Cosmic Acceleration from Topological Considerations

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In this work we explore the possibility that the dynamics of the universe can be reproduced choosing appropriately the global topology of the cosmos. We explore two concentric three-dimensional spherical branes immersed in a five-dimensional space-time. Before to the collision, in the interior sphere there exist only a spin-zero fundamental field (scalar field), in the exterior one there exist only fundamental spin-one interactions and spin-two interactions in the bulk. In this model, like in the Ekpyrotic, the Big Bang is caused for the collision of the branes and generate all the fields predicted by the standard model in the exterior brane (our universe). In the interior brane the scalar field behaves like scalar field dark matter. We discuss two different regimens where the energy density and the brane tension are compared, with the aim to obtain the dynamics of the universe after and before the collision. Finally we discuse the perturbations in the modified Einstein equations of the scalar field dark matter in the inner brane and the consequence in the high energy universe dynamics and the corrections in the standard general relativity.

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I. INTRODUCTION.

We are living exiting times in cosmology, the two main components of the universe were discovered only recently and the nature of them remain till now unknown. This is a time to propose candidates to explain the nature of the matter of the universe. The last data arising from observations confirm that our universe is accelerating due to the existence of some unknown type of energy and also requires the existence of an unknown field that permeates the universe and dictates the formation and evolution of the structures at large scales. Both dark components of the universe are not predicted by the standard model or the general theory of relativity eather. This opens the possibility of extend these theories to limits beyond the current ones and even to formulate new paradigms which predicts new physics, under the condition that in the appropriated limits they reproduce the current standard paradigms. At this moment the best accepted candidate to be the dark energy of the universe is the cosmological constant Λ. It is well-known that the observations of the Cosmic Microwave Background Radiation (CMB) and galaxies surveys fit very well with a small value for Λ but there exist some problems with the interpretation. Exist other proposals where the space-time contains subspaces embedded in a higher dimensional one, for example the branes theory [7], the DGP model [4], Ekpyrotic [8] or cyclic universe [6] in which it is proposed the existence of a four dimensional manifolds embedded in a five dimensional bulk. It is important to mention that the last two models [8] and [6] the branes move through the bulk and collide against each other giving origin to the Big Bang. The main goal of this work is to explore the possibility that the dynamics of the universe follows from its global topology and answer the question: can a initial global topology explain the actual state of our universe? For doing so there are different ways, but the idea here is the following:

1. We start with two three dimensional branes embedded in a five-dimensional space-time, where the five-dimensional Einstein equations with cosmological constant, follow. For facility we will suppose that the branes are three dimensional concentric and spherically symmetric, but the results will be valid for any kind of branes.

2. We assume that the interior brane has the scale factor a_1 and the exterior one has the scale factor a_2, with $a_1 < a_2$.

3. The matter content of the interior brane is of a spin-zero field, i.e., a scalar field with a very small mass.

4. The matter content of the exterior brane are spin-one fields.

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5. The exterior space-time, the bulk, is surrounded by a vacuum such that its spectation value is zero but exist a pure geometrical 5-D cosmological constant.

With these hypothesis it follows:

1. The scalar field content of the interior brane provokes that this brane inflates on the beginning, even when the scalar field has a very small mass, and collides with the exterior one. This inflationary period causes that the quantum fluctuations of the scalar field on this brane grow very fast. From the collision follows the big bang like in the Ekpyrotic models.

2. After the inflationary period the scalar field on the interior brane behaves as dark matter with a small mass [13].

3. The two branes expand together, the exterior brane behaves as our universe with the physics of the standard model of particles. In the interior brane the scalar field fluctuations grow and provoke the potential wells that give rise to the structure of the universe.

The problem of the model presented in [13] is the reheating period, but here the reheating is not necessary because the heat is due to the collision, as in the Ekpyrotic model. The interior brane contains the scalar field that influence the dynamics of our universe but obviously it is not possible to detect them, as it is the case of the dark matter. Thus, in this model, the dark matter field is in the interior brane being this the reason why we are not able to detect it.

II. THE MODEL.

In this section we write the most important feature of the model in a general way. Suppose two concentric 3-dimensional branes embedded in a 5-dimensional bulk. From this we use natural units ($c = \hbar = 1$). The shape of the action to model this physical structure is given by

$$S[G,g] = \int dX^5 \sqrt{-G}\, m^3_{(5)} \left(R_{(5)} + \Lambda\right) - \sum_{\pm} \int_{\pm} dx^4 \sqrt{-g^{\pm}} \left(2m^3_{(5)} K^{\pm} + \nabla^{\pm}_{matter}\right), \tag{1}$$

being $\pm$ the exterior or interior region of the brane respectively, G is the determinant of the five-dimensional (5D) metric and g the determinant of the four-dimensional (4D) one, $m_{(5)}$ is the 5D Planck mass, $R_{(5)}$ is the 5D scalar curvature, K is the extrinsic curvature and Λ is the 5D cosmological constant. Following the steps of [1] the equations of motion of the brane can be expressed as

$$[K]\, g_{\mu\nu} - [K_{\mu\nu}] = k_{(5)} \widetilde{T}_{\mu\nu}, \tag{2}$$

where $[K_{\mu\nu}] = K^{+}_{\mu\nu} - K^{-}_{\mu\nu}$ and $\widetilde{T}_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{k}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)$. But from here up we suppose that there is no curvature on the brane, hence $\widetilde{T}_{\mu\nu} = T_{\mu\nu}$ is the regular momentum-energy tensor. On the other hand, the bulk can be described with the metric of Anti de Sitter-Schwarzschild (AdS-$S_{(5)}$) in the form [3]

$$ds^2_{(5)} = -f(a)_{\pm} dt^2 + \frac{1}{f(a)_{\pm}} da^2 + a^2 d\Omega^2, \tag{3}$$

Now we solve the 5D Einstein equations $R_{(5)AB} = \Delta G_{AB}$ with $\Delta = \frac{4\Lambda^{\pm}}{6}$ $(A, B = 1, \cdots, 5)$ for the function $f(a)_{\pm}$, we obtain

$$f(a)_{\pm} = k - \frac{2M^{\pm}}{m^3_{(5)} a^2} - \frac{\Lambda^{\pm}}{6} a^2, \tag{4}$$

where $\frac{2M^{\pm}}{m^3_{(5)}}$ is the mass parameter of the AdS-$S_{(5)}$ black hole [6] and $\frac{\Lambda^{\pm}}{6}$ is the 5D cosmological constant in the bulk, with " $+$ " and " $-$ " corespond, respectively, to the brane exterior and interior. On the other hand, if we consider the location of the 3-brane by $t = t(\tau)$, $a = a(\tau)$ parameterized by the time τ in the brane, we have the induced four dimensional metric given by

$$ds^2_{(4)} = -d\tau^2 + a^2(\tau)d\Omega^2, \quad (5)$$

The no null $K_{\mu\nu}$ can be written as

$$K^{\pm}_{tt} = -\frac{\left(\ddot{a} + \frac{1}{2}\frac{\partial A(a)_\pm}{\partial a}\right)}{\left(\dot{a}^2 + A(a)_\pm\right)^{\frac{1}{2}}}, \quad (6)$$

$$K^{\pm\chi}_{\chi} = K^{\pm\theta}_{\theta} = K^{\pm\varphi}_{\varphi} = \frac{\left(\dot{a}^2 + A(a)_\pm\right)^{\frac{1}{2}}}{a}. \quad (7)$$

If we use the equation (2) we can found the next equation

$$\left(\dot{a}^2 + f(a)_-\right)^{\frac{1}{2}} - \left(\dot{a}^2 + f(a)_+\right)^{\frac{1}{2}} = \frac{a}{3}k_{(5)}\rho, \quad (8)$$

where we have suppose that the matter content of the branes can be represented as fluids with density ρ [1]. We will use the last equations to find an interpretations of the model.

III. THE MODIFIED FRIEDMANN EQUATIONS.

In this model we distinguish three vacuum regions, the first one inside the interior brane, the second one between the branes and the third region surrounds the exterior. Between the branes there exists a huge spectation value for the vacuum, like in the two flat parallel plates with opposite charges in electrodynamics. The analogy is as follows, for two flat parallel plates with opposite charges, in the inner region there exist a electric field induced by the charged flat regions while in the surrounded region the electric field vanishes. This analogy with the electrodynamic problem gives an extreme difference between the spectation value for the vacuum in the interior and zero in the exterior region (at least in a flat geometry). For the interior region of the branes (Region I) equation (4) becomes

$$f(a)_- = k - \frac{\Lambda_1}{6}a_1^2, \quad (9)$$

since there do not exist a gravitational potential inside of the most internal brane, like a spherical conductor with a electrical field. But in the second region, the equation (4) can be written as

$$f(a)_+ = k - \frac{2M_1}{m^3_{(5)}a_1^2} - \frac{\Lambda_2}{6}a_1^2, \quad (10)$$

where M_1 is the mass of the interior brane. If we substitute the equations (9) and (10) into equation (8), we have

$$\frac{\dot{a}_1^2}{a_1^2} + \frac{k}{a_1^2} = \frac{k^2_{(5)}\rho_1^2}{36} + \frac{\Lambda_1+\Lambda_2}{12} + \frac{M_1}{m^3_{(5)}a_1^4} + \frac{(\Lambda_2-\Lambda_1)^2}{16\rho_1^2 k^2_{(5)}} + \frac{9M_1^2}{m^6_{(5)}k^2_{(5)}\rho_1^2 a_1^8} + \frac{3M_1(\Lambda_2-\Lambda_1)}{2m^3_{(5)}a_1^4 k^2_{(5)}\rho_1^2}. \quad (11)$$

On the other hand, inside of the exterior brane (Region II) we have

$$f(a)_- = k - \frac{2M_1}{m^3_{(5)}a_2^2} - \frac{\Lambda_2}{6}a_2^2, \quad (12)$$

finally outside of the exterior brane, the total mass is $M = M_1 + M_2$. Therefore we have

$$f(a)_+ = k - \frac{2(M_1+M_2)}{m^3_{(5)}a_2^2} - \frac{\Lambda_3}{6}a_2^2. \quad (13)$$

Again we substitute the equations (12) and (13) into the equation (8), to obtain

$$\frac{\dot{a}_2^2}{a_2^2}+\frac{k}{a_2^2}=\frac{k_{(5)}^2\rho_2^2}{36}+\frac{\Lambda_3+\Lambda_2}{12}+\frac{2M_1+M_2}{m_{(5)}^3a_2^4}+\frac{(\Lambda_3-\Lambda_2)^2}{16\rho_2^2k_{(5)}^2}+\frac{9M_2^2}{m_{(5)}^6k_{(5)}^2\rho_2^2a_2^8}+\frac{3M_2(\Lambda_3-\Lambda_2)}{2m_{(5)}^3a_2^4k_{(5)}^2\rho_2^2}.$$

One can see (11) and (14) that there is no direct interaction between the brane 1 and brane 2 because one of them behaves like a stealth brane [1]. Expressions (11) and (14) are the Friedman equations on the interior and exterior branes respectively. In both equations, (11) and (14), the third, fourth, fifth and sixth terms are present only if the assumption of $\mathbb{Z}_2$-symmetry is dropped out. Both equations, (11 and 14), are in agreement with Ida et. al [3], except for the third, fourth, fifth and sixth terms which are characteristic of the proposed model. According to the brane world scenario, it is natural to assume that the matter component consists of the brane tension σ and the ordinary fields $\rho_{dm,}$ and ρ_m, such that the brane density are given by

$$\rho_1 = \rho_{dm}+\sigma, \qquad \rho_2= \rho_m+\sigma, \tag{14}$$
$$P_1 = p_{dm}-\sigma, \qquad P_2 = p_m-\sigma, \tag{15}$$

where ρ_m and P_m denote the energy density and the pressure of ordinary fields like baryons, radiation, neutrinos, ρ_{dm} and P_{dm} denote the energy density and the pressure of dark matter (assuming that it behaves like a scalar field) respectively and σ = constant [3]. Now we assume two scenarios in the branes, the first one corresponds to very big densities and the second one when they are very small in comparison with the tension σ, that is $\rho_i\gg\sigma$ and $\rho_i\ll\sigma$, $i = dm, m$. First we analyse the branes when $\rho_i\gg\sigma$, $i = dm, m$. This scenario corresponds to a very early universe (before to the collision). Further, if we assume the fine tuning conditions on the interior brane like a $\frac{1}{6}\Lambda_1 = \frac{2}{3}\Lambda_{(5)}$, $\frac{1}{6}\Lambda_2 = -\frac{2}{3}\lambda_{(5)}$ with $\kappa_{(4)} = \frac{8\pi}{m_{(4)}^2}$, $\kappa_{(5)} = \frac{8\pi}{m_{(5)}^3}$ and $k_{(5)}^2= 36\frac{\kappa_{(4)}}{3}\frac{1}{2\sigma}$ where $\lambda_{(5)}\sim(10^{18}GeV)^4$ is the order of quantum fluctuations of vacuum predicted by the standard model [2]. In the limit we are considering it follows that

$$\rho_{dm}^2 \sim 2\sigma\rho_{dm}\left(1+\frac{\rho_{dm}}{2\sigma}\right)+\sigma^2,$$

and the energy conservation implies $M_1\equiv -M$, $M_2\equiv M$. Then imposing $\rho_{dm}\gg\sigma$ and proposing $\kappa_{(4)}\sigma\approx\lambda_{(5)}$ we obtain

$$\frac{\dot{a}_1^2}{a_1^2}+\frac{k}{a_1^2}\approx\frac{\kappa_{(4)}^2}{3}\frac{\rho_{dm}^2}{2\lambda_{(5)}}+\frac{\overset{(4)}{\Lambda}_1}{3}+\frac{M}{m_{(5)}^3a_1^4}, \tag{16}$$

with

$$\overset{(4)}{\Lambda}_1 = -\frac{\lambda_{(5)}}{2}. \tag{17}$$

In the same way for the exterior brane we can set the fine tuning conditions $\frac{1}{6}\Lambda_2= -\frac{2}{3}\lambda_{(5)}$, $\frac{1}{6}\Lambda_3 = \frac{2}{3}\Lambda_{(5)}$, with $\kappa_{(4)} = \frac{8\pi}{m_{(4)}^2}$, $\kappa_{(5)} = \frac{8\pi}{m_{(5)}^3}$ and $k_{(5)}^2 = 36\frac{\kappa_{(4)}}{3}\frac{1}{2\sigma}$. Again, if we have set the limit condition and $M_1\equiv -M$, $M_2\equiv M$. In the limit $\rho_m\gg\sigma$ and $\kappa_{(4)}\sigma\approx\lambda_{(5)}$ we obtain

$$\frac{\dot{a}_2^2}{a_2^2}+\frac{k}{a_2^2}\approx\frac{\kappa^2{}_{(4)}}{3}\frac{\rho_m^2}{2\lambda_{(5)}}+\frac{\overset{(4)}{\Lambda}_2}{3}-\frac{M}{m_{(5)}^3a_2^4}, \tag{18}$$

with

$$\overset{(4)}{\Lambda}_2= -\frac{\lambda_{(5)}}{2}. \tag{19}$$

These relations (16) and (18) are the Friedmann equations for the early universe of the model. If ρ_{dm} is the energy density of a scalar field, even with a very small scalar field mass this brane inflates and collides with the exterior one, where the matter content is only spin one particles in early ages. In the same way we analyse the regime $\rho_i \ll \sigma$, $i = dm, m$ for the late universe using the same conditions as in the previous regime. We will use $\rho_i^2 = 2\sigma\rho_i\left(1+\frac{\rho_i}{2\sigma}\right)+\sigma^2, (i = dm, m)$ and $M_1 \equiv -M$, $M_2 \equiv M$. Then imposing $\rho_i \ll \sigma$, $i = dm, m$ and $\kappa_{(4)}\sigma \approx \lambda_{(5)}$ we obtain

$$\frac{\dot{a}_1^2}{a_1^2}+\frac{k}{a_1^2} \approx \frac{\kappa_{(4)}}{3}\rho_{dm}+\frac{\overset{(4)}{\Lambda}_1}{3}+\frac{3M^2}{2m_{(5)}^6\lambda_{(5)}a_1^8}, \tag{20}$$

$$\frac{\dot{a}_2^2}{a_2^2}+\frac{k}{a_2^2} \approx \frac{\kappa_{(4)}}{3}\rho_{m}+\frac{\overset{(4)}{\Lambda}_2}{3}+\frac{3M^2}{2m_{(5)}^6\lambda_{(5)}a_2^8}, \tag{21}$$

where $\overset{(4)}{\Lambda}_{1,2} = \Lambda_{(5)} \sim (10^{-12}GeV)^4$ is the order of classical observed 4-dimensional cosmological constant [2]. These two previous relations (20) and (21) are the Friedmann equations for the late universe and give the dynamics of the two branes respectively.

IV. THE MODIFIED EINSTEIN EQUATIONS IN THE BRANE

In this section we analyze the behavior of the perturbations of the scalar field as dark matter in the inner brane. First we assume that the exterior brane is far enough of the inner brane and for simplicity we suppose the existence of Z_2 symmetry in the bulk. We propose that Einstein′s equations in five dimensions are valid:

$$G_{AB}^{(5)} + \Lambda^{(5)}g_{AB} = k_{(5)}^2 T_{AB}^{(5)}, \tag{22}$$

where $k_{(5)}$ is the five dimensional gravitational constant and $\Lambda^{(5)}$ is the five dimensional cosmological constant. Then using the junction conditions (2) and the Darmoise-Gauss equations we obtain the modified Einstein equation from the view of the brane.

$$G_{\mu\nu} + \Lambda_{(4)}g_{\mu\nu} = k_{(4)}^2 T_{\mu\nu} + k_{(5)}^4 \Pi_{\mu\nu} - \xi_{\mu\nu}, \tag{23}$$

where

$$\Lambda_{(4)} = \frac{\Lambda_{(5)}}{2} + \frac{k_{(5)}^4}{12}\lambda^2, \tag{24}$$

$$k_{(4)}^2 = 8\pi G_N = \frac{k_{(5)}^4}{6}\lambda, \tag{25}$$

$$\begin{aligned} \Pi_{\mu\nu} &= -\frac{1}{4}T_{\mu\alpha}T_\nu^\alpha + \frac{1}{12}TT_{\mu\nu} \\ &+ \frac{1}{24}(3T_{\alpha\beta}T^{\alpha\beta} - T^2)g_{\mu\nu}, \end{aligned} \tag{26}$$

and the conservations law equations can be obtained by $T^\mu_{\nu;\mu} = 0$.

V. SCALAR FIELD DARK MATTER IN THE BRANE AND PERTURBED EINSTEIN FIELD EQUATIONS

In this section we write the scalar cosmological perturbations of the modified Einstein equations. The perturbed space-time metric in the conformal time with signature sign$(- + ++)$ can be written as [25], [28], [29].

$$ds^2 = -a(\tau)^2(1+2\Psi(\tau,\vec{x}))d\tau^2 + a(\tau)^2\delta_{ij}(1-2\phi(\tau,\vec{x}))\,dx^2, \quad (27)$$

where $a(\tau)$ is the scale factor, $\Psi(\tau,\vec{x})$ correspond to the Newtonian potential and $\phi(\tau,\vec{x})$ correspond to the spatial curvature perturbation. We suppose that the scalar field with a potential $V(\Phi) = \frac{1}{2}m_\Phi\Phi^2$ behaves as dark matter [18], [19], [21] where m_Φ is the mass of the scalar field with a mass given approximately by $m_\Phi \backsimeq 10^{-23}\mathrm{eV} \backsimeq 9\times10^{-52}m_{Pl}$ [21]. From the perturbed scalar field $\Phi(\tau,\vec{x}) = \Phi^{(0)}(\tau) + \delta\Phi(\tau,\vec{x})$ we obtain the perturbed energy-momentum tensor as

$$\begin{aligned}
\delta T^0_0 &= -a(\tau)^{-2}(\dot{\Phi}^{(0)}\delta\dot{\Phi} - \Psi\dot{\Phi}^{(0)2}) - V_{,\Phi}\delta\Phi = -\delta\rho, && (28)\\
\delta T^j_i &= a(\tau)^{-2}(\dot{\Phi}^{(0)}\delta\dot{\Phi} - \Psi\dot{\Phi}^{(0)2})\delta^j_i - V_{,\Phi}\delta\Phi\delta^j_i = \delta P, && (29)\\
\delta T^0_i &= -a(\tau)^{-2}(\dot{\Phi}^{(0)}\delta\Phi_{,i}), && (30)
\end{aligned}$$

where $\delta\pi^j_i = 0$ because we are dealing with a scalar field and we consider only a AdS bulk with no black hole mass. In the other hand the quadratic energy momentum tensor is given by

$$\begin{aligned}
\delta\Pi^0_0 &= -\frac{1}{12}(\dot{\Phi}^{(0)2} + 2V^{(0)})(a(\tau)^{-2}(\dot{\Phi}^{(0)}\delta\dot{\Phi} - \Psi\dot{\Phi}^{(0)2}) - V_{,\Phi}\delta\Phi), && (31)\\
\delta\Pi^0_i &= -\frac{1}{12}(\dot{\Phi}^{(0)2} + 2V^{(0)})(a(\tau)^{-2}(\dot{\Phi}^{(0)}\delta\Phi_{,i})), && (32)\\
\delta\Pi^j_i &= \frac{1}{12}(\frac{3}{4}\dot{\Phi}^{(0)4} - V^{(0)2} + \dot{\Phi}^{(0)2}V^{(0)} + 2\dot{\Phi}^{(0)2}\delta\rho + (\dot{\Phi}^{(0)2} + 2V^{(0)})\delta P)\delta^j_i, && (33)
\end{aligned}$$

It follows that the projected Weyl Tensor is

$$\delta\xi^\nu_\mu = 0. \quad (34)$$

this is again because we choose a bulk with AdS topology. The perturbed Klein-Gordon equation [26], [27] is given by

$$\delta\ddot{\Phi} + 2\frac{\dot{a}}{a}\delta\dot{\Phi} - \dot{\Psi}\dot{\Phi} - 3\dot{\Phi}\dot{\phi} + a^2V_{,\Phi\Phi}\delta\Phi + 2a^2\Psi V_{,\Phi} - \bigtriangledown^2\delta\Phi = 0. \quad (35)$$

On the other hand, the perturbed Einstein field equations can be written as

$$\delta G^\nu_\mu + \Lambda_{(4)}\delta^\nu_\mu = k^2_{(4)}\delta T^\nu_\mu + k^4_{(5)}\delta\Pi^\nu_\mu. \quad (36)$$

Putting all the results together and using equation (36) we can write the field equations for the perturbations as

$$\begin{aligned}
6H(\phi_{,0} + H\Psi) - \frac{2}{a^2}\bigtriangledown^2\phi - \Lambda_{(4)} &= -(k^2_{(4)} + \frac{k^4_{(5)}}{12}[a^2\Phi^{(0)2}_{,0} + 2V^{(0)}])(\Phi^{(0)}_{,0}\delta\Phi_{,0} - \Psi\Phi^{(0)2}_{,0} + V_{,\Phi}\delta\Phi), && (37)\\
2(H\Psi + \phi_{,0})_{,i} - a\Lambda_{(4)} &= (k^2_{(4)} + \frac{k^4_{(5)}}{12}[a^2\Phi^{(0)2}_{,0} + 2V^{(0)}])\Phi^{(0)}_{,0}\delta\Phi_{,i}, && (38)\\
2[\phi_{,00} + H(\Psi_{,0} + 2\phi_{,0}) + \\
(2\dot{H} - H^2)\Psi - \frac{1}{3a^2}\bigtriangledown^2(\phi - \Psi)] + \Lambda_{(4)} &= k^2_{(4)}[\Phi^{(0)}_{,0}\delta\Phi_{,0} - \Psi\Phi^{(0)2}_{,0} - V_{,\Phi}\delta\Phi] \\
&+ \frac{k^4_{(5)}}{12}[a^4\Phi^{(0)4}_{,0} - V^{(0)2} + a^2\Phi^{(0)2}_{,0}V^{(0)} \\
&+ 2a^2\Phi^{(0)2}_{,0}[\Phi^{(0)}_{,0}\delta\Phi_{,0} - \Psi\Phi^{(0)2}_{,0} + V_{,\Phi}\delta\Phi] \\
&+ (a^2\Phi^{(0)2}_{,0} + 2V^{(0)})[\Phi^{(0)}_{,0}\delta\Phi_{,0} \\
&- \Psi\Phi^{(0)2}_{,0} - V_{,\Phi}\delta\Phi]], && (39)\\
-\frac{2}{3a^2}(\phi - \Psi)^j_i + \Lambda_{(4)}\delta^j_i &= k^2_{(4)}\delta T^j_i + k^4_{(5)}\delta\Pi^j_i \qquad (i \neq j), && (40)
\end{aligned}$$

and the respective Klein-Gordon equation (35) as

$$\delta\Phi_{,00} + 2H\delta\Phi_{,0} - \Psi_{,0}\Phi_{,0} - 3\Phi_{,0}\phi_{,0} + V_{,\Phi\Phi}\delta\Phi + 2\Psi V_{,\Phi} - \frac{1}{a^2}\bigtriangledown^2\delta\Phi = 0, \tag{41}$$

where H is the Hubble parameter. The relations (37)-(41) are very similar as the corresponding for the scalar field dark matter paradigm, where we know the pertubed equations reproduce the observations quite well [34]. The only one difference is the terms where the fifth dimensional Planck mass $k_{(5)}$ appears. Nevertheless, this mass is extremely small, thus all these terms contribute very few or almost nothing to the structure formation. Then, it is expected that this model reproduce the structure formation observations as well. In other words, the model presented here can reproduce the observed cosmology and the observed structure formation of the universe putting as initial condition the topology of the universe.

VI. DISCUSION AND CONCLUSIONS

The question we are facing here is the following; is it possible that the universe has the dynamics we observe because of its global topology? This question make sense if we start from the most general Einstein equations in vacuum, $R_{\mu\nu} - 1/2\, g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 0$. This equation is completely geometric, there is no matter content in it. The Friedman equation for this case is

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\kappa^2}{3}\Lambda = 0,$$

or, in a more convenient form for our goal here, we can write it as

$$\frac{1}{2}\dot{a}^2 - \frac{\kappa^2}{6}\Lambda\, a^2 = -\frac{1}{2}k,$$

which can be seen as a $(1/2)\dot{a}^2 + V = E$ dynamical equation. We observe that even when there is no matter, the space-time contains a dynamic. Even if there is no cosmological constant $\Lambda = 0$, it is enough that $k \neq 0$ to have a dynamical universe. In other words, it is sufficient that at the beginning, the global topology is fixed to have a specific dynamic. In this work we choose a special global topology of the universe, two concentric branes, and some content of matter interactions in each region of the universe: fundamental interactions of spin zero confined in the interior brane, fundamental interactions of spin one confined in the exterior brane (the brane that will lead to the standard model) and fundamental spin two interaction in the bulk. We find that the spin zero interactions inflate the interior brane, making this brane collide with the exterior one, the branes heat because of a small interactions constant between the spin zero particles and the spin one particles. After the collision, both branes expand together, the scalar field fluctuations build potential wells which are seen as dark matter in the exterior brane. In the exterior brane the collision provoke the existence of a particle with fractional spin and spin one, giving the zoo of particles predicted by standard model. The universe expands as we see it, but the particles of the standard model cannot interact with the scalar field particles because they are confined in the interior brane and the standard model ones are confined in the exterior one. The five-dimensional cosmological constant Λ is pure geometrical and can be seen as a second free constant of this model. The extremely difference between the spectation values in the interior and exterior values can be explained using an analogy with the electrodynamics. If the fields in the 5D bulk produced by the branes, behaves like a electrodynamic field, remember that for the weak field, linearised Einstein equations are analogous to the electromagnetic one, then there exist a hugh spectation value inside of the region between the two branes which is associated with λ, while in the exterior region the spectation value is zero. During the collision, the primordial structure in the exterior brane is created like in a Epkyrotic model, in the interior brane the dark matter will play the fundamental role with the structure formation of the exterior brane through the gravitational force. During the collision it is impossible to analyse all the interactions with the classical theory of general relativity due to the quantum-gravitational effects and the interactions between the fields that permeates both branes. In future works it is necessary to analyse the perturbations in the metric to understand the large scale structure, CMB, Sachs-Wolfe effect [10] and other cosmological parameters with the aim of understand the fate of the universe from this brane model point of view.

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