Alternative interpretation for the moduli fields of string theories

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Abstract. In this work we provide a basis for studying the cosmologies derived from superstring theory. Distinct features of these cosmologies are the presence of an axion field, and the interaction of the dilaton field with all the other matter fields. We make a first study of the equations of motion and write them as an autonomous dynamical system. The fixed points of the equations and their corresponding stability are determined in turn. We then discuss the viability of the string fields as dark energy and dark matter.

1. Introduction

Doubtless one of the main problems in physics now is the nature of dark matter and the understanding of the accelerated expansion of the universe. There are a number of observations supporting the existence of dark matter [1] and the accelerated expansion of the universe as well [2]. On the other hand, one of the main problems of superstring theory is that there is no real phenomenology that can support the theory. Usually, superstring theory is supported only by its mathematical and internal consistency, but not by real experiments or observations. One way in which superstring theory can make contact with phenomenology is through cosmology [3]. In the last years, a number of new observations have given rise to a new cosmology and to a new perception of the universe (see for example [4]). The discovery of two kind of substances in the universe, one of them attractive, called dark matter and the other one repulsive, called dark energy, have changed our paradigm on the cosmos. On the other hand, the superstring theory contains a set of fields and particles that have not been seen in nature. In particular, two fields, the *dilaton* and the *axion*, are two very important components of the theory which can not be easily fixed. In fact, one should find a physical interpretation for these fields or give an explanation of why we are not able to see them in nature. Some explanations for the present non-existence of the moduli fields is that there exists a mechanism for eliminating these fields during the evolution of the universe [5]. One of the most popular interpretations for the dilaton

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field is that it can be the dark energy of the universe, *i.e.* a Quintessence field [6], thus after a non trivial compactification, the dilaton field acquires an effective potential [6].

Some attempts have been carried out in the past with other dilaton potentials [7]. Here, we will be very specific starting with effective potentials inspired in the type IIB supergravity theory [8].

Moduli stabilization has been used also in string cosmology to fix other moduli fields than the volume modulus including dilaton+axion and Kahler moduli [9]. For a description of more realistic scenarios, see [10].

2. Coupled matter fields: Mathematical background

As pointed out before, the main feature of the low energy superstring theory are the dilaton and the axion fields. Let us start with a generic Lagrangian of the low energy superstring theory given by

$$\mathcal{L} = \sqrt{-g}\left(R - \mathcal{L}_\phi - e^{\alpha_\psi \phi}\mathcal{L}_\psi - e^{\alpha_\gamma \phi}\mathcal{L}_\gamma\right) \,, \tag{1}$$

where the α's are the couplings of the dilaton field ϕ to the axion ψ and other matter fields. The individual Lagrangians for the dilaton and axion fields and the perfect fluid are, respectively,

$$\mathcal{L}_\phi = \frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V_\phi \,, \quad \mathcal{L}_\psi = \frac{1}{2}\partial^\sigma\psi\partial_\sigma\psi + V_\psi \,, \qquad \mathcal{L}_\gamma = \rho_\gamma \,. \tag{2}$$

The perfect fluid pressure is defined through the equation $p_\gamma = (\gamma - 1)\rho_\gamma$, where $0 < \gamma < 2$ is its barotropic equation of state. On the other hand, the equations of motion for the scalar fields are

$$\Box\phi - \frac{\partial V_\phi}{\partial\phi} = \alpha_\psi e^{\alpha_\psi\phi}\mathcal{L}_\psi + \alpha_\gamma e^{\alpha_\gamma\phi}\mathcal{L}_\gamma \,, \quad \Box\psi - \frac{\partial V_\psi}{\partial\psi} = -\alpha_\psi\partial^\mu\phi\partial_\mu\psi \,, \tag{3}$$

where $\Box$ is the d'Alembertian operator. Notice that the corresponding Klein-Gordon equations are not homogeneous as in the standard case, but the non-zero r.h.s.'s are proportional to the coupling coefficients.

We also need the Einstein field equations, which now read

$$G_{\mu\nu} = \kappa^2\left[T^{(\phi)}_{\mu\nu} + e^{\alpha_\psi\phi}T^{(\psi)}_{\mu\nu} + e^{\alpha_\gamma\phi}T^{(\gamma)}_{\mu\nu}\right] \,. \tag{4}$$

Do notice that each of the energy-momentum tensors on the r.h.s. has the standard form for the corresponding matter field. The Bianchi identities, $G^{\mu\nu}{}_{;\nu} = 0$, are still valid, and then we find in this case the conservation of the *total* energy-momentum tensor

$$T^{(\phi)\mu\nu}{}_{;\nu} + \left[e^{\alpha_\psi\phi}T^{(\psi)\mu\nu} + e^{\alpha_\gamma\phi}T^{(\gamma)\mu\nu}\right]_{;\nu} = 0 \,. \tag{5}$$

Unlike the standard case, it does not follow from Eq. (3) that each individual energy-momentum tensor is conserved. One can show, from Eqs. (5), that the conservation equation for the dilaton and the matter fields are, respectively,

$$T^{(\phi)\mu\nu}{}_{;\nu} = \sum_i \partial^\mu\left(e^{\alpha_i\phi}\right)\mathcal{L}_{(i)} \,, \quad \left(e^{\alpha_i\phi}T^{(i)\mu\nu}\right)_{;\nu} = -\partial^\mu\left(e^{\alpha_i\phi}\right)\mathcal{L}_{(i)} \,, \tag{6}$$

where $i = \psi$ and γ, expressions which are then consistent with Eq. (2).

For a flat Friedmann-Robertson-Walker metric, the equations of motion are the Einstein equations

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 e^{\alpha_\psi \phi} + \rho_\gamma e^{\alpha_\gamma \phi} + V_\phi + V_\psi e^{\alpha_\psi \phi} + \rho_L\right) , \tag{7}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \dot{\psi}^2 e^{\alpha_\psi \phi} + \gamma\rho_\gamma e^{\alpha_\gamma \phi}\right) , \tag{8}$$

together with the equation of the matter fields

$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_\phi}{\partial \phi} &= \alpha_\psi e^{\alpha_\psi \phi}\left(\frac{1}{2}\dot{\psi}^2 - V_\psi\right) - \alpha_\gamma e^{\alpha_\gamma \phi}\rho_\gamma , \\
\ddot{\psi} + 3H\dot{\psi} + \frac{\partial V_\psi}{\partial \psi} &= -\alpha_\psi \dot{\phi}\dot{\psi} , \\
\dot{\rho_\gamma} + 3H\gamma\rho_\gamma &= 0 , \\
\dot{\rho_L} &= 0 ,
\end{aligned} \tag{9}$$

where the dot stands for the derivative with respect to the cosmological time, H is the Hubble parameter $H = \dot{a}/a$, and $\kappa^2 = 8\pi G/c^4$.

In order to analyze the behavior of this cosmology, we define new variables as follows

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H} , \quad y \equiv \frac{\kappa\sqrt{V_\phi}}{\sqrt{3}H} , \quad \Omega_\gamma \equiv \frac{\kappa^2\rho_\gamma}{3H^2}e^{\alpha_\gamma \phi} , \tag{10}$$

$$u \equiv \frac{\kappa\dot{\psi}}{\sqrt{6}H}e^{\frac{1}{2}\alpha_\psi \phi} , \quad v \equiv \frac{\kappa\sqrt{V_\psi}}{\sqrt{3}H}e^{\frac{1}{2}\alpha_\psi \phi} , \quad z \equiv \frac{\kappa\sqrt{\rho_L}}{\sqrt{3}H} \tag{11}$$

Observe that z can also be taken as an auxiliary variable and not as real one. With these definitions equations (9) transform into

$$x' = -3x + \tilde{\lambda}y^2 + \tilde{\alpha}_\psi\left(u^2 - v^2\right) - \tilde{\alpha}_\gamma\Omega_\gamma + \Pi x , \tag{12}$$

$$y' = -\tilde{\lambda}xy + \Pi y , \tag{13}$$

$$u' = -3u - \tilde{\alpha}_\psi xu - zv + \Pi u , \tag{14}$$

$$v' = \tilde{\alpha}_\psi xv + zu + \Pi v , \tag{15}$$

$$\Omega_\gamma' = 2\left(\tilde{\alpha_\gamma}x + \Pi - \frac{3}{2}\gamma\right)\Omega_\gamma , \tag{16}$$

$$z' = \Pi z. \tag{17}$$

where prime stands for the derivative with respect to the number of e-foldings $N \equiv \ln(a)$, Π is the function

$$\Pi = -\frac{\dot{H}}{H^2} = 3\left(x^2 + u^2 + \frac{\gamma}{2}y^2\right) \tag{18}$$

and we have also defined the constants

$$\begin{aligned}
&\tilde{\alpha}_\psi \equiv \sqrt{\frac{3}{2}}\frac{\alpha_\psi}{\kappa} , \quad \tilde{\alpha}_\gamma \equiv \sqrt{\frac{3}{2}}\frac{\alpha_\gamma}{\kappa} , \\
&\tilde{\lambda} \equiv -\sqrt{\frac{3}{2}}\frac{1}{\kappa V_\phi}\partial_\phi V_\phi , \quad z \equiv \frac{\sqrt{2}}{H}\partial_\psi\sqrt{V_\psi} .
\end{aligned} \tag{19}$$

Finally, note that the Friedmann equation (7) now reads

$$x^2 + y^2 + u^2 + v^2 + \Omega_\gamma + z^2 = 1 , \tag{20}$$

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3. Exponential dilatonic potential and massive axion field

There is one possibility for which the definitions of $\tilde{\lambda}$ and z can take their simplest form. If $\tilde{\lambda} =$ const., the dilatonic potential is an exponential of the form $V_\phi = V_0 e^{-\lambda_\phi \phi}$. As for variable z, we choose $\partial_\psi \sqrt{V_\psi}=$ const. $= m/\sqrt{2}$, where m would be the mass of the axion field, and the axion potential is then $V_\psi = (1/2)m^2\psi^2$; in consequence, $z = m/H$ and it represents the ratio between the Hubble distance H^{-1} and the axion Compton lenght $\lambda_C = m^{-1}$.

3.1. Fixed points and stability

The dynamical system (17) is of the form $\mathbf{x}' = \mathbf{f}(\mathbf{x})$, where $\mathbf{x} \equiv \{x, y, u, v, \Omega_\gamma, z\}$, and $\mathbf{f} \equiv \{f_x, f_y, f_u, f_v, f_{\Omega_\gamma}, f_z\}$ represent, respectively, the different functions on the r.h.s. of equations (17), together with constrain (20). Thus, the fixed points $\mathbf{x}_c$ are solutions to the equations $\mathbf{f}(\mathbf{x}_c) = 0$.

The stability of the fixed points is determined by taking small perturbations $\mathbf{x} = \mathbf{x}_c + \delta\mathbf{x}$ around them, so that the equations of motion become, at leading order,

$$\delta\mathbf{x}' = \mathcal{M}\,\delta\mathbf{x}\,, \tag{21}$$

where the stability matrix has components given by

$$\mathcal{M}_{ij} \equiv \left.\frac{\partial f_i}{\partial x_j}\right|_{\mathbf{x}_c} . \tag{22}$$

The stability of each fixed point is determined by the eigenvalues ω of the stability matrix; in general, if $\mathcal{R}e(\omega) < 0$ the fixed point is called *stable*, whereas the fixed point is called *unstable* if $\mathcal{R}e(\omega) > 0$. Clearly, we are interested in knowing the stable points, as they are the attractor solutions of the dynamical system, and represent the asymptotic state of the original cosmological variables.

It is easy to show that z is an ever growing function whenever the Hubble parameter decreases, which is usually the case in an expanding universe. This also can be seen from the equation of motion for z,

$$z' = -\frac{\dot{H}}{H} z\,. \tag{23}$$

We write the fixed points of the dynamical system (17) which fulfil the constraint (20) in the following list.

- Kinetic dilaton domination $\{1, 0, 0, 0, 0, 0\}$, and $\{-1, 0, 0, 0, 0, 0\}$.
- Axion and kinetic dilaton domination

$$\left\{-\frac{\tilde{\alpha}_\psi}{3}, 0, 0, \pm\sqrt{1 - \frac{\tilde{\alpha}_\psi^2}{9}}, 0, 0\right\} \tag{24}$$

- Dilatonic domination

$$\left\{\frac{\tilde{\lambda}}{3}, \pm\sqrt{1 - \frac{\tilde{\lambda}}{9}}, 0, 0, 0, 0\right\} \tag{25}$$

- Dilaton and kinetic axion domination

$$\left\{-\frac{3}{\tilde{\alpha}_\psi - \tilde{\lambda}}, \pm\sqrt{\frac{\tilde{\alpha}_\psi}{\tilde{\alpha}_\psi - \tilde{\lambda}}}, \pm\frac{\sqrt{\tilde{\lambda}^2 - \tilde{\lambda}\tilde{\alpha}_\psi - 9}}{\tilde{\alpha}_\psi - \tilde{\lambda}}, 0, 0, 0\right\} \tag{26}$$

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- Kinetic dilaton and axion domination

$$\left\{-\frac{3}{2\tilde{\alpha}_\psi}, 0, \pm\frac{\sqrt{2\tilde{\alpha}_\psi^2 - 9}}{2\tilde{\alpha}_\psi}, \pm\frac{1}{\sqrt{2}}, 0, 0\right\} \tag{27}$$

- Kinetic dilaton and perfect fluid domination

$$\left\{\frac{2\tilde{\alpha}_\gamma}{3(\gamma-2)}, 0, 0, 0, -\frac{4\tilde{\alpha}_\gamma^2 - 36 + 36\gamma - 9\gamma^2}{9(\gamma-2)^2}, 0\right\} \tag{28}$$

- Kinetic dilaton, potential axion and perfect fluid domination

$$\left\{-\frac{3\gamma}{2(\tilde{\alpha}_\psi - \tilde{\alpha}_\gamma)}, 0, 0, \pm\frac{\sqrt{4\tilde{\alpha}_\gamma^2 - 9\gamma^2 - 4\tilde{\alpha}_\psi\tilde{\alpha}_\gamma + 18\gamma}}{2(\tilde{\alpha}_\psi - \tilde{\alpha}_\gamma)}, -\frac{2\tilde{\alpha}_\psi\tilde{\alpha}_\gamma + 2\tilde{\alpha}_\psi^2 - 9\gamma}{2(\tilde{\alpha}_\psi - \tilde{\alpha}_\gamma)^2}, 0\right\} \tag{29}$$

- Kinetic dilaton, kinetic axion and perfect fluid domination

$$\left\{\frac{3(\gamma-2)}{2(\tilde{\alpha}_\gamma + \tilde{\alpha}_\psi)}, 0, \pm\frac{\sqrt{4\tilde{\alpha}_\gamma^2 - 36 + 36\gamma - 9\gamma^2 + 4\tilde{\alpha}_\psi\tilde{\alpha}_\gamma}}{2(\tilde{\alpha}_\gamma + \tilde{\alpha}_\psi)}, 0, \frac{\tilde{\alpha}_\psi}{\tilde{\alpha}_\psi + \tilde{\alpha}_\gamma}, 0\right\} \tag{30}$$

- Scaling solution for dilaton and perfect fluid

$$\left\{\frac{3\gamma}{2(\tilde{\alpha}_\gamma + \tilde{\lambda})}, \pm\frac{\sqrt{4\tilde{\alpha}_\gamma^2 - 9\gamma^2 + 4\tilde{\lambda}\tilde{\alpha}_\gamma + 18\gamma}}{2(\tilde{\alpha}_\gamma + \tilde{\lambda})}, 0, 0, \frac{2\tilde{\lambda}\tilde{\alpha}_\gamma + 2\tilde{\lambda}^2 - 9\gamma}{2(\tilde{\alpha}_\gamma + \tilde{\lambda})^2}, 0\right\} \tag{31}$$

- Kinetic dilation, perfect fluid and cosmological constant domination

$$\left\{\frac{3\gamma}{2\tilde{\alpha}_\gamma}, 0, 0, 0, \frac{-9\gamma}{2\tilde{\alpha}_\gamma^2}, \pm\frac{\sqrt{4\tilde{\alpha}_\gamma^2 + 18\gamma - 9\gamma^2}}{2\tilde{\alpha}_\gamma}\right\} \tag{32}$$

All the fixed points above are unstable under small perturbations, but the physical system can reach them if given the appropriate intitial conditions. In other words, the universe can go through any of the above solutions, and they may be detected, if realistic, in cosmological observations.

A stable point is $\{0, 0, 0, 0, 0, \pm 1\}$, which corresponds to *cosmological constant domination*. This is not surprising, as a cosmological constant corresponds also to the extreme case of a perfect fluid with an equation of state $\gamma = 0$.

What is surprising is the existence of the points

$$\left\{0, \pm\sqrt{\frac{\tilde{\alpha}_\psi}{\tilde{\alpha}_\psi + \tilde{\lambda}}}, 0, \pm\sqrt{\frac{\tilde{\lambda}}{\tilde{\alpha}_\psi + \tilde{\lambda}}}, 0, 0\right\} \tag{33}$$

which could be dubbed *potential dilaton and axion domination.* These are stable under the conditions ($\tilde{\lambda} < 0$ and $\frac{9}{8\tilde{\lambda}} \leq \tilde{\alpha}_\psi \leq 0$), or ($\tilde{\lambda} > 0$ and $0 < \tilde{\alpha}_\psi \leq \frac{9}{8\tilde{\lambda}}$).

Notice that the scalar field potentials dominate the universe, under the condition that $\tilde{\alpha}_\psi \sim 1/\tilde{\lambda}$, which represents a very special kind of universe, where the axion coupling constant is of the order of the inverse of the potential coupling constant.

4. General discussion

We have shown the simplest cases for the selection of the dilaton and axion potentials in which the equations of motion for a homogeneous and isotropic universe can be written as an autonomous system. This form of the equations eased the analysis of the interesting solutions such a universe can go through.

The dilatonic field acts as a kind of quintessence field with an exponential potential, but its couplings to the different matter fields make its evolution to differ significantly from the standard case [11]. For instance, we see that the typical scalar field domination and the scaling solution are part of the fixed points in of the dynamical system. However, those points are unstable, though it is not clear in our analysis if such instability is intrinsic or it is imposed by the presence of the cosmological constant.

We have seen that in the generic case, the dilaton field is subdominant in the presence of a cosmological constant, unless the axion coupling constant is of the order of less than the inverse of the dilaton potential coupling constant. In that case the universe reaches a condition where the axion and dilaton potentials dominate the universe. This behavior seems to be generic in the dilaton-axion theory of relativity.

There is one point that deserves a more careful study we expect to tackle in a future publication. The dynamical system we studied does not, and cannot, have a fixed point corresponding to perfect fluid domination. The reason for this is the non-zero dilatonic coupling to the perfect fluid, α_γ, see Eq.(12).

Such non-existence of a perfect fluid domination fixed point, if generic, can be a serious failure of the model, as the standard Big Bang model requires the existence of well defined radiation and matter domination eras. As said before, it is to be investigated whether this is a strong failure or not of any string-inspired dark energy model.

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