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COMPLEX SCALAR FIELD DARK MATTER

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In this proceeding we show that a model in wich the dark matter of the Universe is modeled by a complex scalar field Φ endowed with a scalar potential $U(|\Phi|^2) = U_0\left[\cosh\left(\lambda\sqrt{\kappa_0}\sqrt{\Phi\Phi^*}\right) - 1\right]$ does not have exactly the same evolution equations than that of a real scalar field with the potential $U(\Phi) = U_0\left[\cosh\left(\lambda\sqrt{\kappa_0}\Phi\right) - 1\right]$. We find that the $U(1)$ symmetry of the potential implies that the scalar field should be of the form $\Phi = \sigma(t)e^{i\theta}$ while the scalar perturbations $\Phi \to \Phi + \phi$ are determined by the evolution of $\varphi_R = Re(e^{-i\theta}\phi)$. The cosmological solution is inflationary with a equation of state $< \omega_\Phi >= -1/3$ and the scalar density contrast is $\delta_\Phi = (2/3)\delta_{CDM}$. This scenario differs from that of a real scalar field.

1 Introduction

Scalar fields has become very important in Cosmology in the recent years. In particular, one can find the proposals of presence of complex scalar fields in some stages of the evolution of the Universe, like during inflation[1], or at late times forming boson stars[2]. These boson stars has been proposed to be good candidates for the center of galaxies[3]. On the other hand, it was shown that a real scalar field could be a reliable model for the dark matter in the Universe[4]. In this proceeding, we analyze the evolution equations for a Universe in wich the dark matter is a complex scalar field endowed with a potential

$$U(|\Phi|^2) = U_0\left[\cosh\left(\lambda\sqrt{\kappa_0}\sqrt{\Phi\Phi^*}\right) - 1\right], \tag{1}$$

with $m^2 = U_0\kappa_0\lambda^2$ being the mass of the particle. This particular potential has a $U(1)$ global symmetry and it has been anlyzed in both the framework of boson stars[3] and in the case of real scalar field dark matter[4,5].

2 The cosmological model

We will consider a Universe with ordinary matter, a complex scalar field Φ and a cosmological constant Λ. The action is

$$S = \int d^4x \left\{\sqrt{-g}\left[\frac{R}{\kappa_0} - \frac{1}{2}g^{\mu\nu}\left(\partial_\mu\Phi^*\partial_\nu\Phi + \partial_\mu\Phi\partial_\nu\Phi^*\right) - U(|\Phi|^2) - \Lambda\right] - \mathcal{L}_M\right\} \tag{2}$$

where $\kappa_0 = 8\pi G$. Notice that the action (2) has also a $U(1)$ global symmetry. The potential (1) satisfies[1]

$$\frac{\partial U}{\partial \Phi} = \Phi^* \frac{dU}{d(|\Phi|^2)},$$

$$
\begin{aligned}
\frac{\partial^2 U}{\partial \Phi^2} &= \Phi^{*2} \frac{d^2 U}{d(|\Phi|^2)^2}, \\
\frac{\partial^2 U}{\partial \Phi \partial \Phi^*} &= \frac{dU}{d(|\Phi|^2)} + |\Phi|^2 \frac{d^2 U}{d(|\Phi|^2)^2}
\end{aligned} \tag{3}
$$

and the respective complex conjugate expressions.

Considering a flat, homogeneus and isotropic Universe, we take the Friedmann-Robertson-Walker (FRW) metric $ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2)$. The evolution of the Universe is completely determined by the Friedmann equation

$$
H^2 \equiv \left(\frac{\dot{a}}{a}\right) = \frac{\kappa_0}{3}\left(\rho + \frac{1}{2}\dot{\Phi}\dot{\Phi}^* + U(|\Phi|^2)\right), \tag{4}
$$

ρ being the energy density for radiation, baryons, neutrinos, a cosmological constant, etc., and the Klein Gordon (KG) equation

$$
\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU}{d|\Phi|^2}\Phi = 0, \tag{5}
$$

and its complex conjugate. It follows that the set of equations is invariant under the transformation $\Phi \leftrightarrow \Phi^*$, a direct consequence of the $U(1)$ symmetry of potential (2). Thus, the complex scalar field should be of the form

$$
\Phi = \sigma(t) e^{i\theta} \tag{6}
$$

θ being an arbitrary constant phase. This solution is the asymptotic one for the complex escalar field[1]. In particular, for the potential (1) we find that

$$
\begin{aligned}
\sigma \frac{\partial U}{\partial(|\Phi|^2)}(|\Phi| = \sigma) &= \frac{1}{2} U_0 \sqrt{\kappa_0}\, \lambda \sinh(\lambda\sqrt{\kappa_0}\,\sigma) = \frac{1}{2}\frac{dU(\sigma)}{d\sigma}, \\
\sigma^2 \frac{\partial^2 U}{\partial(|\Phi|^2)^2}(|\Phi| = \sigma) &= \frac{m^2}{4}\left[\cosh(\lambda\sqrt{\kappa_0}\,\sigma) - \frac{\sinh(\lambda\sqrt{\kappa_0}\,\sigma)}{\lambda\sqrt{\kappa_0}\,\sigma}\right] \\
&= \frac{1}{4}\frac{d^2U(\sigma)}{d\sigma^2} - \frac{1}{4\sigma}\frac{dU(\sigma)}{d\sigma},
\end{aligned} \tag{7}
$$

where the scalar potential is $U(\sigma) = U_0\left[\cosh(\lambda\sqrt{\kappa_0}\,\sigma) - 1\right]$. Therefore, the evolution equations can be written as

$$
\begin{aligned}
H^2 &= \frac{\kappa_0}{3}\left(\rho + \frac{1}{2}\dot{\sigma}^2 + U(\sigma)\right), \\
\ddot{\sigma} &= -3H\dot{\sigma} - \frac{1}{2}\frac{dU(\sigma)}{d\sigma}.
\end{aligned} \tag{8}
$$

Notice that Eq. (8) is not the usual KG equation for a real scalar field. Following the real case[4], one can find the solution for the function σ once it oscillates around the minimum of the potential. The mean scalar equation of state is $< \omega_\Phi >= -1/3$, that is, it is inflationary[1]. This result differs from that one found for a real scalar

field[4]. Also, because of the $U(1$
given by

$J^\mu =$

but in this case (6) $J^\mu = 0$.

3 Lineal Perturbation Th

Working in the synchronus
$a^2\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$
$\Phi \to \Phi + \phi$ in the k-space, th

$\delta\rho_\Phi = \frac{1}{2a}$

$\delta p_\Phi = \frac{1}{2a}$

The perturbed KG equation

$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + k^2\phi$

and its respective complex c
Eqs. (3), we can rewrite Eq.

$e^{-i\theta}\ddot{\phi} + 2\mathcal{H}e^{-i\theta}\dot{\phi} + k^2 e^{-}$

and its complex conjugate
$\varphi = \varphi_R + i\varphi_I$ with $\varphi_{R,I}$ be

and the equations for the

$\ddot{\varphi}_R + 2$

field[4]. Also, because of the $U(1)$ global symmetry in (2), there is a conserved current given by

$$J^{\mu} = \frac{i}{2}\sqrt{-g}\, g^{\mu\nu}\left[\Phi^*\partial_\nu\Phi - \Phi\partial_\nu\Phi^*\right] \tag{9}$$

but in this case (6) $J^{\mu} = 0$.

3 Lineal Perturbation Theory

Working in the synchronus gauge formalism, where the line element is $ds^2 = a^2\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$ where h_{ij} is the perturbation of the space. Taking $\Phi \to \Phi + \phi$ in the k-space, the perturbed scalar energy density and pressure are

$$\begin{aligned}
\delta\rho_\Phi &= \frac{1}{2a^2}\left(\dot{\Phi}\dot{\phi}^* + \dot{\Phi}^*\dot{\phi}\right) + \frac{\partial U}{\partial\Phi^*}\phi^* + \frac{\partial U}{\partial\Phi}\phi, \\
\delta p_\Phi &= \frac{1}{2a^2}\left(\dot{\Phi}\dot{\phi}^* + \dot{\Phi}^*\dot{\phi}\right) - \frac{\partial U}{\partial\Phi^*}\phi^* - \frac{\partial U}{\partial\Phi}\phi.
\end{aligned} \tag{10}$$

The perturbed KG equation is

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + k^2\phi + a^2\left(\frac{\partial^2 U}{\partial\Phi^{*2}}\phi^* + \frac{\partial^2 U}{\partial\Phi\partial\Phi^*}\phi\right) + \frac{1}{2}\dot{\Phi}\dot{h} = 0, \tag{11}$$

and its respective complex conjugate. Recalling the unperturbed result Eq. (6) and Eqs. (3), we can rewrite Eq. (11) as

$$\begin{aligned}
e^{-i\theta}\ddot{\phi} + 2\mathcal{H}e^{-i\theta}\dot{\phi} + k^2 e^{-i\theta}\phi + a^2\left[\sigma^2\frac{d^2U}{d(|\Phi|^2)^2}(|\Phi| = \sigma)\right]\left(e^{-i\theta}\phi + e^{i\theta}\phi^*\right) \\
+ a^2\left[\frac{dU}{d(|\Phi|^2)}(|\Phi| = \sigma)\right]e^{-i\theta}\phi + \frac{1}{2}\dot{\sigma}\dot{h} = 0,
\end{aligned}$$

and its complex conjugate as equation for ϕ^*. Considering Eqs. (7) and $e^{-i\theta}\phi = \varphi = \varphi_R + i\varphi_I$ with $\varphi_{R,I}$ being real functions, we find

$$\begin{aligned}
\delta\rho_\Phi &= \frac{1}{a^2}\dot{\sigma}\,\dot{\varphi}_R + \frac{dU}{d\sigma}\varphi_R, \\
\delta p_\Phi &= \frac{1}{a^2}\dot{\sigma}\,\dot{\varphi}_R - \frac{dU}{d\sigma}\varphi_R,
\end{aligned} \tag{12}$$

and the equations for the perturbations become

$$\ddot{\varphi}_R + 2\mathcal{H}\dot{\varphi}_R + k^2\varphi_R + \frac{a^2}{2}\frac{d^2U}{d\sigma^2}\varphi_R + \frac{1}{2}\dot{\sigma}\dot{h} = 0, \tag{13}$$

$$\ddot{\varphi}_I + 2\mathcal{H}\dot{\varphi}_I + k^2\varphi_I + \frac{a^2}{2\sigma}\frac{dU}{d\sigma} = 0. \tag{14}$$

The imaginary part φ_I always decays. Then, the cosmological evolution only depends on the evolution of the real functions σ and φ_R. We can manage the first of Eqs. (14) in a similar way as in the real scalar field case[4]. Then, we find that $\delta_\Phi = (\delta p_\Phi / \delta \rho_\Phi) = (2/3)\delta_{CDM}$, with δ_{CDM} being the density contrast for standard cold dark matter.

4 Conclusions

The $U(1)$ global symmetry of the potential (1) makes the complex scalar field be of the form $\Phi = \sigma(t)e^{i\theta}$, with $\sigma(t)$ a real function. In consequence, the scalar perturbations depends on the evolution of the quantity $\varphi_R = Re(e^{-i\theta}\phi)$, ϕ being the complex scalar field perturbation. Even if the evolution equations are only for real functions, these equations differs from those of the real scalar field case, that is, we do not recover the real scalar field cosmology by setting $\theta = 0$. In the real scalar field case, the important features appear once the scalar field oscillates around the minimum of the potential, and we can treat the complex case in a similar manner. However, for the latter, we find an inflationary solution ($< \omega_\Phi >= -1/3$) and that the scalar density contrast is not the cold dark matter one, $\delta_\Phi = (2/3)\delta_{CDM}$, as it happens in the real scalar field case. This seems to rule out a model of cosmological complex scalar field dark matter with potential (1). Further analysis will be published elsewhere.

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