Non Cuspy Galactic Halos from Scalar Field Dark Matter

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Abstract. The Lambda Cold Dark Matter model has recently shown an enormous predictive power. Nevertheless, the CDM paradigm predicts a density profile which corresponds to the Navarro-Frenk-White profile which is cusp and singular in the center of the galaxies. However, observations in LSB galaxies show a density profile which seems to follow an almost constant density profile. In this work we show that a flat central density profile naturally arises within the scalar field dark matter hypothesis. This adds evidence in favor that the cold dark matter in galaxies can be identified with the scalar field.

1. INTRODUCTION

Dark matter in the universe seems to be one of the most important puzzles science has to face in this moment. These component account for the 23% of the total material content of the universe [1, 2] and it existence has been only detected trough gravitational interactions. The fact that the dark matter does not interact with the rest of the matter components of the universe implies an important puzzles for ths Standard Model of Particles (SMP) as well.

Addressing the dark matter problem The Lambda Cold Dark Matter model (LCDM) is very successful at cosmological level. Nevertheless, some problems of this model at galactic level have arisen, (Moore [3], Burkert [4] and Tyson et al [5]). The CDM paradigm predicts a density profile which corresponds to the Navarro-Frenk-White (NFW) profile [6] given by

$$\rho_{\rm NFW} = \frac{\rho_0}{\frac{r}{r_0}\left(\frac{r}{r_0}+1\right)^2}. \tag{1}$$

This cusp density profile seems to have some differences with the observed profiles of LSB galaxies, see de Blok et al [7, 8, 9] and Simon et al [10]. The evidence points to the fact that, in the central regions, galaxies prefer to follow an almost flat density profile. In this work we review the main feature of the Scalar Field Dark Matter hypothesis (SFDM) which recover the success of CDM at cosmological level. We also review some analytical solutions of the flat and weak field limit of the Einstein- Klein-Gordon equations within the SFDM model. With this solutions we show that its scalar field density profile corresponds to a halo with an almost flat central density and that this halo coincides with the CDM model in a large outer region. Such a result could solve the

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problem of the cusp dark matter halo in galaxies without extra hypothesis, adding to the viability of the SFDM model.

2. THE SCALAR FIELD DARK MATTER MODEL

We work within the specific context of the so–called 'strong self–interacting scalar field dark matter' hypothesis that has recently been developed by several authors, Matos and Ureña-López [11, 12, 13], Matos and Guzmán [14], Alcubierre et al [15, 16], Ureña-López [17] (see also Peebles [18]).

The SFDM scenario can be explained as follows. We suppose that the dark matter responsible for structure formation in the Universe is a real scalar field, Φ, minimally coupled to Einstein gravity with self-interaction parameterized by a potential energy. Therefore, we propose the following effective Lagrangian for the SFDM model,

$$\begin{aligned} \mathscr{L}_{\text{L-SFDM}} &= \mathscr{L}_{\text{GR}} + \mathscr{L}_{\text{B}} + \mathscr{L}_{\Lambda} - \sqrt{-g}\left[\Phi^{,\mu}\Phi_{,\mu} + 2V(\Phi)\right], \\ V(\Phi) &= \frac{m_\Phi^2}{8\pi G\lambda^2}\left[\cosh\left(\sqrt{8\pi G}\lambda\Phi\right) - 1\right] \end{aligned} \tag{2}$$

where λ and m_Φ are the free parameters of the model; the latter is recognized as the mass of the scalar field. The values of λ and m_Φ can be fixed by imposing that the SFDM model should reproduce the success of CDM at cosmological scales [11, 12], and then

$$\lambda \sim 20, \qquad m_\Phi \sim 10^{-23}\,\text{eV}. \tag{3}$$

The behavior of the SFDM obtained from the lagrangian (2) is the following. After and during inflation, the universe is dominated by the radiation component, the scalar potential is exponential and is far from the minimum. The large value of λ makes the scalar matter sub dominant and behave as part of the radiation fluid. After inflation, the universe has cooled and the scalar field approaches the minimum of the potential, ceases to follow the radiation fluid, and begins to behave as a dust fluid (see M. S. Turner [19]). From this time on, the evolution of the homogeneous and isotropic universe proceeds exactly as in the LCDM model.

The latter is also true for the development of scalar fluctuations, which grow as in the CDM case, except that all structure on scales smaller than $\lambda_\Phi = m_\Phi^{-1} \sim 10\,\text{pc}$ is suppressed. This fact eventually prevents the formation of too much substructure like dwarf galaxies in clusters. At the other hand, the collapse of the scalar field form bound objects of the size of a galaxy. Numerical simulations for the collapse of a single scalar fluctuation, which ends as a self-gravitating scalar object called *oscillaton*[20, 15, 16, 21] have shown that this objects avoid the formation of gravitationally bounded objects with a cusp density profile, the energy density profile of oscillatons is regular everywhere. Oscillatons are stable scalar configurations if their mass is below the critical value[16]

$$M_{\text{crit}} \sim 0.6\frac{m_{\text{Pl}}^2}{m_\Phi}. \tag{4}$$

For the scalar mass value (3), we get $M_{\rm crit} \sim 10^{12} M_\odot$, which is roughly the mass content of a typical galaxy. Also, numerical studies within the weak-gravity regime have shown that a galaxy scalar fluctuation rapidly virializes[22], which seems to be in accord with observations of the oldest galaxies in the universe[23].

We want to underline the fact that the scalar field has no interaction with the rest of the matter, thus, it does not follow the standard lines of reasoning for the particle-like candidates for dark matter. The scalar field was not thermalized, that is, the scalar field forms a Bose condensate, and thus behaves strictly as cold dark matter from the beginning (see J. Lee and I. Koh [24]). In what follows we briefly describe the SFDM at galactic level.

3. DENSITY PROFILES FROM A FLAT SPACE-TIME

For the scenario of galactic formation with the SFDM hypothesis, when the scalar field fluctuations reaches the non-linear regime, the scalar field collapses in a different way as the standard CDM. In a normal dust collapse, as for example in CDM, there is in principle nothing to avoid that the dust matter collapse all the time. In the scalar field paradigm this collapse is different. The radial and angular pressures are two natural components of the scalar field which stops the collapse, avoiding the cusp density profiles in the centers of the collapsed objects. The pressures play an important roll in the SFDM equilibrium [25].

Galaxies have a weak gravitational field. In this way, their space-time is almost flat. We will suppose first that the space-time of the galaxy is flat. This is a crude approximation but it shows us with great clarity the physics taking place in the oscillatons. In the next section we perform the weak field limit to the Einstein-Klein-Gordon (EKG) equations and show that the density profile remains almost flat and regular in the center of the galaxy.

Imposing spherical symmetry, we work with the line element

$$ds^2 = -e^{2\nu} dt^2 + e^{2\mu} dr^2 + r^2 d\Omega^2, \tag{5}$$

with $\mu = \mu(r,t)$ and $\nu = \nu(r,t)$, being this last function the Newtonian potential. The energy momentum tensor of the scalar field is

$$T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \frac{g_{\mu\nu}}{2}\left[\Phi^{,\alpha}\Phi_{,\alpha} + 2V(\Phi)\right]. \tag{6}$$

In the Minkowski background $\mu \sim \nu \sim 0$ and making the approximation of the potential (2) to $V = \frac{m_\Phi^2}{8\pi G\lambda^2}\left[\cosh\left(\sqrt{8\pi G}\lambda\Phi\right) - 1\right] \sim \frac{1}{2}m_\Phi^2\Phi^2$ (we will see later that it is not a so bad approximation neither in the flat or weak field cases) the Klein-Gordon equation $\mathscr{D}\Phi - dV/d\Phi = 0$, where $\mathscr{D}$ stands for the DAlambertian, reads

$$\Phi'' + \frac{2}{r}\Phi' - m_\Phi^2\Phi = \ddot{\Phi} \tag{7}$$

where over-dot denotes $\partial/\partial t$ and prime denotes $\partial/\partial r$. The exact general solution for the scalar field Φ is

$$\Phi(t,r) = \frac{e^{\pm ikr}}{r} e^{\pm i\omega t} \tag{8}$$

where we obtain the dispersion relation $k^2 = \omega^2 - m_\Phi^2$. For $\omega > m_\Phi$ the solution is non-singular and vanishes at infinity. We will restrict ourselves to this case. It is more convenient to use trigonometric functions and to write the particular solution in the form

$$\Phi(t,x) = \Phi_0 \frac{\sin(x)}{x} \cos(\omega t) \tag{9}$$

where $x = kr$.

Even though it is not a solution to the Einstein equations as we are neglecting the gravitational force provoked by the scalar field, the solution is analytic and it helps us to understand some features that appear in the non-flat oscillatons. The scalar field oscillates in harmonic manner in time. and it spreads over all space, *i.e.*, it is not confined to a finite region, as we are neglecting the gravitational interactions. In Ureña-López, [17], and Ureña-López, Matos and Becerril [26] has been shown that when the gravitational interaction is taken into account, the oscillaton gets confined in a finite region. The analytic expression for the scalar field energy density derived from Eq. (9) is

$$\rho_\Phi = \frac{1}{2}{\Phi_0}^2 \left(\left(\frac{x\cos(x) - \sin(x)}{x^2} \right)^2 - \frac{\sin^2(x)}{x^4} \right) k^4 \cos^2(\omega t) \tag{10}$$

$$+\frac{1}{2}{\Phi_0}^2 \frac{\omega^2 k^2 \sin^2(x)}{x^2} \tag{11}$$

which oscillates with a frequency 2ω. Observe that close to the central regions of the object, the density of the oscillaton behaves like

$$\rho_\Phi \sim \frac{1}{2}{\Phi_0}^2 k^2 \left[\omega^2 - k^2 \cos^2(\omega t)\right] + O(x^2) \tag{12}$$

which implies that the density is almost constant in the central regions, i.e. when $x \to 0$ the central density oscillates around a fixed value. Recall that this is an exact solution of the Klein Gordon field equation, so this behaviors arise naturally.

On the other hand, the asymptotic behavior when $x \to \infty$, is such that $\rho_\Phi \sim 1/x^2$, *i.e.* far away from the center, in this approximation, the flat oscillaton density profile behaves like the isothermal one. Nevertheless, if the gravitational interaction is taken into account, the object must be confined (see Ureña-López, [17], and Ureña-López, Matos and Becerril [26]) and a more realistic profile should change this behavior. Obviously, the mass function oscillates around $M \sim x$. In this approximation the integrated mass of the scalar field gives an infinite value.

In order to understand what is happening within the object, observe that the KG equation can be rewritten in a more convenient form in terms of the energy density, as

$$\frac{\partial \rho_\Phi}{\partial t} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \mathscr{P}_\Phi\right) = 0. \tag{13}$$

where $\mathscr{P}_\Phi$ is the momentum density T_{01}. This last equation has a clear interpretation: Since its form looks like the conservation equation, $\dot{\rho}+\nabla\cdot\vec{J}=0$, equation (13) represents the conservation of the scalar field energy. It also tells us that there is a scalar field current given by

$$\begin{aligned}\vec{J}_\Phi &= -\mathscr{P}_\Phi\vec{r}\\ &= \Phi_0^2\frac{k\omega}{2}\left[x\cos(x)-\sin(x)\right]\frac{\sin(x)\sin(2\omega t)}{x^3}\vec{r}.\end{aligned}$$

Although the flux of scalar radiation at large distances does not vanish, there is not a net flux of energy, as it can be seen by averaging the scalar current on a period of a scalar oscillation. We also see that the only transformation process is that of the scalar field energy density into the momentum density, and vice versa. For the realistic values this transfer is very small.

In the next section we will see that if the gravitational force is taken into account, the oscillaton is more confined (see also Ureña-López, [17], Ureña-López, Matos and Becerril [26] and Alcubierre et al [16])

4. DENSITY PROFILES FROM THE WEAK-FIELD LIMIT

In this section we make the analysis of the oscillaton behavior in the presence of a weak gravitational field. In the weak field limit the space-time metric can be written as

$$g_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta} \tag{14}$$

were $\eta_{\alpha\beta}$ is the Minkowski metric and $|h_{\alpha\beta}|\ll 1$ is a small correction to it. Considering spherical symmetry (5) the non zero perturbations will be

$$h_{00} = -2\nu(r,t) \tag{15}$$
$$h_{11} = 2\mu(r,t) \tag{16}$$

In this case the dynamics of the scalar field is given by the linearized EKG equations

$$R_{\alpha\beta} = \kappa_0 S_{\lambda\beta} \tag{17}$$
$$\mathscr{D}\Phi-m_\Phi^2\Phi = 0 \tag{18}$$

where $R_{\alpha\beta}$ is the first order Ricci tensor in $h_{\alpha\beta}$ and $\kappa_0=8\pi G$ the equation (18) is the same as in the flat space (see S. Weinberg [27]), then the solution of (18) will be in the form (9). Following Ureña-López [17] and Ureña-López, Matos and Becerril [26] we propose for μ and ν

$$\mu(t,r) = \mu_0(r)+\mu_2(r)\cos(2\omega t) \tag{19}$$
$$\nu(t,r) = \nu_0(r)+\nu_2(r)\cos(2\omega t) \tag{20}$$

Replacing this anzats we obtain a closed system of linear equations for μ_0, μ_2, ν_0, ν_2 and Φ. It is convenient to introduce the scaled variables

$$x=m_\Phi r,\quad \tau=m_\Phi t,\quad \Omega=\frac{\omega}{m_\Phi} \tag{21}$$

In terms of these new variables the solutions for the scalar field and the r–dependent functions of the perturbations to the metric are

$$\Phi = \frac{\sigma_0}{\sqrt{\kappa_0}}\frac{\sin(x\sqrt{\Omega^2-1})}{x}\cos(\Omega\tau) \tag{22}$$

$$\mu_2 = \frac{1}{8}\sigma_0^2\left[-\frac{\sin^2(\sqrt{x\Omega^2-1})}{x^2}+\frac{1}{2}\frac{\sqrt{\Omega^2-1}\sin(2x\sqrt{\Omega^2-1})}{x}\right] \tag{23}$$

$$\mu_0 = \frac{1}{8}\sigma_0^2\left[-\frac{1}{2x^2}+\frac{1}{2}\frac{\cos(2x\sqrt{\Omega^2-1})}{x^2}-\frac{1}{2}\frac{\sin(2x\sqrt{\Omega^2-1})}{x\sqrt{\Omega^2-1}}+\Omega^2\right]+\frac{C_1}{x} \tag{24}$$

$$\nu_2 = \frac{1}{8}\sigma_0^2\left[\frac{1}{2}\frac{\sqrt{\Omega^2-1}\sin(2x\sqrt{\Omega^2-1})}{x}+Ci\left(2x\sqrt{\Omega^2-1}\right)-\ln(2x\sqrt{\Omega^2-1})\right]+C_2 \tag{25}$$

$$\nu_0 = \frac{1}{8}\sigma_0^2(2\Omega^2-1)\left[\frac{\sin(2x\sqrt{\Omega^2-1})}{2\sqrt{\Omega^2-1}x}-Ci\left(2x\sqrt{\Omega^2-1}\right)+\ln(2x\sqrt{\Omega^2-1})\right]+C3 \tag{26}$$

Observe that the final solution for the space-time metric and the scalar field depend only on two parameters: the scalar field amplitude σ_0 and the quotient $\Omega = \omega/m_\Phi$. If we require a non-singular and asymptotically flat solution for the x-dependent scalar field part we have to restrict $\Omega > 1$. On the other hand remember that the weak field limit is maintained only if $|h_{\alpha\beta}| \ll 1$. Both $\mu_0(x)$ and $\mu_2(x)$ are bounded and have an oscillatory nature. On the contrary $\nu_0(x)$ and $\nu_2(x)$ are increasing unbounded functions however it is possible to find a space range, starting from the origin, where $|h_{00}| \ll 1$ and $|h_{11}| \ll 1$ (see A. Bernal,T. Matos and D. Nuñez [25] and C. Misner, k.Thorne and J. Wheeler [28]). Those conditions are satisfied if

$$\Omega \approx 1 \tag{27}$$

$$\sigma_0^2 \sim 10^{-6} \tag{28}$$

The analytic expression for the scalar field energy density is given by $\rho_\Phi = -T^0{}_0$. In our case it reads as

$$\begin{aligned}\rho_\Phi = {} & \frac{1}{2}M^2(1+2\nu)^{-1}\sigma^2\Omega^2 \\ & +\frac{1}{2}M^2\left[-(1+2\nu)^{-1}\sigma^2\Omega^2+(1+2\mu)^{-1}\sigma'^2+\sigma^2\right]\cos^2(\Omega\tau)\end{aligned} \tag{29}$$

where we have defined $M^2 \equiv m_\Phi^2/\kappa_0$, σ is the spatial part of the scalar field and prime denote $\partial/\partial x$. As in the flat space case when $x \to 0$ the central density oscillates around a fixed value, this is because both $\nu(x)$ and $\mu(x)$ are regular at the origin.

In the figure 1 we compare the density profile from the weak field which is analytical, and the density profile from a relativistic oscillaton obtained with the numerical

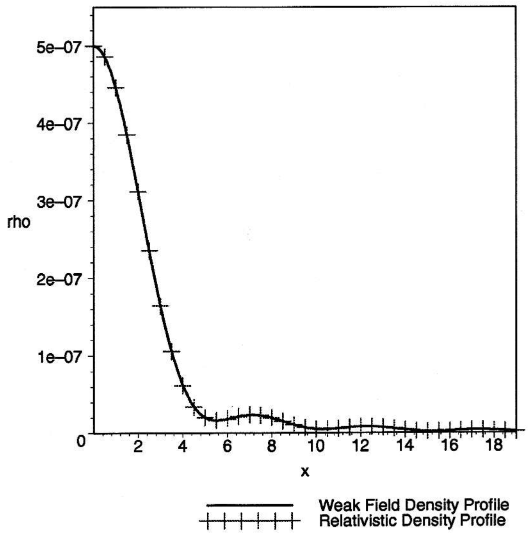

FIGURE 1. The input values for the relativistic oscillaton are specified in A. Bernal, T. Matos and D. Nuñez [25]. The output value for Ω is 1.18100, given this, the value for σ_0 is $1.59156e^{-3}$. The rescaled x does not have units because the change of variables (21) and the density's, ρ, units are in terms of M^2, for this graphic $M^2 = 1$. For these values of σ_0 and Ω the flat density profile is equal to the weak field density profile.

solutions in Ureña-López, Matos and Becerril [26]. Ω is an output value of the relativistic oscillaton. Taking this Ω, in the weak field, the lastly free parameter σ_0 is obtained scaling the central scalar field amplitude to the central scalar field amplitude for the relativistic oscillaton (see [25]).

It so important to note that because the change of variables made (21) the units and the magnitude of the energy–momentum density (29) are in terms of $M^2 \equiv m_\Phi^2/\kappa_0$. In A. Bernal, T. Matos and D. Nuñez [25] the scalar field mass is taken as (3) and the fits with some LSB galaxies density profiles are made. Finally we have to point that we have been working with an approximation of the SFDM potential (2) however we found that in the weak field limit the argument of the cosh goes like $\sqrt{\kappa_0}\lambda\Phi \sim 10^{-2}$ so it is valid to take just a quadratic potential in this limit. Further work account for study the complete potential.

5. DISCUSSION

There are many theoretical proposals that go beyond the standard model of particles, among which the most successful are the superstring theory and the brane models. A common feature of all of these super-theories is the existence of *scalar fields*: the so called dilatons, radions, etc., which are key elements by themselves. The presence of scalar fields in these models is surprising, since no fundamental scalar field has ever been detected, nor even the so-needed Higgs particle of the SMP. There are only four fundamental interactions in Nature: gravitational, electromagnetic, strong and weak. Gravitation is a spin-2 interaction, and the others are spin-1. As there is not evidence for the existence of a fundamental scalar field nor of a spin-0 interaction, it seems that our super-theories have to have an internal mechanism to suppress their own scalar fields. But, the evidence is not at all conclusive and we should ask: *did Nature forget the most simple interaction?*

On one hand, we have to face the theoretical existence of fundamental scalar fields; on the other hand, we have to face the problem of the missing matter in the universe. A deal seems possible: could it be possible that Nature did include the spin-0 interaction and made it the dominant one as part of what we called dark energy and dark matter?

We have seen in this work how the SFDM scenario offers the same results as the concordance LCDM model at large scales but it is different at galactic level. The SFDM provides a new paradigm for galaxy formation that seems to be in accord with actual observations. If this model is realistic, it could give us a solution of the dark matter problem and give us a connection between our supertheories, the simple spin-0, long range interaction and the actual observations of the universe.

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