Structure Formation with Scalar Field Dark Matter

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In this work we calculate the different ingredients needed to obtain the matter density profile of the large-scale structure we observe now a days in a qualitative way. The key difference of the work lays on the fact that we study the formation of structure in the Universe assuming that dark matter can be described by a scalar field (SF) Φ with a quadratic potential $V(\Phi) = m^2\Phi^2/2$. We derive the evolution equations in the linear regime of perturbations, here the scalar perturbations have an oscillating growing mode that could lead to the formation of gravitational structures. We then solve the obtained set of equations like a dynamical system in which we introduce new dimensionless parameters that describe the cosmology of the different studied quantities.

One of the fundamental problems in modern Cosmology is to know the nature of dark matter. The standard model of cosmological structure formation in the Universe is Lambda cold dark matter (ΛCDM), many cosmological observations support it,like the spectrum of CMB,but this paradigm has some problems at galactic scales.

We study Scalar Field Dark Matter (SFDM) in order to investigate if this model is able to explain the formation of structure. SFDM supposes that dark matter is a minimally coupled to gravity real SF with a potential that interacts only gravitationally with the rest of the matter. The motivation of this model is that our SF with quadratic potential can reproduce the cosmological evolution of the Universe, so it behaves like CDM and therefore solving many problems of the standard model. For models that involve SFDM, it is usually assumed that the SF does not show density fluctuations on cluster scales or below. If the mass of the SF does feel the fluctuations, we then must take into account non-linear perturbations, that could modify the evolution of the dark matter, and affect the evolution of structure formation. In this work dark matter is a SF that has a potential, $V(\Phi) = \frac{1}{2}m_\Phi^2\Phi^2$, where the mass of the SF is defined as $m_\Phi = V''|_{\Phi=0}$ and dark matter is affected by radiation only through the gravitational potential. If we want to study the anisotropies we need to know how the perturbations that act upon the dark matter evolve. The evolution of these fluctuations with specified magnitude are defined by several fundamental parameters,like:the cosmic density Ω,the cosmological constant Λ and the relative contributions of radiation, and densities of dark and visible matter in the universe. We then define the relative fluctuations on the mass density as $\delta \equiv \frac{\delta\rho}{\rho}$ where ρ is the mass density in the Universe. We develop the theory and the numerical simulations to obtain such results, and making reference to the non-perturbed model (without fluctuations) as the *background* from which we evolve the small perturbations to what we see today.

We use the Friedmann-Lemaître-Robertson-Walker (FLRW) metric with scale factor $a(t)$. Our background Universe is composed by SFDM (Φ_0) endowed with a potential $V \equiv V(\Phi_0)$, radiation (z), neutrinos (ν), baryons (b), and a cosmological constant (Λ) as dark energy. From the energy-momentum tensor for a SF, the energy density and pressure are given respectively by

[*] Part of the Instituto Avanzado de Cosmología (IAC) collaboration http://www.iac.edu.mx/

CP1256, *Gravitational Physics: Testing Gravity from Submillimeter to Cosmic Scale*
edited by H. A. Morales-Técotl, L. A. Ureña-López, R. Linares-Romero, and H. H. García-Compeán

$$T^0_0 = -\rho_{\Phi_0} = -\left(\frac{1}{2}\dot{\Phi}_0^2 + V\right), \tag{1}$$

$$T^i_j = p_{\Phi_0} = \left(\frac{1}{2}\dot{\Phi}_0^2 - V\right)\delta^i_j, \tag{2}$$

where the dots stand for the derivative with respect to the cosmological time and δ is the Kronecker delta. Here the Equation of State (EoS) for the SF is $p_{\Phi_0} = \omega_{\Phi_0}\,\rho_{\Phi_0}$.

From the Einstein's and Klein-Godrdon equations we have that the Universe can be described by (in units $c = \hbar = 1$)

$$\begin{aligned}
\dot{H} &= -\frac{\kappa^2}{2}\left(\dot{\Phi}_0^2 + \frac{4}{3}\rho_z + \frac{4}{3}\rho_\nu + \rho_b\right), && (3)\\
\ddot{\Phi}_0 &+ 3\,H\dot{\Phi}_0 + V_{,\Phi_0} = 0, && (4)\\
\dot{\rho}_i &+ 4\,H\rho_i = 0, && (5)\\
& && (6)
\end{aligned}$$

$i = z,\, \nu,\, b$, being $\kappa^2 \equiv 8\pi G$, $H \equiv \dot{a}/a$ the Hubble parameter and the commas stand for the derivative with respect to SF. To solve the system of equations (6), the following dimensionless variables are defined

$$\begin{aligned}
x &\equiv \frac{\kappa}{\sqrt{6}}\frac{\dot{\Phi}_0}{H}, \quad u \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{V}}{H}, \quad z \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_z}}{H},\\
\nu &\equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_\nu}}{H}, \quad b \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_b}}{H}, \;\; l \equiv \frac{\kappa}{\sqrt{3}}\frac{\sqrt{\rho_\Lambda}}{H}.
\end{aligned} \tag{7}$$

Here the mass, m, of the ultra-light boson particle is $\sim 1 \times 10^{-23}$eV. Using these variables, the set of equations become;

$$\begin{aligned}
x' &= -3\,x - su + \frac{3}{2}\Pi\,x, \quad u' = sx + \frac{3}{2}\Pi\,u, && (8)\\
& && (9)\\
z_i' &= \frac{3}{2}\left(\Pi - \gamma_i\right)\,z_i, \quad s' = s_0\,s^{-k}, && (10)\\
& && (11)
\end{aligned}$$

$z_i = z,\, \nu,\, b$; $\gamma_z = \gamma_\nu = 4/3$, $\gamma_b = 1$, where the prime denotes a derivative with respect to the e-folding number $N = \ln a$, and Π is defined as

$$-\frac{\dot{H}}{H^2} = \frac{3}{2}(2x^2 + b^2 + \frac{4}{3}z^2 + \frac{4}{3}\nu^2) \equiv \frac{3}{2}\Pi. \tag{12}$$

The variable $s \equiv C_0/H$ is introduced as a control parameter for the dynamics of H, C_0 is a constant and in equation (11) s_0 is a constant and the exponent k is ≤ 0 [1].

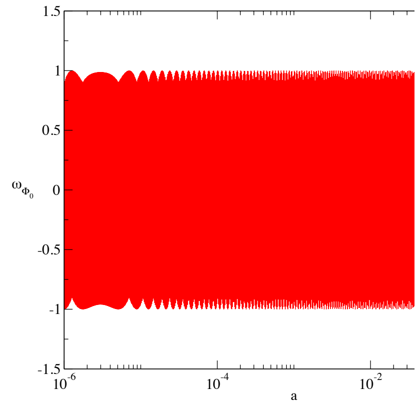

FIG. 1: Evolution of the Φ^2-dark matter equation of state for the background Universe.

Then the EoS of the SF can be written as $\omega_{\Phi_0} = \frac{x^2-u^2}{\Omega_{\Phi_0}}$. Since ω_{Φ_0} is a function of time, if its temporal average tends to zero, this would imply that Φ^2-dark matter can mimic the EoS of CDM.

The system of equations (11) and (12) are solved numerically for the background Universe with a four order Adams-Bashforth-Moulton method. The best estimates from WMAP values [2] are taken as initial conditions $\Omega_\Lambda^{(0)} = 0.73$, $\Omega_{DM}^{(0)} = 0.22994$, $\Omega_b^{(0)} = 0.04$, $\Omega_z^{(0)} = 0.00004$, $\Omega_\nu^{(0)} = 0.00002$ and the exponent $k = 0$. Figure [5] of reference [4] shows the numerical evolution of the density parameters in our model. At early times, radiation dominates the evolution of the Universe. Later on, the Universe has an epoch where the energy density of radiation is equal to the dark matter density, at z_{eq}, then dark matter begins to dominate. The recombination era in Φ^2-dark matter model occurs at $z \sim 1000$. At later times, the cosmological constant dominates the dynamics at $z_\Lambda \sim 0.5$. The behavior is the same as in the ΛCDM model.

Figure 1 shows the evolution of the EoS for the SF. Although the EoS varies with time (oscillations), the temporal average, $\langle\omega_{\Phi_0}\rangle$, tends to zero. Therefore, Φ_0^2 behaves as CDM at cosmological scales [3].

Now we compute the growth of the Φ^2-dark matter overdensities $\delta\rho_\Phi$ at the stage when the density contrast $\delta \equiv \delta\rho_\Phi/\rho_{\Phi_0}$ is much smaller than unity. After introducing the perturbed metric tensor in the FLRW background, we only consider scalar perturbations. We then have a perturbed scalar metric to first order in terms of four scalars ψ (lapse function), ϕ (gravitational potential), B (shift) and E (anisotropic potential), from this we get the most general perturbed line element

$$ds^2 = a(\eta)^2[(1+2\psi)d\eta^2 + 2B_{,i}\,d\eta dx^i + [(1-2\phi)\delta_{ij} + 2E_{,ij}]dx^i dx^j], \tag{13}$$

where δ_{ij} is the background metric, and equation (13) is given in conformal time η.

We consider the evolution equations within the Newtonian gauge that is defined when $B = 0$, and $E = 0$ and applies only to the scalar modes of the metric perturbations, implying that vector and tensorial modes are not taken into account.

Now we derive the perturbed evolution equations for the scalar perturbation $\delta\Phi$ and the scalar potential ψ. From the perturbed energy-momentum tensor, we get

$$\delta T^0_0 \;=\; -\delta\rho_\Phi = -(\dot\Phi_0\delta\dot\Phi - \dot\Phi_0^2\psi + V_{,\Phi_0}\,\delta\Phi), \tag{14}$$

$$\delta T^0_i \;=\; -\frac{1}{a}(\dot\Phi_0\delta\Phi_{,i}\,), \tag{15}$$

$$\delta T^i_j \;=\; \delta p_\Phi = (\dot\Phi_0\delta\dot\Phi - \dot\Phi_0^2\psi - V_{,\Phi_0}\,\delta\Phi)\delta^i_j. \tag{16}$$

Here again the dots denote differentiating with respect to cosmological time t.

In the Newtonian gauge, the metric tensor $g_{\mu\nu}$ becomes diagonal and from this, in the trace of Einstein's equations, ψ plays the role of the gravitational potential $\psi - \phi = 0$. Usually this equation contains a term of anisotropic stress, which vanishes in the case of a SF.

In order to solve the evolution equations, we will turn to Fourier's space, where k will denote the wave number defined as $k = 2\pi/\lambda$, and λ will be the length scale of the perturbation. Altogether, the perturbed Eintein's equations $\delta G^i_j = \kappa^2\delta T^i_j$ to first order for a SF in the Newtonian gauge read

$$8\pi G(3H\dot\Phi_0\delta\Phi_k) + \frac{2k^2}{a^2}\phi \;=\; -8\pi G(\dot\Phi_0\delta\dot\Phi_k - \phi\dot\Phi_0^2 + V_{,\Phi}\,\delta\Phi_k), \tag{17}$$

$$2(H\phi + \dot\phi) \;=\; 8\pi G\dot\Phi_0\delta\Phi_k, \tag{18}$$

$$2[\ddot\phi + 3H\dot\phi + (2\dot H + 3H^2)\phi] \;=\; 8\pi G(\dot\Phi_0\delta\dot\Phi_k - \phi\dot\Phi_0^2 - V_\Phi\delta\Phi_k), \tag{19}$$

and the Klein-Gordon equation transforms into

$$\delta\ddot\Phi_k + 3H\delta\dot\Phi_k - 4\dot\phi\dot\Phi_0 + V_{,\Phi\Phi}\,\delta\Phi_k + 2\phi V_{,\Phi} + \frac{k^2}{a^2}\delta\Phi_k = 0. \tag{20}$$

Eq. (18) makes reference to the evolution of the energy density, eq. (19) to the evolution of the gravitational potential and finally, eq. (21) refers to the perturbations over the SF. From the time derivative of (14), we have

$$\delta\dot\rho_\Phi \;=\; (\ddot\Phi_0 + V_\Phi)\delta\dot\Phi_k + (\delta\ddot\Phi_k + V_{,\Phi\Phi}\,\delta\Phi_k - \dot\Phi_0\dot\phi)\dot\Phi_0 - 2\phi\dot\Phi_0\ddot\Phi_0. \tag{21}$$

Performing a Fourier transformation to the above equation, then combining it with the unperturbed Klein-Gordon equation and equation (21), and finally with the use of equation (19) we arrive at

$$\delta\dot\rho_\Phi = -6H\dot\Phi_0\delta\dot\Phi_k + 6\phi\dot\Phi_0^2 H - \frac{2k^2}{a^2\kappa^2}(H\phi + \dot\phi) + 3\dot\phi\dot\Phi_0^2. \tag{22}$$

So,

$$\delta\dot\rho_\Phi = -3H(\delta p_\Phi + \delta\rho_\Phi) - \frac{2k^2}{a^2\kappa^2}(H\phi + \dot\phi) + 3\dot\phi\dot\Phi_0^2. \tag{23}$$

This last equation can be expressed in terms of the density contrast making use of the equations from the background,

$$\dot\delta_\Phi + 3H(\frac{\delta p_\Phi}{\delta\rho_\Phi} - \omega_\Phi)\delta_\Phi = 3\dot\phi(1 + \omega_\Phi) - G_\phi, \tag{24}$$

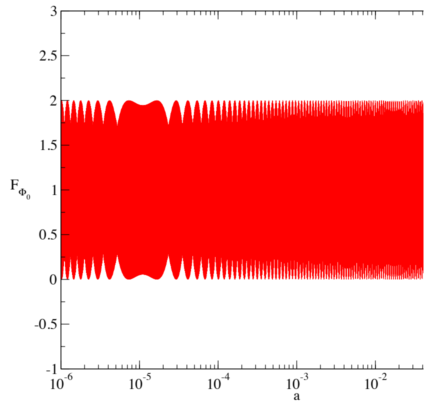

FIG. 2: Evolution of the average of the F_Φ term involved on the right-hand side of equation (25).

where $G_\phi = \frac{2k^2}{a^2\kappa^2}\frac{\dot{\phi}+H\phi}{\rho_{\Phi_0}}$.

Now, taking the average of equation (25) for the radiation and matter dominated eras the first term in the right-hand-side goes as $\langle\delta p_\Phi\rangle / \langle\delta\rho_\Phi\rangle \approx 0$, see [3], also $\langle\omega\rangle_\Phi = 0$ and finally since we are using post-newtonian approximation, G_ϕ can be neglected (see Fig. 2). So, the SF Φ changes completely to standard CDM and so do its perturbations.

When computing the temporal average of F_Φ and G_ϕ of equation (25), we obtain that $\langle F_\Phi\rangle$ tends to one. On the other hand, $\langle G_\phi\rangle$ drops to zero, meaning that the second term dissapears. Together with the fact that $\langle\omega_{\Phi_0}\rangle \to 0$, we find that eq. (25) resembles the equation for the density contrast as in the standard CDM models (see [1]).

We have investigated the evolution of the perturbations in the scalar field Φ around the universe.The cosmological limits here imposed are model dependent and therefore relay on the assumption of our theoretical model of structure that,even if in agreement with current data,may need further key ingredients and analysis to explain mysteries and inconsistencies such as dark energy.

With this important facts at hand, we conclude that the different perturbations on the scalar field, depend deeply on what we assume for our initial conditions of the matter that now dominate the universe. However, further laboratory experiments will certainly test our cosmological results

This work was partially supported by CONACyT México, under grants 49865-F and I0101/131/07 C-234/07, Instituto Avanzado de Cosmologia (IAC) collaboration.

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