v^i = (\dot{r}, \dot{\theta}, \dot{\phi})$, from where it arises an expression for A in terms of the metric functions

$$A = \frac{r^2 + b^2}{\sqrt{f_0(r^2 + b^2) - v^2/c^2}} \sim \sqrt{\frac{r^2 + b^2}{f_0}} = R \tag{8}$$

since $v^2 \ll c^2$. Using (6) and (8) we obtain an estimation for the angular momentum $B = v_{DM} R$, which implies $B \sim f_0 c R$, and therefore $v_{DM} \sim f_0 c = constant$ in the regions where the dark matter dominates; we have labeled v_{DM} the contribution of dark matter to the velocity of test particles. This remarkable result qualitatively agrees with observations, it means that the circular velocity of a star far away from the center of the galaxy v_{DM} does not depend on the distance R. Let us now see what happens when adding the luminous matter. The first of equations (5) is the second Newton's law for particles travelling into the dark matter background. We can interpret

$$\frac{\partial^2 R}{\partial \tau^2} = R\left(\frac{\partial \phi}{\partial \tau}\right)^2 - f_0^2 c^2 R\left(\frac{\partial t}{\partial \tau}\right)^2 = \frac{B^2}{R^3 f_0^2} - c^2\frac{A^2}{R^3} = \frac{c^2}{R} - c^2\frac{A^2}{R^3} \tag{9}$$

as the force due to the dark matter background, i.e. $F_{DM} = c^2/R - c^2 A^2/R^3$. At the other hand, the Newtonian force due to the luminous matter is given by $F_L = GM_L(R)/R^2 = v_L^2/R = B_L^2/R^3$, where v_L is the circular velocity of the test particle due to the contribution of the luminous matter and B_L is its corresponding angular momentum per unit of mass. The total force acting on the test particle is then $F = F_\Phi + F_L$. For circular trajectories $\partial^2 R/\partial \tau^2 = F = 0$, and therefore the following relation is valid

$$\frac{B_L^2}{R^2} - c^2\frac{A^2}{R^2} = -c^2 \tag{10}$$

The constants of motion are the total energy per unit of mass of the test particle and its angular momentum per unit of mass

$$A_L^2 = \frac{f_0^2 c^4 (r^2+b^2)^2}{f_0(r^2+b^2) - v^2/c^2}, \qquad B_L^2 = \frac{v^2(r^2+b^2)}{f_0(f_0(r^2+b^2) - v^2/c^2)} \tag{11}$$

given $B = B_L$, and $A_L = c^2 f_0 A$. Since $v^2 \ll c^2$ an important result arises:[3]

$$v_{DM} = f_0 l/m \tag{12}$$

i.e. the velocity is independent of the radius as observed at large radii, where the dark matter dominates, it is $< v_C >$.

What is remarkable is that B_L is the angular momentum the luminous matter imprints over the test particles and that equals $< v_C >$ except by a factor f_0 which is different for different galaxies described by line elements like (4) and plays the role of the amplitude of the dark matter profile because the temporal component of the enrgy momentum tensor of (4) satisfies $T^0{}_0 = 2f_0/(8\pi G(r^2+b^2)$.[3] Therefore property (12) is physically equivalent to the Tully-Fisher realtion (3).[1]

[1]If the reader needs an expression of f_0 in terms of the luminosity (in solar units) it is $\frac{f_0}{r_0} = 0.05177 L^{-0.3173}$ valid for a sample of 10 SGs we have workerd with.

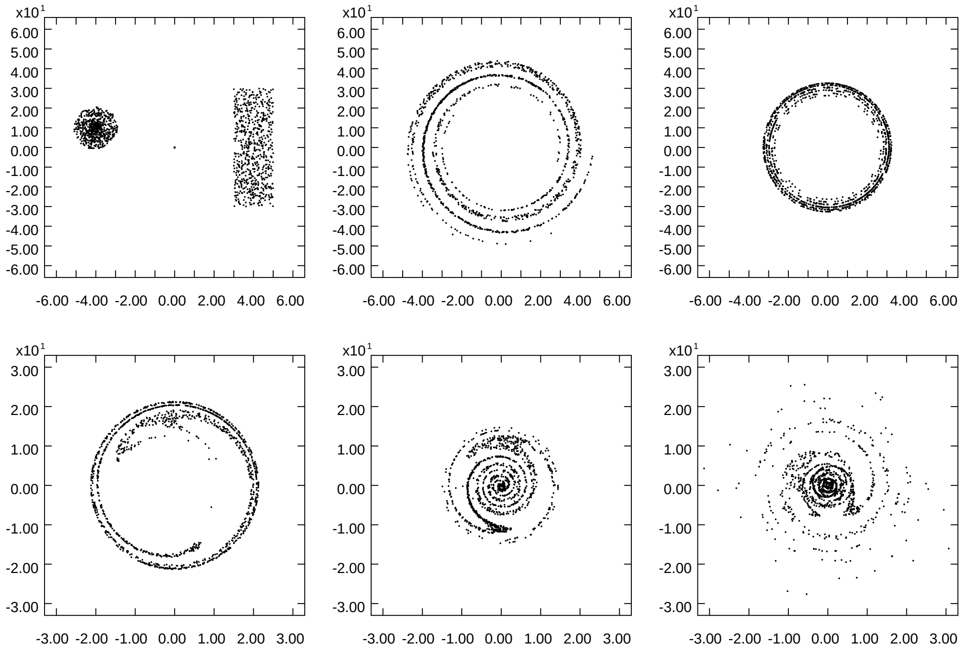

Fig. 1: Evolution of 4000 non-interacting particles falling into the potential corresponding to NGC 2903, as obtained in the rotation curve fitting.[3] Periods between steps is of 40 machine lapses. Units in axes are in KPc.

The Tully-Fisher relation is intimatelly related to the formation of galaxies because $< v_C >$ is associated to the velocity the dust arrived with to the region of the cosmological fluctuation in the process of formation, in fact galaxies still evolving.[4]

Now, let us see if the space-time of our example permits the formation of spirals. In order to do so, we have designed a code to simulate the free falling of particles into the space-time ($l = 1$). The potential obtained from the geodesic equations is

$$V = -\frac{f_0(r^2+b^2)}{2r^2}(f_0^2(r^2+b^2) - B^2) + \frac{f_0 c^2 (r^2+b^2)^3}{2r^2} \tag{13}$$

We start the evolution with two very arbitrary clouds of particles falling into the potential (13) and the evolution appears in the Figure 1,from wich it is evident that spirals can be formed.

Tully-Fisher relation is valid for any SG, and we now show that *our* Tully-Fisher relation (12) can be interpreted in the same sense as the traditional one.

In (4) it is possible to absorb f_0 for the case we are dealing with ($l = 1$), so that the line element reads

$$ds^2 = \frac{1}{f}[e^{2k}(d\rho^2 + d\zeta^2) + W^2 d\phi^2] - f d\hat{t}^2 \tag{14}$$

which is nothing but that of a universal galaxy, including the way the time has to be meassured $\hat{t} \to f_0 t$, and should be different for different galaxies. It is shown in Figure 2,the simulation ran for $f_0 = 0.00844$, 0.000844 and 0.0844 (this last being the parameter that fits with the rotation curve of NGC 2903), for 2000, 200, and 20 machine steps respectively.

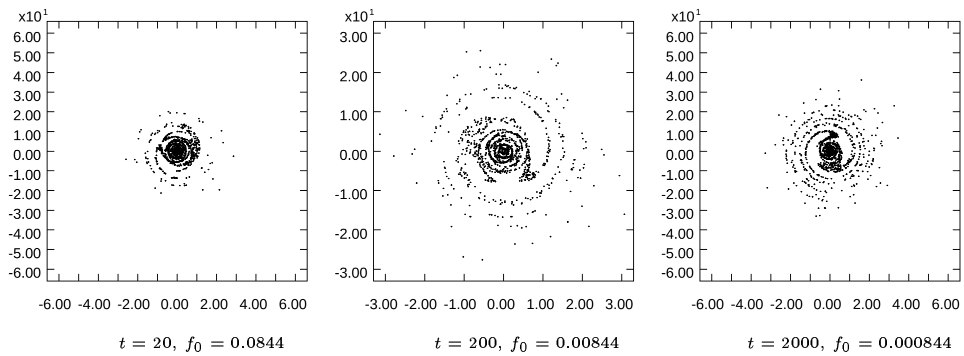

Fig. 2 For distinct f_0 and execution lapses. For an increasing order of magnitude in f_0 the lapse is reduced in one order and visceversa.

The reader surely remembers that the steps between one and another box of Figure 1was 40 and in this case these are an oder of magnitude but with the time modified and in the three cases the galaxies appear in an equivalent stage of evolution. Thus it arises an important question: are all galaxies the same thing, and the differences between one and another is the way the time is meassured? This is the subject of further investigations.

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