Cosmic Braneworld and Ultralight Bosonic Dark Matter

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Abstract. In this paper we study a gas of ultra light bosons as a candidate for dark matter in a higher dimensional framework. We briefly introduce the formalism of brane theory (BN) and and relevant aspects of ultra light bosonic dark matter (ULBDM) with the aim to combine both ideas in a single model. In this model, we assume two branes (visible and hidden brane) inside of the bulk and study the behavior of ULBDM in the hidden brane for an observer in the visible one. We reformulate the Friedmann equations for our Universe and the Friedmann equation concerning to the hidden brane. Thus, we can stablish the mutual interaction between the branes through the Hubble parameter, which measures the rate of expansion of our Universe. Finllay, we discuss that the hypothesis of ULBDM in the context of the braneworld theory is an interesting idea regarding the nature of dark matter.

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INTRODUCTION.

Recently, astronomical measurements point to the need of new components in the cosmological models. They are not predicted neither by the standard model of particles nor the theory of general relativity. One possibility is the existence of *dark* components of matter which govern the dynamics of the Universe, being more than the 95% of the total matter.

Abundant new ideas seek to explain the nature of dark energy (DE) and dark matter (DM). The most accepted are the cosmological constant (Λ) for DE and weakly interacting massive particles (WIMPs) for DM. However, these candidates contain some inconsistencies of both theoretical and observational kind [1], opening the alternative to propose new and somewhat more radical explanations of these dark components.

Some authors reformulate the basic ideas claiming the question: What is the real topology of the universe and the nature of dark matter? New ideas have been proposed attempting to answer this question. For instance, in the work of developed in [2]-[4], it is considered a 4-dimensional space-time (the brane) immersed in a 5-dimensional manifold (the bulk). This new degree of freedom provides non local effects through the Weyl tensor and quadratic corrections to the energy density of matter.

The addition of another brane in the topology (hidden brane) generates constraints in the energy-momentum tensor for an observer in our Universe, obtaining the dynamics related with the expansion rate of the visible brane [5].

For DM, an interesting alternative is a model of ultralight particles in a Bose-Einstein (BE) condensate (BEC) state. While the thermal fraction scales as radiation ($\sim a^{-4}$) [1], the BEC fraction does as matter ($\sim a^{-3}$), allowing for formation of large scale structures.

More precisely, these models consist in the assumption of some species of ultralight bosonic dark matter (ULBDM) in local thermodynamic equilibrium (LTE) with the primeval fireball [6]. At early stages, the temperature of the Universe is extremely high ($>$ MeV), thus, the bosonic gas of ULBDM lies in the relativistic regime $m/T << 1$. Nevertheless, BE statistics allows a phase transition to a condensate state, whose dynamics is described by the momentum distribution f given by,

$$f(\mathbf{p}) = n_{bec}^{(d)}\delta^3(\mathbf{p}) + \frac{1/h^3}{e^{(E-\mu)/T_d} - 1} \tag{1}$$

In this paper and for simplicity, we will assume that the BEC is already formed when the bosonic gas decouple from the thermal bath. Thus, at decoupling, $n_{bec}^{(d)} = n_{bec}(t)a^{-3}(t)$ is the number density of particles on the ground state, $T_d = T(t)a^{-1}(t)$ is the temperature, δ^3 is the 3-dimensional Dirac delta function, $\hbar$ is the Planck constant and μ is the chemical potential, whose magnitude, under BE condensation approaches to the mass of the particle $\mu \to m$. Notice that the singular (or "blow-up") term is the signature of BE condensation in phase space; the narrow distribution of momenta describes the coherent behavior of the BEC fraction of particles. The absence of the Planck constant and internal degrees of freedom in the first term expresses the macroscopic manifestation of the condensed fraction.

In this paper we focus on cosmological scales, assuming 5-dimensional topologies. We introduce two branes (visible and hidden brane) inside of the bulk and study the behavior of ULBDM in the hidden brane for an observer in the visible one, assuming a general equation of state for ULBDM. We work in units in which $c = h = k_B = 1$, unless explicitly written.

The article is organized as follows. In sections and we briefly review some key points of the brane theory formalism, necessary for our forward discussion. Basic physical quantities of ULBDM are calculated in section to be generalized in section , into the brane theory framework. Finally, in section we summarize and briefly discuss the framework and results developed in this preliminar approach.

CONSTRAINT EQUATIONS

In this section we formulate the basic equations, assuming two branes (visible and hidden) embedded in a 5-dimensional manifold. We start writing the 5-dimensional action in the following way

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[1] here $a(t)$ is the expansion scale factor of the Universe

$$S=-\frac{1}{2\kappa_{(5)}^2}\int d^5x\sqrt{-g_{(5)}}R_{(5)}+\int d^5x\sqrt{-g_{(5)}}(\pounds+\pounds_*), \tag{2}$$

where $g_{(5)}$ is the metric, $\kappa_{(5)}$ is the gravitational constant, related to the Planck mass $\kappa_{(5)}^2 \sim M_{(5)}^{-3}$. $R_{(5)}$ is the Ricci scalar, $\pounds$ and $\pounds_*$ are the scalar field Lagrangian of the visible (our universe) and hidden brane, respectively. The Einstein equation can be written in the following way,

$$G_{AB}=\kappa_{(5)}^2(T_{AB}|_{bulk}+\widetilde{T}_{AB}|_{branes}), \tag{3}$$

where $A,B=0,1,2,3,4$. The tensor $T_{AB}|_{bulk}$ is for the bulk and $\widetilde{T}_{AB}|_{brane}=T_{AB}+T_{AB*}$ for the visible brane and the hidden brane, respectively. Following the proposal, it is possible write a general flat metric as

$$ds^2=-n^2(t,y)dt^2+a^2(t,y)\delta_{ij}dx^idx^j+b^2(t,y)dy^2. \tag{4}$$

where $n(t,y)$, $a(t,y)$ and $b(t,y)$ are arbitrary functions and y is the fifth coordinate. In the same way, we choose no energy-momentum tensor in the bulk $T_{AB}|_{bulk}=0$ and we fix the energy-momentum tensor for the branes in the following way

$$T_{AB}=\frac{\delta(y)}{b}diag(-\rho,\vec{p},0), \tag{5}$$

$$T_{AB*}=\frac{\delta(y-1/2)}{b}diag(-\rho_*,\vec{p}_*,0), \tag{6}$$

the visible brane is fixed in $y=0$ and the hidden brane in $y=1/2$ in the orbifold. Using G_{AB}, it is possible to obtain the following dynamical equations.

$$\frac{[a']_0}{a_0b_0}=-\frac{\kappa_{(5)}^2}{3}\rho, \tag{7}$$

$$\frac{[a']_{1/2}}{a_{1/2}b_{1/2}}=-\frac{\kappa_{(5)}^2}{3}\rho_*, \tag{8}$$

$$\frac{[a']_0}{b_0}=-\frac{[a']_{1/2}}{b_{1/2}}, \tag{9}$$

being ρ and ρ_* the energy density of the visible and hidden brane respectively. Using (7), (8) in (9) we obtain the first constrain equation for the different energy densities in both branes

$$a(t,y)_0\rho=-a(t,y)_{1/2}\rho_*. \tag{10}$$

A similar case it is found for "$n(t,y)$" and it is possible to obtain the second constraint obtained in [5] as

$$(3p+2\rho)n(t,y)_0 = -(3p_* + 2\rho_*)n(t,y)_{1/2}, \tag{11}$$

being p and p_* the pressure for the visible and hidden brane respectively.

The last equations (10), (11) are very important to understand the relations between the equation of state of the hidden and visible brane as we show in the next section.

THE ω_* FUNCTION FOR AN OBSERVER IN OUR BRANE.

In this section, we obtain an expression to calculate the equation of state ω_* of the hidden brane and the relation with the equation of state and the evolution of the visible brane. It is possible write the most general solution for $a(t,y)$ and $n(t,y)$ in the following way

$$a(t,y) = a_0(t) f(\lambda|y|), \tag{12}$$

$$n(t,y) = n_0(t) f(\mu|y|), \tag{13}$$

$$b(t,y) = b_0, \tag{14}$$

where f is a generalized function of $|y|$ and $\lambda = -b_0 \kappa^2_{(5)} \frac{\rho}{6A}$ and $\mu = b_0 \kappa^2_{(5)} \frac{\rho(2+3\omega)}{6\widehat{A}}$ where $A(|y|) = \frac{df(\lambda|y|)}{d(\lambda|y|)}$ and $\widehat{A}(|y|) = \frac{df(\mu|y|)}{d(\mu|y|)}$. If we use the equations (10), (11) and replace (12) and (13) it is possible to generalize ω_* as

$$\omega_* = \frac{1}{3}\left((2+3\omega)\frac{f(\lambda/2)}{f(\mu/2)} - 2\right), \tag{15}$$

where we introduce $p = \omega\rho$ and $p_* = \omega_*\rho_*$. Where we choose $A = \widehat{A}$. Then, the equation (15) can be written as

$$\omega_* = \frac{1}{3}\left((2+3\omega)\frac{f(\lambda/2)}{f(-\lambda(2+3\omega)/2)} - 2\right). \tag{16}$$

With this expression it is possible to relate the fluids in the hidden brane with the visible brane and understand the behavior of the components in the hidden brane.

With the useful equations obtained in the last part, it is possible to apply the linear solution obtained by Binetruy *et al* [5] to the generalized function of ω_* (16)

$$a(t,y) = a_0(t)\left(1+\lambda|y|\right), \tag{17}$$

$$n(t,y) = n_0(t)\left(1+\mu|y|\right), \tag{18}$$

$$b(t,y) = b_0, \tag{19}$$

and $A = \widehat{A} = 1$ for the linear solution. Using (17), (18) and (19) and replacing in (16) we obtain ω_* for the linear solution

$$\omega_* = \frac{1}{3}\left((2+3\omega)\frac{2-b_0 H}{2+b_0 H(2+3\omega)} - 2\right), \tag{20}$$

similarly the equation for ρ_* is

$$\rho_* = -\rho(1-b_0 H)^{-1}, \tag{21}$$

and the pressure $P_* = \omega_* \rho_*$. Equations (20) relates ω_* with ω and the expansion rate H of the universe. If the universe is static $\omega_* = \omega$. Similar case happens with ρ_* and ρ in equation (21).

ULTRA LIGHT BOSONIC DARK MATTER.

The equation of state ω_B of the ultra light bosonic dark matter (ULBDM) gas can be calculated at the moment of thermodynamic decoupling from the primeval fireball. At this stage, it is a very good approximation to take the ultrarelativistic limit $m/T_d << 1$. This is so, because of the extremely low mass of ULBDM particles. We recall that their bosonic nature allows a phase transition to a BEC state, whose dynamics in LTE are described in phase space by the momentum distribution of eqn. (1).

The total number density is calculated by integrating $f(\mathbf{p})$ over all momenta, that is $n_B = n_0 + n_1$. In the ultrarelativistic limit has an exact solution,

$$n_B(t) = n_0(t) + \frac{\zeta(3) T^3(t)}{\pi^2}, \tag{22}$$

with $\zeta(3) \approx 1.2$. This result defines the critical temperature,

$$T_c(t) = \left(\frac{\pi^2 n_B(t)}{\zeta(3)}\right)^{1/3}. \tag{23}$$

If at decoupling, the temperature of ULBDM is such that $T_d < T_c$, an important fraction of particles starts to populate the ground state, that is, the BEC is formed.

The energy density is also defined in terms of $f(\mathbf{p})$,

$$\begin{aligned} \rho(t) &= m n_0(t) + \frac{T^4(t)}{2\pi^2} I_\rho[x], \\ I_\rho[x] &= \int_x^\infty \frac{(u^2-x^2)^{1/2} u\, du}{e^{(u-x)}-1}. \end{aligned} \tag{24}$$

Interestingly, the BEC does not contribute to the presure of ULBDM gas as a whole,

$$\begin{aligned} P(t) &= \frac{1}{3}\frac{T^4(t)}{2\pi^2} I_P[x], \\ I_P[x] &= \int_x^\infty \frac{(u^2-x^2)^{3/2}\, du}{e^{(u-x)}-1}, \end{aligned} \tag{25}$$

where we write $E/T \equiv u$ and $x \equiv m/T_d$.

Now, the equation of state $\omega_B(t) \equiv P(t)/\rho(t)$ is straightforward. In the ultra relativistic limit, $\omega_B(t)$ scales as a^{-1}, as expected for a mixture of matter and radiation.

BRANE THEORY WITH ULBDM GAS

On cosmic scales, we assume the existence of a hidden brane embedded in the bulk. This new topological imposition generates constraints in the equation of state, energy density and pressure of the fluid content in the hidden brane for an observer in our universe (20), (21).

In this section we analyze ULBDM gas that fills the hidden brane and write the respective modified equation of state using the equation associated in the last section. Then, we rewrite the equation (20) at the moment of decoupling as,

$$\begin{aligned}\omega_{B*} &= \frac{1}{3}(2+3\omega_B(t)a(t)) \\ &\quad \times \left\{ \frac{2-b_0H(t)}{2+b_0H(t)[2+3\omega_B(t)a(t)]} - 2 \right\}. \end{aligned} \tag{26}$$

In brane theory $H(t) = \frac{\kappa^2_{(5)}}{6}\rho(t)$. Following this expression, it is possible to study the equation of state for the hidden brane in the moment of decoupling with radiation domination $\rho(t) \sim a(t)^{-4}$.

$$\begin{aligned}\omega_{B*} &= \frac{1}{3}(2+3\omega_B(t)a(t)) \\ &\times \left\{ \frac{2-b_0\frac{\kappa^2_{(5)}}{6}a(t)^{-4}}{2+b_0\frac{\kappa^2_{(5)}}{6}a(t)^{-4}(2+3\omega_B(t)a(t))} - 2 \right\}. \end{aligned} \tag{27}$$

The last equation corresponds to the equation of state for ULBDM gas in the hidden brane. Clearly if the fifth dimension is zero or the rate expansion $H(t) \sim 0$ we recover the equation of state in the hidden brane $\omega_{B*} = \omega_B$.

For the Friedmann equation, the constraints plays the similar role and the Friedmann equation for the hidden brane can be written in the following way

$$\begin{aligned}\left(\frac{\dot{a}_{hide}}{a_{hide}}\right)^2 &= \frac{\kappa^2_{(5)}}{36}\left\{ \frac{(2+b_0\frac{\kappa^2_{(5)}\rho_B}{6}(2+3\omega_B))^2}{(2-b_0\frac{\kappa^2_{(5)}\rho_B}{6})^2}\rho_B^2 \right\} \\ &+ \frac{\kappa^2_{(5)}}{36}\left\{ \sum_i H_i^2 \frac{(2+b_0H_i(2+3\omega_i))^2}{(2-b_0H_i)^2} \right\}, \end{aligned} \tag{28}$$

where it is possible to define $H_{hide} = \dot{a}_{hide}/a_{hide}$. The first brace content the information of ULBDM gas in the hidden brane and the second brace content the other kind of fields. The equivalent energy conservation equation for ULBDM in the hidden brane is

$$\dot{\rho}_{B*} + 3\left(\frac{\dot{a}_{hide}}{a_{hide}}\right)^2 (1+\omega_{B*})\rho_{B*} = 0. \tag{29}$$

CONCLUDING REMARKS.

Remarkable new ideas arise from the implementation of the ULBDM model and the braneworld theory. We have shown that a gas of ultralight particles at very high temperatures scales as a fluid of matter and radiation in a Universe in expansion. This would be attainable only if they are bosons and reside in a BE condensate state. BEC and thermal particles act as standar cold and hot DM, respectively.

We also treated the modification of the equation of state (27) and Friedmann equation (28) due to the braneworld topology assumed in our approach. This effect comes from constraints on the branes and bulk. The result is a bound impossed by the rate expansion H and the fifth dimension b_0. High energy corrections are provided by the expression (27) and (28) bounded by the 5-dimensional gravitational constant $\kappa_{(5)}$. However, the conservation equation for ULBDM (29) is preserved, provided there is no exchange between the energy-momentum tensors of the brane and the bulk.

We stress that the classical GR result is recovered straightforward in the regime of low energy $\rho/\lambda \to 0$ and no higher dimensions $b_{(0)} \to 0$. Similarly, in the case when the rate expansion of our Universe is null $H \to 0$, the success of GR is preserved, provided a lower limit on the brane tension is imposed at $\lambda > (1MeV)^4$.

Finally, it is important to mention that the extension of ULBDM gas to higher dimensions shows new contributions to the GR theory and provides a different approach to the problem of dark matter and dark energy. New incoming observational evidences from polarization of the CMB photons may be useful to endorse or refute these new ideas.

Some url test `http://www.world.universe`.

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