RELATIVISTIC DARK MATTER IN SPIRAL GALAXIES

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It is presented a discussion about the meaning of newtonian dark matter (like dust) in galaxies. On the basis of a geometric approach it is shown that the dark matter could be not only relativisic, but even exotic like quintessence, provided that the state equation $p = \omega d$ of the dark matter which is assumed to be a perfect fluid satisfies the relation $\omega \in (-1, -1/3) \cup (0, 1)$ in the dark matter dominated region of disc galaxies.

Since the formulation of a possible quintessence field in 1998 that should fill the Universe in a great proportion [1], it has been possible to go deeper in the study of such kind of exotic components sorrounding ourselves, say the quintessence (Q) or the Cosmological Constant (Λ) corresponding to a background energy in the cosmos. The success (need?) of such components started to be relevant since the observation of the luminosity of Supernovae type Ia located at redshift near to one [2], which required an accelerated expansion of the Universe, and even if such observations were not correct, the recent observations of the angular power spectrum of the CMBR requires the Universe to be geometrically flat, in which case such kind of vacuum energy is required, one kind of energy that can not colapse to form structure, but that is there as the background energy, the so called dark energy [3].

It is well known that the vacuum energy for instance, satisfies a "state equation" $p = -d$, being p the analog of the pressure in a fluid and d its energy density. Such relation is said to correspond to the presence of a cosmological constant. The quintessence is nothing but a relaxation over the caracteristics of such vacuum energy, requiring the state equation $p = \omega d$ with $-1 < \omega < -1/3$, where the fluid is made of a scalar field (ϕ) with a potential of self interaction ($V(\phi)$) containing as a particular case the Λ case with ϕ and $V(\phi)$ being constants, corresponding to $\omega = -1$. This caracteristic by itself of the Q and Λ models ($|p| \sim |d|$) makes the Universe to be a relativistic system since it is gravitationally dominated by the dark energy.

Therefore the dynamics of the cosmos can be explained with a general relativistic approach, but what happens at galactic scale?

There are some standarized relativistic phenomea in galaxies, like the emission of X-ray jets in active nuclei that involve the study of the geometry of the space time in the innermost region of a disc galaxy. Despite of such phenomena, which in fact do not concern the observations of rotation curves in galaxies, i. e. the problem of dark matter, we will consider that the dark matter is related to a special structure of the space time background of a galaxy, which provides an explanation of the flatness of rotation curves at large radii, where the dark component is the main contributor to the kinematics of any test particle, like those atoms of neutral hydrogen from which the observations of rotation curves are extracted. Moreover, for the shake of simplicty it will be assumed that the dark halo is spherically symetric.

In this way, the general static line element of a spherically symmetric space time is given by the metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \tag{1}$$

from which it is possible to write down the geodesic equation for a test particle travelling there:

$$\dot{r}^2 - \frac{1}{A}\left[\frac{E^2}{B} - \left(L_\theta^2 + \frac{L_\varphi^2}{\sin^2\theta}\right)\frac{1}{r^2} - 1\right] = 0 \tag{2}$$

where E and $L^2 = L_\theta^2 + \frac{L_\varphi^2}{\sin^2\theta}$ are two conserved quantities, the energy per unit of mass and the total angular momentum per unit of mass of the test particle; a dot stands for derivative with respect to the propper time of the test particle, which is assumed to freely fall. If one is looking for circular and stable orbits of test particles the following conditions should be fulfilled: a) $\dot{r} = 0$, (circular trajectories), b)$\frac{\partial V(r)}{\partial r} = 0$, (extreme) and c)$\frac{\partial^2 V(r)}{\partial r^2}|_{extr} > 0$, (and stable), being $V(r) = -\frac{1}{A}\left[\frac{E^2}{B} - (L_\theta^2 + \frac{L_\varphi^2}{\sin^2\theta})\frac{1}{r^2} - 1\right]$ (see ref 4).

After impossing such conditions it is easy to find an expression for the tangential velocity of test particles travelling in such well behavied orbits, restricting the metric function B in the following way

$$v^\varphi = \sqrt{\frac{r^2}{B}\left[\left(\frac{\dot{\theta}}{\dot{t}}\right)^2 + \sin^2\theta\left(\frac{\dot{\varphi}}{\dot{t}}\right)^2\right]} = \sqrt{\frac{rB_{,r}}{2B}} \tag{3}$$

providing a way to prescribe the test function for a model of the velocity of the particles. In this point I will only consider that the rotation curve is flat, i. e. that the velocities of test particles is radii independent, which is valid in the outer parts of many disk galaxies; in any case, such condition could be relaxed. Thus if the velocity is independent of the radii an expression for the metric function B is available:

$$B(r) = B_0 r^m \tag{4}$$

being $m = 2(v^{(\varphi)})^2$, which is called the *flat curve condition* [4].

Now let us consider as in the quintessence case at cosmological level, that the dark matter (in this case) can be represented by a perfect fluid, whose stress energy tensor is $T_{\mu\nu} = (d+p)\, u_\mu\, u_\nu + g_{\mu\nu}\, p$ with the four velocity given by $u^\mu = (u^0, 0, 0, 0)$ with $u^0 = \frac{1}{\sqrt{B_0}} r^{-(v^\varphi)^2}$.In this way, we get that the non zero components of the static spherically symmetric perfect fluid are:

$$
\begin{aligned}
T_{tt} &= d\, B_0\, r^{2\,(v^\varphi)^2}, \\
T_{rr} &= p\, A(r), \\
T_{\theta\theta} &= pr^2 && (5) \\
T_{\varphi\varphi} &= pr^2 \sin^2\theta. && (6)
\end{aligned}
$$

Substituting these expressions into the Einstein equations $G_{\mu\nu} = \kappa_0 T_{\mu\nu}$ for the line element (1), provided the flat curve condition (4), one obtains:

$$
\begin{aligned}
\frac{rA_{,r} + A(A-1)}{A^2 r^2} &= \kappa_0 d \\
\frac{m+1-A}{r^2 A} &= \kappa_0 p \\
-\frac{1}{4}\frac{(2+m)rA_{,r} - m^2 A}{A^2 r^2} &= \kappa_0 p
\end{aligned}
$$

for which it is not difficult to find the solution, which is given by

$$
\begin{aligned}
A(r) &= \frac{b/2}{(A_0 r^{-b/a} - 1)} && (7) \\
p(r) &= \frac{2}{\kappa_0 b r^2}[(2(v^\varphi)^2 + 1)A_0 r^{-b/a} - (v^\varphi)^4] \\
d(r) &= \frac{2}{\kappa_0 b r^2}[(\frac{b}{2} + 1) + (\frac{b}{a} - 1)A_0 r^{-b/a}]
\end{aligned}
$$

with $b = 2((v^\varphi)^4 - 2(v^\varphi)^2 - 1)$ y $a = (v^\varphi)^2 + 1$. Then two cases for the state equation $p = \omega d$ deppending on the value of the constant of integration A_0 arise. For instance, if $A_0 = 0$:

$$
\omega = \frac{(v^\varphi)^2}{2 - (v^\varphi)^2} \tag{8}
$$

with the physical restriction $0 < (v^\varphi)^2 < 1$. It is possible to recover the newtonian limit in the following sense: the dark matter could be dust ($\omega \simeq 0$), and in the other hand the physics is the same of Newton's laws, i. e., just recall that the Newtonian potential ψ is defined as $g_{00} = -\exp(2\psi) = -1 - 2\psi - \cdots$. On the other side, the observed rotational curve profile in the dark matter dominated region is such that the rotational velocity v^φ of the stars is constant, the force is then given by $F = -(v^\varphi)^2/r$, which respective Newtonian potential is $\psi = (v^\varphi)^2 \ln(r)$. If we now read the Newtonian potential from the metric (1,4), we just obtain the same result (see [4]). Nevertheless, it is also physically meaningful the case of stiff matter which is an extremaly relativistic case, and that is now evidenced by this analysis.

In the other case, $A_0 > 0$ (the situation concerning $A_0 < 0$ is not considered here since it does not guarantee the positiveness of $A(r)$) the following expression is obtained

$$\omega = \frac{(2(v^{\varphi})^2 + 1)A_0 r^{-b/a} - (v^{\varphi})^4}{(\frac{b}{2} + 1) + (\frac{b}{a} - 1)A_0 r^{-b/a}} \tag{9}$$

with the restriction $0 < (v^{\varphi})^2 < 1$ once again. Note that a variation of the parameter A_0 is equivalent to a variation of the radius r, altough ω depends rigorously on a combination of both. If the relation $A_0 r^{-b/a} \gg (v^{\varphi})^2$ holds (i.e. for large radii) it is easy to show that $-1 < \omega < -1/3$, which corresponds to the same "state equation" than quintessence. At this stage, the dark fluid is not only possibly a relativistic one, since $|p| \sim |d|$, but also exotic, like quintessence provided that $\omega < 0$.

The usual approach in the study of galaxtic dark matter considering that the dark matter is dust, the cold dark matter model, is now falling down. There are studies based on n-body simulations that show that there should be a self-interacting dark matter (very different from dust) in order to avoid the formation of structure smaller than dwarf galaxies [5]. In a recent study the scalar field is presented as a candidate to be the galactic dark matter, and it is found that the components of the stress energy tensor of such field provides a relation $p/d \sim 10^6$ which evidently is not dust [4,6]. Based on this last hypothesis, but at cosmological scale, it is found that the scalar dark matter should have a non zero cross section of dispersion in order to form the observed structure, putting forward the idea of a self interacting dark matter [7,8]. Thus, as a main conclusion it should be stated that it was only presented the geometric approach of a very well know phenomenon.

Acknowledgments

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References

1. R.R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80** (1998) 1582
2. S. Perlmutter et al., ApJ **483** (1997) 565
3. A. Balbi, et al., ApJ Lett., in press. Preprint astro-ph/0005124
4. T. Matos, F. S. Guzmán and D. Núñez, Phys. Rev. **D 62** (2000) 061301
5. C. Firmani, E. D'Onghia, V. Avila-Reese, G. Chincarini, X. Hernandez, MNRAS **315** (2000) L29
6. F. S. Guzmán and T. Matos, Class. Quantum Grav. **17** (2000) L9
7. T. Matos and L. A. Ureña López, Class. Quantum Grav. **17** (2000) L75
8. T. Matos and L. A. Ureña López. To be published. Preprint astro-ph/0010226