Empirical testing of Tsallis' Thermodynamics as a model for dark matter halos

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Abstract. We study a dark matter halo model from two points of view: the "stellar polytrope" (SP) model coming from Tsallis' thermodynamics, and the one coming from the Navarro-Frenk-White (NFW) paradigm. We make an appropriate comparison between both halo models and analyzing the relations between the global physical parameters of observed galactic disks, coming from a sample of actual galaxies, with the ones of the unobserved dark matter halos, we conclude that the SP model is favored over the NFW model in such a comparison.

Keywords: dark matter, galaxy dynamics

INTRODUCTION

An alternative formalism to the micro-canonical ensemble treatment, that allows non-extensive forms for entropy and energy under simplified assumptions, has been developed by Tsallis [1] and applied to self–gravitating systems [2, 3, 4] under the assumption of a kinetic theory treatment and a mean field approximation. As opposed to the Maxwell–Boltzmann distribution that follows as the equilibrium state associated with the usual Boltzmann–Gibbs entropy functional, the Tsallis' functional yields as equilibrium state the "stellar polytrope" (SP), characterized by a polytropic equation of state with index n.

On the other hand, high precision N–body numerical simulations based on Cold Dark Matter (CDM) models, perhaps the most powerful method available for understanding gravitational clustering, lead to the famous results of Navarro, Frenk and White (NFW model) [5] that predicts density and velocity profiles which are roughly consistent with observations, however, it also predicts a cuspy behavior at the center of galaxies that is not observed in most of the rotation curves of dwarf and LSB galaxies [6, 7, 8, 9, 10, 11]. The significance of this discrepancy with observations is still under dispute, nevertheless, the NFW model of collision–less WIMPs remains as a viable model to account for DM in galactic halos, provided there is a mechanism to explain the

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discrepancies of this model with observations in the center of galaxies.

Since gravity is a long–range interaction and virialized (i.e in virial equilibrium) self–gravitating systems are characterized by non-extensive forms of entropy and energy, it is reasonable to expect that the final configurations of halo structure predicted by N–body simulations must be, somehow, related with states of relaxation associated with non-extensive formulations of Statistical Mechanics; therefore, a comparison between the SP and NFW models is both, possible and interesting. The main purpose of our analysis is to make a dynamical analysis of two halo models, one based on the NFW paradigm, and other based on the SPs derived from Tsallis' non-extensive thermodynamics, compare them and test both with observational results coming from a sample of disk galaxies.

SP AND NFW HALO MODELS

For a face space given by $(\mathbf{r}, \mathbf{p})$, the kinetic theory entropy functional associated with Tsallis' formalism is [2, 3], and [4]

$$S_q = -\frac{1}{q-1} \int (f^q - f)\, d^3\mathbf{r}\, d^3\mathbf{p}, \tag{1}$$

where f is the distribution function and $q > 1$ is a real number. In the limit $q \to 1$, the functional (1) leads to the usual Boltzmann–Gibbs functional, corresponding to the isothermal sphere. The condition $\delta S_q = 0$ leads to the distribution function that corresponds to the SP model characterized by the equation of state $p = K_n \rho^{1+1/n}$, where K_n is a function of the polytropic index n, and can be expressed in terms of the central parameters: $K_n = \frac{\sigma_c{}^2}{\rho_c{}^{1/n}}$. The polytropic index, n, is related to the Tsallis' parameter $q > 1$ by: $n = \frac{3}{2} + \frac{1}{q-1}$. Inserting the equation of state into Poisson's equation, the Lane-Emden equation [12] is obtained, based on it we can obtain density, mass and velocity profiles for the SP model.

NFW numerical simulations yield the following expression for the density profile of virialized galactic halo structures [5, 13]:

$$\rho_{\text{NFW}} = \frac{\delta_0\, \rho_0}{y\,(1+y)^2}, \tag{2}$$

where: $\delta_0 = \frac{\Delta c_0^3}{3[\ln(1+c_0) - c_0/(1+c_0)]}$, $\rho_0 = \rho_{\text{crit}} \Omega_0 h^2 = 253.8\, h^2\, \Omega_0 \frac{M_\odot}{\text{kpc}^3}$, $y = c_0 \frac{r}{r_v}$, and Ω_0 is the ratio of the total density to the critical density of the Universe. Using equation (2) its easy to obtain mass and velocity profiles.

COMPARISON OF SP AND NFW MODELS

In order to compare both halo models, it is important to make various physically motivated assumptions. First, we want both models to describe a halo of the same scale, which means same virial mass M_{vir}. Secondly, both models must have the same maximal value for the rotation velocity. This is a plausible assumption, as it is based on the

TABLE 1. Parameters characterizing the polytropes while being compared to NFW halos

$\log_{10}(M_{\rm vir}/M_\odot)$	$\rho_c\,[M_\odot/{\rm pc}^3]$	σ_c [Km/s]	n	q	K_n	$v_{\rm max}$ [Km/s]	$r_{\rm vir}$ [kpc]
15	3.7×10^{-4}	982	4.93	1.29	4873.4	1504	2606.2
12	7.5×10^{-4}	108	4.87	1.30	478.94	164	260.6
11	9.0×10^{-4}	52	4.83	1.30	221.82	79.1	120.9
10	1.2×10^{-3}	25	4.82	1.30	100.68	38.2	56.1

Tully–Fisher relation [14], a very well established result that has been tested successfully for galactic systems, showing a strong correlation between the total luminosity of a galaxy and its maximal rotation velocity. Our third assumption is that the polytropic and NFW halos, complying with the previous requirements, also have the same total energy evaluated at the cut–off scale $r = r_{\rm vir}$. The main justification for this assumption follows from the fact that the total energy is a fixed quantity in the collapse and subsequent virialized equilibrium of dark matter halos [15]. Since the SP model has three free parameters (contrary to only one parameter of the NFW model), and we have selected three comparison criteria, we have a mathematically closed problem once $M_{\rm vir}$ is specified.

Following the guidelines described above, we proceed to compare NFW and polytropic halos for $M_{\rm vir}$ ranging from 10^{10} up to 10^{15} solar masses. From this comparison we find the "best-fit" values for the free parameters of the SP model. The results are displayed explicitly in table 1. The comparison between both models in velocity profiles is shown in the left panel of figure 1. SP and NFW models have both the same virial mass, $M_{\rm vir} = 10^{12}\,M_\odot$. For other values of $M_{\rm vir}$ the velocity profiles are qualitatively similar to the one displayed in figure 1 (left panel). The detailed description and results of the method of comparison presented above will appear in an article that is being prepared [16].

So far we have analyzed only the global structure of the dark matter halo without considering the effects of the luminous galaxy within. If one wishes to test a given model with observational results, it is necessary to add the galactic baryonic disk as a dynamical component of the model. In order to do so we followed the method described in [13, 17]. Then to compare both models with observational results we used the prescription presented in [18].

Using such a prescription, we may define the ratio of maximum disk velocity to maximum total velocity, $V_{d,m}/V_{c,m}$, which is a global quantity that can be directly compared with theoretical predictions. This ratio is not defined at a given radius, but it can be related to the total mass to disk mass ratio M_t/M_d, defined at an specific radius. In particular at radius r_m where the total rotational curve has its maximum we have [18]:

$$\left(\frac{M_t}{M_d}\right)_{r_m} \propto \left(\frac{V_{d,m}}{V_{c,m}}\right)^{-2} \tag{3}$$

The use of $V_{d,m}/V_{c,m}$ instead of M_t/M_d is suitable because it can be obtained directly from observational parameters, without the assumptions needed to calculate M_t/M_d.

One of the principal results obtained in the work [18] is that the ratio $V_{d,m}/V_{c,m}$ correlates principally with the disk surface density Σ_d of galaxies. Therefore we will

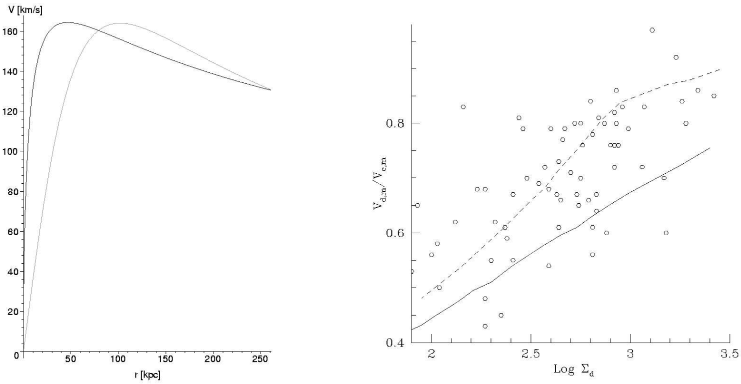

FIGURE 1. Left panel: velocity profiles for the NFW halo (solid line) and its compared SP one (dashed curve). Right panel: luminous to total dark matter content vs central surface density; open circles correspond to observational data, the dashed line represents the NFW model and the solid one to SP model. Both models have a virial mass of $M_v = 10^{12} M_{\odot}$.

use this result to compare the NFW and the SP models with the observational results coming from the sample. We will take $M_{vir} = 1 \times 10^{12} M_{\odot}$, which is a characteristic value for the mass of dark matter halos. For simplicity, we will assume that the baryonic mass fraction of the disk, f_d, has the same value for all disks.

In the right panel of figure 1 it is shown the ratio $V_{d,m}/V_{c,m}$ vs $Log\Sigma_d$ for the sample of galaxies used (open circles). An almost linear trend can be seen between this quantities. HSB (high surface brightness) galaxies (corresponding approximately to values of $Log\Sigma_d$ greater than 2.5) have greater values of $V_{d,m}/V_{c,m}$ than LSB (low surface brightness) galaxies. This means that the luminous matter content is greater for HSB than for LSB galaxies. The shown picture is consistent with a well known result: LSB galaxies are dark matter dominated systems within the optical radius.

The value of the graphic presented is that it allow us to bound statistically the possible values of the $V_{d,m}/V_{c,m}$ ratio that galaxies with a given surface density can have. As was proved by [18], the size of this range of values (associated with dispersion on the graphic) is mainly due different virial mass that galaxies with the same Σ_d have.

Right panel of figure 1 also show clearly that NFW models can not reproduce at satisfaction the results obtained for the compiled sample without introducing unrealistic values for the virial mass. This is one of the results that lead us to the possibility of seeking an alternative to the NFW paradigm. The curves shown in figure 1 (right panel) represent both models with average values for their respective parameter; it's clear from the figure that the SP model follows in better agreement the average behavior of the observational sample than the NFW model.

CONCLUSIONS

Motivated by the fact that SPs are the equilibrium state in Tsallis' non-extensive entropy formalism, we have found the structural parameters of those SPs that allows us to compare them with NFW halos of virial masses in the range $10^{10} < M_{vir}/M_{\odot} < 10^{15}$; the results are displayed in Table 1. It is shown in the left panel of figure 1 that the velocity profile of the SP model is much less steep in the same region than that of the NFW halo. These features are consistent with the fact that NFW profiles predict more dark matter mass concentration than what is actually observed in a large sample of galaxies [19, 8, 18].

We have also shown that the SP model is favored over the NFW model regarding the dark matter content in disk galaxies (within the optical radius) which is shown by the average behavior of the observational sample in figure 1 (right panel). These results show that the NFW halo model can be enhanced with the use of alternative paradigms in Statistical Mechanics, which seems to solve a recurrent item which throws a shadow in such an excellent description as the NFW model is.

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