GALAXIES FORMATION FROM THE SCALAR FIELD DARK MATTER MODEL

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Abstract In recent works, we presented a model for the dark matter in cosmos supposing that dark matter is a scalar field endowed with a cosh scalar potential. We obtained that the model fit well the observed mass and angular power spectrums, is consistent with the observed acceleration of the expansion of the Universe and contains and natural cutoff for small structure at $\sim 0.1\ Mpc^{-1}$. In other works we presented a model for the dark matter in galaxies supposing that dark matter is a scalar field endowed with an exponential scalar potential. We found that the effective energy density goes like $1/(r^2+b^2)$ and the resulting circular velocity profile of tests particles in it is in good agreement with the observed one in spiral galaxies. In this work we give a connection between both models, supposing that the cosmological scalar field fluctuations are the halos of the galaxies and discuss some of the physical consequences of the model.

Keywords: Cosmology, Galaxy Formation, Dark Matter

1. Introduction

Doubtless we are living very exiting times in cosmology. At this end of Millennium many new crucial observations have been carry out giving the human beings a new vision of the Universe. Among others, observations of the luminosity–redshift relation of Ia Supernovae suggest that distant galaxies are moving slower than predicted by Hubble's law, implying an accelerated expansion of the Universe[1, 2]. These observations open the possibility to the existence of an energy component in the Universe with a negative equation of state, $\omega < 0$, being $p = \omega\rho$, called dark energy. Current observations of CMBR anisotropy by BOOMERANG [3] and MAXIMA[4], could implies a flat, homogenous and isotropic Universe. The counting of the distribution of cluster and supercluster of galaxies gives us a new vision of the large scale structure of the universe [5]. The most successful cosmological model until now seems to be the Λ Cold Dark Matter (ΛCDM). This model consists of cold dark matter $\sim 25\%$, whose nature is unknown, and $\sim 75\%$ of cosmological constant Λ.

Nevertheless, in addition to the old problems of the cosmological constant, the Λ CDM model over predicts subgalactic structure and singular cores for the halos of galaxies[6]. Some problems of the cosmological constant paradigm can be ameliorated by Quintessence: a fluctuating, inhomogeneous scalar field (Q) rolling down a scalar potential $V(Q)$. However, it has not been agreement about which scalar potential $V(Q)$ is the correct one. It is assumed that flat models with $\Omega_M = 0.33 \pm 0.05$ and $\omega_Q = -0.65 \pm 0.07$ are the most consistent with all observations[7].

The matter component Ω_M of the Universe decomposes itself in baryons, neutrinos, etc. and dark matter. Observations indicate that stars and dust (baryons) represent something like 0.3% of the whole matter of the Universe. The new measurements of the neutrino mass indicate that neutrinos contribute with a same quantity like matter. On other words, say $\Omega_M = \Omega_m + \Omega_{DM} = \Omega_b + \Omega_\nu + \cdots + \Omega_{DM} \sim 0.05 + \Omega_{DM}$, where Ω_{DM} represents the dark matter part of the matter contributions which has a value of $\Omega_{DM} \sim 0.25$. This value of the amount of baryonic matter is in concordance with the limits imposed by nucleosynthesis (see for example [8]). But we do not know the nature of the dark matter component Ω_{DM}.

The flat profile of the rotational curves is maybe the main feature observed in many galaxies. There are some particles with nice features in super-symmetric theories which could be candidates to be the dark matter, they are called WIMP'S (Weak Interacting Massive Particles). However, since these candidates behave just like standard CDM, they can not explain the observed scarcity of dwarf galaxies and the smoothness of the galactic-core matter densities [11]. This is the reason why we need to look for alternative candidates that can explain both the structure formation at cosmological level, the observed amount of dwarf galaxies, and the dark matter density profile in the core of galaxies.

2. Scalar field as cosmological dark matter

We have proposed a scalar field model for dark matter in galaxies [12]. This model has created a great expectation for solving the problem of the nature of dark matter [13, 14]. The scalar field not only gives the correct energy density for the required matter in galaxies to predict the rotation curves of stars, but it is obtained the correct distribution of dark matter in galaxies as well. It also has the advantage to be a particle predicted by fundamental theories like superstrings, Kaluza–Klein, etc. At the galactic level, attention has been put on the quadratic potential Φ^2, because of the well known fact that it behaves as pressureless matter due to its oscillations around the minimum of the potential[15], implying that $\omega_\Phi \simeq 0$, for $< p_\Phi >= \omega_\Phi < \rho_\Phi >$. Following an analogous procedure to the one used in particle physics, we may write a phenomenological Lagrangian with all the terms we need in order to reproduce the observed Universe. In particular, for modelling the dark matter of the Universe we use a minimally coupled real scalar field Φ with a self–interaction potential of the form ($\kappa_0 = 8\pi G$, we use natural units such that $\hbar = c = 1$)[16, 17]

$$\begin{aligned} V(\Phi) &= V_o\left[\cosh\left(\lambda\sqrt{\kappa_o}\Phi\right) - 1\right] \qquad (1) \\ &= \begin{cases} \frac{1}{2}m_\Phi^2\Phi^2 + \frac{1}{24}\lambda^2 m_\Phi^2\kappa_o\Phi^4 & |\lambda\sqrt{\kappa_o}\Phi| \ll 1 \\ (V_o/2)\exp\left(\lambda\sqrt{\kappa_o}\Phi\right) & |\lambda\sqrt{\kappa_o}\Phi| \gg 1 \end{cases} . \end{aligned}$$

The mass of the scalar field Φ is defined as $m_\Phi^2 = V''|_{\Phi=0} = \lambda^2\kappa_o V_o$. In this case, we have a massive scalar field. The components of the Universe are baryons, radiation, three species of light neutrinos, etc., and two minimally coupled and homogenous scalar fields Φ and Ψ, which represent the dark matter and the dark energy, respectively.

The scalar field has been proposed as a viable candidate, since it mimics standard CDM above galactic scales very well, reproducing most of the features of the standard (ΛCDM) model [17, 18, 13]. However, at galactic scales, the scalar field model presents some advantages over the standard ΛCDM model. For example, it can explain the observed scarcity of dwarf galaxies since it produces a sharp cut–off in the Mass Power Spectrum. Also, its self-interaction can, in principle, explain the smoothness of the energy density profile in the core of galaxies [17, 19].

At the cosmological scale, it is found that the mass of the boson is not the only parameter that determines the power spectrum. The self-interaction of the scalar field is also important. The free parameters of the scalar potential, V_0 and the scalar field mass $m_\Phi = \lambda\sqrt{V_0\kappa_0}$, can be fitted by cosmological observations. Doing this one finds that [17]

$$\begin{aligned} \lambda &\simeq 20.28\,, \qquad (2) \\ V_0 &\simeq (3\times 10^{-27}\, m_{Pl})^4\,, \qquad (3) \\ m_\Phi &\simeq 9.1\times 10^{-52}\, m_{Pl} = 1.1\times 10^{-23} eV\,, \qquad (4) \end{aligned}$$

with $m_{Pl} \equiv G^{-1/2}$ the Planck mass.

3. Scalar field dark matter in galaxies

The formation of galaxies through gravitational collapse of dark matter is not an easy problem to understand. A good model for galaxy formation has to take into account all the observed features of real galaxies. There are some ideas in this respect when dealing with a scalar field. Our main aim in this section is to present a plausible scenario for galaxy formation under the scalar field dark matter (SFDM) hypothesis. Through a gravitational cooling process [23, 24, 25], a cosmological fluctuation of the scalar field collapses to form a compact oscillaton by ejecting part of the field. The key idea consists precisely in assuming that such final object could distribute as galactic dark matter does. The final configuration then should consist of a central object (a core), i.e. an oscillaton, surrounded by a diffuse cloud of scalar field, both formed at the same time due to the same collapse process. The idea follows the standard idea of galaxy formation, namely that scalar field (dark matter) fluctuations are the responsible for the origin of the galaxies. In the case of the scalar field potential $\cosh(\lambda\Phi)$, we have used in [17] scalar field fluctuations of the form $\cosh(\lambda\Phi) \to \cosh(\lambda(\Phi + \delta\Phi)) \sim \exp(\lambda\delta\Phi) = \exp(\alpha\phi)$ for the regions where the scalar field fluctuation dominates $\delta\Phi > \Phi$. Of course, as in the standard theory of galaxy formation, the dark matter fluctuations are of different size in different regions of the Universe, for different galaxies. Therefore, at galactic level, we have a scalar field potential which depends on the local variable α. Thus, the exponential potential approximates the cosh potential in some regimes of the scalar field and we could develop some interesting aspects of a scalar dark matter halo in galaxies and although all this work is fully relativistic, it was done assuming staticity in galaxies [12, 26].

The question we are facing now is whether there is a dynamical mechanism that could provide a realistic scenario of galaxy formation using the scalar field dark matter hypothesis. First of all, a complete evolution of galactic and under galactic fluctuations belong to the non–linear regime of perturbations. The right answer would be provided by numerical evolutions of Einstein's equations in order to see the galaxy formation from the cosmological context. Fortunately, a partial answer is given in numerical research on Einstein's equations developed since 1990. In particular, the collapse of a scalar field has been studied deeply in [23, 24, 25] and it was found that there are final equilibrium and stable configurations for collapsed scalar field particles: boson stars (when the scalar field is complex) and oscillatons (when the scalar field is real and time-dependent), both of them being formed through a process called gravitational cooling[24]. Let us draw a possible physical picture of a galaxy collapse with dark matter being a scalar field endowed with the potential (1).

4. Scalar field collapse

In a realistic model the metric and fields should depend on space and time, thus a complete study would involve numerical calculations within and beyond General Relativity. One alternative is to study the behavior of the galaxy numerically with all the hypotheses stated above. In this section we will adopt this alternative, i.e. we will perform numerical simulations and in the next section we will perform semi–analytical calculations.

If a galaxy is an oscillaton, i.e. an oscillating soliton object, it must correspond to coherent scalar oscillations around the minimum of the scalar potential (1). For the scalar field collapse, the critical value for the mass of an oscillaton (the maximum mass for which a stable configuration exists) will depend on the mass of the boson. Roughly speaking, if we take $m_\Phi = 1.1 \times 10^{-23} eV$, and use the formula for the critical mass of the oscillaton corresponding to a scalar field with a Φ^2 potential (i.e. just a mass term), we expect the critical mass to be [23, 24]

$$M_{crit} \sim 0.6 \frac{m_{Pl}^2}{m_\Phi} \sim 10^{12} M_\odot \,. \tag{5}$$

This is a surprising result: the critical mass of the model shown in [17] is of the order of magnitude of the dark matter content of a standard galactic halo.

In order to study this situation for the case of a potential of the form (1), we present a numerical simulation of Einstein's equations in which the energy momentum tensor is that of a real scalar field. The scenario of galactic formation we assume is as follows: a sea of scalar field particles fills the Universe and forms localized primordial fluctuations that could collapse to form stable objects, which we will interpret as the dark matter halos of galaxies.

The numerical simulations suggest that the critical mass for the case considered here, using the scalar potential (1), is approximately [25, 28]

$$M_{crit} \simeq 0.1 \frac{m_{Pl}^2}{\sqrt{\kappa_0 V_0}} = 2.5 \times 10^{13} M_\odot \,. \tag{6}$$

The results of the numerical simulations are as follows. Essentially, we have found three different types of behavior for the scalar field collapse. In the first case, a generic feature is that scalar field distributions with an initial mass slightly larger than the critical mass collapse very violently and form a black hole. In the second type of behavior, fluctuations with an initial mass significantly smaller than the critical mass can not form stable oscillatons: the scalar field is completely ejected out as the system evolves [28]. The third behavior corresponds to a case where a fraction of the initial density is spread out, leaving an oscillating object that appears to be stable. This situation happens in a narrow window of initial conditions, between $0.05 - 1 \times M_{crit}$ [28].

From the cosmological point of view, the narrow window of initial conditions means that not all fluctuations will collapse into stable objects. Moreover, the collapsed objects will have masses of the same order of magnitude $M_{final} \sim 10^{12} M_{\odot}$, as it seems to be precisely the case for galaxies.

Summarizing, from the results of the numerical simulations of the collapse of the real scalar field with a cosh potential we find many similarities with the structure of the halos of galaxies. The scalar field density profile is not singular at the center. This fact, and the values of the final masses obtained using the cosmological values (3) and (4) for the parameters of the self-interaction potential, could correspond to objects like realistic galaxies. Moreover, it is in agreement with the observational constraints related to the phenomenological maximum galactic mass pointed out by Salucci and Burkert [29]. Therefore, we expect that fluctuations of this scalar field, due to Jeans instabilities, will in general collapse to form objects of the order of the mass of the halo of a typical galaxy.

We have shown before [17] that the SFDM model could be a good model for the universe at cosmological level, here we see that the scalar field could also be a good candidate for the dark matter content of individual galaxies (as suggested in [12, 26, 30]).

5. The galactic model with scalar field dark matter

5.1. A long exposition photograph of a galaxy

Summarizing, we have considered two working hypothesis up to now. First, we identify the formation of a central compact object and a halo with the gravitational collapse of a scalar field. The compact object could be a) an oscillaton (since we are dealing only with a real scalar field) or b) a Bose condensate. Second, we identify the ejected scalar field with the halo of this galaxy.

In this section we will adopt the second alternative, i.e. we will build a toy model for case a) stated above in purely geometrical terms and considering only the final stage of the collapse. To start with, we support our toy model on the numerical results studied in the previous section (see also [23, 24, 25]). First, since the time–dependence of the metric in an oscillaton is quite small [23], we suppose then that the center of this toy galaxy is an oscillaton which oscillates coherently but considering a *static* metric. This is an approximation because neither the galactic nuclei nor the oscillaton are expected to be static. However, for the purposes of this analytic work, we suppose that the dynamics of the oscillaton can be frozen in time in a way we explain below. Second, we do not expect the scalar halo to possess the same properties than the collapsed oscillaton; in some sense, they must be different. Thus, we will consider the scalar halo as another scalar field. Third, baryonic matter is considered to lie in the galaxy. Thus, we suppose that baryonic matter, which is part of the central object, (and only to the central object) will contribute to the curvature of the space–time of the galaxy.

This matter can be modelled as dust very well. As in previous works [12, 26], we let the luminous matter around the galaxy as test particles, i.e. they do not essentially contribute to the curvature of the space-time. Thus, we will take the baryonic matter surrounding the halo of the galaxy as test particles.

5.2. The analytical solution

Then, starting from a spherically symmetric space–time, using the *harmonic maps ansatz* [31] we were able to find a solution of the system. The exact solution of the averaged Einstein's field equations is

$$ds^2 = -B_0(r^2+b^2)^{v_a^2}\left(1-\frac{2M}{r}\right)dt^2 + \frac{A_0}{(1-\frac{2M}{r})}dr^2 + r^2 d\Omega^2 \tag{7}$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and M is a constant with the interpretation discussed below. This metric is singular at $r = 0$, but it has an event horizon at $r = 2M$. This metric does not represent a black hole because it is not asymptotically flat. Nevertheless, for regions where $r \ll b$ but $r > 2M$ the metric behaves like a Schwarzschild black hole. Inside of the horizon the pressure of the perfect fluid is not zero anymore, thus our toy model is valid only in regions outside of the horizon, where it could be an approximation of the galaxy. Metric (7) in not asymptotically flat, but it has a natural cut off when the dark matter density equals the intergalactic density as mentioned in [26].

5.3. Physical features of the model

In order to understand the other parameters of the metric, let us proceed in the following way. It is believed that in a standard galaxy, the central object has a mass of $M \sim 2-3\times 10^6 M_\odot$ ~some *a.u.* Far away from the center of the galaxy, say from $1pc$ up, the term $2M/r \ll 1$. In this limit metric (7) becomes

$$ds^2 = -B_0(r^2+b^2)^{v_a^2}dt^2 + A_0 dr^2 + r^2 d\Omega^2. \tag{8}$$

This space–time is very similar to metric (18) of reference [26], but now with the potential

$$U(\psi(r)) = \frac{2v_a^2}{\kappa_0\left(1-v_a^2\right)}\frac{1}{\left(r^2+b^2\right)} \tag{9}$$

being both solution the same in the limit $r \to \infty$. This implies that $A_0 = 1 - v_a^4$, recovering in this way the asymptotic results shown in [26]. Parameter b is related to parameter b of metric (21) in reference [12], where it acts as a gauge parameter. Of course this metric is only valid far away from the center of the galaxy. With parameter b it is now possible to fit quite well the rotation curves of spiral galaxies. Therefore metric (7) could not only represent the exterior part of the galaxy, but it could be a good approximation for the core part of it as well. Let us see this point.

The rotation curves v^{rot} seen by an observer at infinity for a spherically symmetric metric are given by $v^{rot} = \sqrt{rg_{tt,r}/(2g_{tt})}$ [26]. For metric (7) such result reads

$$v^{rot}(r) = \sqrt{\frac{v_a^2(r-2M)r^2 + M(r^2+b^2)}{(r-2M)(r^2+b^2)}}$$

formula that allows one to fit observational curves. Now let us explore an ADM–like concept of mass associated to our scenario. This mass can be calculated using the standard form of the metric

$$ds^2 = -e^{2\delta}dt^2 + \frac{dr^2}{(1-\frac{2m}{r})} + (1-\alpha)r^2 d\Omega^2 \tag{10}$$

where $m = m(r)$ is interpreted as the mass function and $\delta = \delta(r)$ as the gravitational potential. This form of the metric is convenient because in this coordinates $m_{,r} = 4\pi r^2 \rho_T$, where ρ_T is the total density of the object. This interpretation is correct in regions where the space–time is almost flat, i.e. far away from the horizon $r = 2M$. Close to the horizon or inside of it, function m is a quantity that should be similar to the mass of the object, but it is not since it contains the contribution of all the components together; in this region, where the curvature of the space-time is huge, the volume element is different from $4\pi r^2 dr$. Furthermore, inside of the horizon we are not able to know the real physics of the object. On the other side, far away from the center of the toy galaxy, this function can be interpreted as the mass of an infinitesimal shell at radius r. Anyway we will call function m the mass function everywhere. Thus, the ADM–like mass is obtained at infinity by $M_{ADM} = \lim_{r\to\infty} m$. We perform the coordinate transformation $\sqrt{A_0}r \to r$, $\sqrt{A_0}b \to b$ in order to compare metrics (7) and (10). We obtain

$$ds^2 = -\frac{B_0}{A_0^{v_a^2}}\left(r^2+b^2\right)^{v_a^2}\left(1-\frac{2M\sqrt{A_0}}{r}\right)dt^2+\frac{dr^2}{(1-2M\sqrt{A_0}/r)}+\frac{r^2}{A_0}d\Omega^2 \tag{11}$$

with $A_0 = 1 - v_a^4$. Thus, $M_{ADM} = \sqrt{1-v_a^4}M$. Probably an observer at infinity would see the mass M_{ADM} at the center of the galaxy. Observe that for $r >> b$ it follows the line element

$$ds^2 = -B_0 r^l dt^2 + A_0 dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \tag{12}$$

where $l = 2\,(v^\varphi)^2$. This result is not surprising. Remember that the Newtonian potential ψ is defined as $g_{00} = -exp(2\psi) = -1-2\psi-\cdots$. On the other side, the observed rotational curve profile in the dark matter dominated region is such that the rotational velocity v^φ of the stars is constant, the force is then given by $F = -(v^\varphi)^2/r$, which respective Newtonian potential is $\psi = (v^\varphi)^2\ln(r)$. If we now read the Newtonian potential from the metric (11), we just obtain the same result. Metric (11) is then the metric of the general relativistic version of a matter distribution, which test particles move in constant rotational curves. Observe that

$$V = -\frac{l}{\kappa_0(2-l)}\frac{1}{r^2} \tag{13}$$

Thus, we recover the very important result, namely the scalar potential goes always as $1/r^2$ for a spherically symmetric metric with the *flat curve condition.* It is remarkable that this behavior of the stress tensor coincides with the expected behavior of the energy density of the dark matter in a galaxy. The effective density depends on the velocities of the stars in the galaxy, $\rho = (v^\varphi)^4/(1-(v^\varphi)^4)\times 1/(\kappa_0 r^2)$ which for the typical velocities in a galaxy is $\rho \sim 10^{-12}\times 1/(\kappa_0 r^2) \sim 1/310^{-12}H_0^{-2}\rho_{crit}/r^2$, while the effective radial pressure is $|P| = (v^\varphi)^2((v^\varphi)^2+2)/(1-(v^\varphi)^2)\times 1/(\kappa_0 r^2) \sim 10^{-6}\times 1/(\kappa_0 r^2)$, *i.e.*, six orders of magnitude greater than the scalar field density. This is the reason why it is not possible to understand a galaxy with Newtonian dynamics. Newton theory is the limit of the Einstein theory for weak fields, small velocities but also for small pressures (in comparison with densities). A galaxy fulfills the first two conditions, but it has pressures six orders of magnitude bigger than the dark matter density, which is the dominating density in a galaxy. This effective pressure is the responsible for the behavior of the flat rotation curves in the dark matter dominated part of the galaxies.

Metric (12) is not asymptotically flat, it could not be so. An asymptotically flat metric behaves necessarily like a Newtonian potential provoking that the velocity profile somewhere decays, which is not the observed case in galaxies. Observe also that the matter density around a galaxy is smaller than the critical density, say $\rho_{around} \sim 0.06\rho_{crit}$, then $r_{crit} \approx 14Kpc$, which correspond to a typical size for galaxies. At infinity, the observer will only measure M_{ADM}, i.e. it will see a Black-Hole-like metric at the center of the galaxy which horizon lies at $r = 2M_{ADM}$.

In Figure 2 of [30] the fit of the curves is done using the observed rotation curves of some dwarf galaxies, whose dark matter contribution is extremely dominating and therefore are considered as the *test of fire* for a dark matter model in galaxies. In general, for disc galaxies, the fit of the rotation curves using this metric is analogous as in reference [12]. It seems then that metric (7) is a good approximation for some late stadium of the space-time of a spiral galaxy; it is a good approximation of a "long exposition photograph" of a galaxy

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