ROTATING GROSS-PERRY-SORKIN MONOPOLE SOLUTION IN 5-DIMENSIONAL KALUZA-KLEIN THEORY

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We present a set of formulae that shows us how to construct a rotating solution from a static solution to the 5-dimensional Kaluza-Klein field equations. As an example, a rotating Gross-Perry-Sorkin solution is constructed.

1 Introduction

In the past few years, a number of stationary, axialsymmetric solutions to the 5-dimensional Kaluza-Klein theory have been constructed [1-3] using Neugebauer's potential formalism [4]. These solutions are written in terms of harmonic maps. By choosing appropriate values of these harmonic maps, the solutions represent monopoles, dipoles, quadrupoles etc. However, all the solutions presented in these references are static. In the next section, we present a set of formulae that shows us how to construct rotating solutions from static ones. We use as a seed metric, the Gross-Perry-Sorkin monopole solution [5], to obtain, as an example, its corresponding rotating metric. We discuss some of its properties.

2 Rotating Solutions

Neugebauer's potential formalism has been widely employed to generate stationary axial symmetric solutions in 5D gravity. The field equations $\hat{R}_{AB} = 0$ (where $\hat{R}_{AB}$ is the 5-dimensional Ricci tensor) written in terms of the potentials $\Psi^A = (f, \epsilon, \psi, \chi, \kappa)$ can be derived from the Lagrangian [4]

$$\mathcal{L} = \frac{\rho}{2f^2}[f_{,i} f^{,i} + (\epsilon_{,i} - \psi\chi_{,i})(\epsilon^{,i} - \psi\chi^{,i})] + \frac{\rho}{2f}\left(\kappa^2\psi_{,i}\psi^{,i} + \chi_{,i}\chi^{,i}\frac{1}{\kappa^2}\right) - \frac{2\alpha}{3\kappa^3}\kappa_{,i}\kappa^{,i} \quad (1)$$

where all quantities depend only on two variables $i = (\rho, \zeta)$. The invariance group of the Lagrangian (1) is $SL(3, R)$. As a result, the invariant transformations can be written as $g \longrightarrow C g_0 C^T$ where g and $C \in SL(3, R)$. In terms of this g, given in a suitable parametrization, the field equations can be cast into a non-linear σ-model form: $\left(\rho g_{,z}\, g^{-1}\right)_{,\bar{z}} + \left(\rho g_{,\bar{z}}\, g^{-1}\right)_{,z} = 0$. We use the invariance transformation to generate rotating solutions from static solutions g_0. As it is known, the 5D metric is given by $d\hat{s}^2 = I^{-1} ds_4^2 + I^2 (A_\alpha dx^\alpha + dx^5)^2$, where ds_4 is the 4-dimensional line element. In Ref. [2] several classes of solutions were found in terms of one harmonic map λ. We choose the class of solutions

$$ds_4^2 = (1+\lambda)^{1/2}\left[\left(1-\frac{2m}{r}+\frac{m^2\sin^2\theta}{r^2}\right)\left(\frac{dr^2}{1-\frac{2m}{r}}+r^2d\theta^2\right)\right.$$
$$\left.+\left(1-\frac{2m}{r}\right)r^2\sin^2\theta d\varphi^2\right]-\frac{dt^2}{(1+\lambda)^{1/2}}, \tag{2}$$

written in Boyer-Lindquist coordinates. After the transformation the metric reads

$$ds_4^2 = T^{1/2}(1+\lambda)^{1/2}\left[\left(1-\frac{2m}{r}+\frac{m^2\sin^2\theta}{r^2}\right)\left(\frac{dr^2}{1-\frac{2m}{r}}+r^2d\theta^2\right)\right.$$
$$\left.+\left(1-\frac{2m}{r}\right)r^2\sin^2\theta d\varphi^2\right]-\frac{(dt-sA_{03}d\varphi)^2}{T^{1/2}(1+\lambda)^{1/2}}, \tag{3}$$

where $T = 1 - s^2\lambda$. Given any explicit harmonic map, the set of Eqs. (2) and (3) gives a static stationary axial symmetric solution and its corresponding rotating solution. As an example, If $m = 0$ and $\lambda = 4M/r$ one obtains the Gross-Perry-Sorkin magnetic monopolesolution. Its correspnding rotating solution reads

$$ds_4^2 = T^{1/2}\left(1+\frac{4M}{r}\right)^{1/2}\left[dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\right]$$
$$-\frac{1}{T^{1/2}(1+\frac{4M}{r})^{1/2}}(dt-4sM(1-\cos\theta)d\varphi)^2 \tag{4}$$

For $M \neq 0$ and $s \neq 0$, this metric is not asymptotically flat, a pure magnetic monopole remains at infinity. The Newtonian potential ϕ is defined as $g_{44} = -e^{2\phi}$. Expanding $\phi \sim -M(1-s^2)/r + O(1/r^2)$, whereupon the mass parameter is found to be $M(1-s^2)$. Therefore, if $s^2 > 1$ the gravitational interaction will be repulsive. Expanding the electromagnetic fields we find $A_3 \sim 4Mq(1-\cos\theta) + 16M^2qs^2(1-\cos\theta)/r + O(1/r^2)$ and $A_4 \sim 4Mqs/r + 16M^2qs^3/r^2 + O(1/r^3)$, which tell us that the metric (4) represents a magnetic monopole with charge $4Mq$ together with a electric monopole of charge $4Mqs$ induced by rotation. Observe that for $s = 1$ this metric represents a rotating dyon without mass parameter.

References

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