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A warning on the determination of the halo mass

Darío Núñez, Alma X. González-Morales*, Jorge L. Cervantes-Cota†,** and Tonatiuh Matos‡,§

*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A.P. 70-543, 04510 México D.F., México
†Depto. de Física, Instituto Nacional de Investigaciones Nucleares, México,
**Berkeley Center for Cosmological Physics, University of California, Berkeley, CA, US
‡Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, A.P. 14-740, 07000 México D.F., México
§Instituto Avanzado de Cosmología, IAC

Abstract. During this talk, I will summarize our studies from the determination of the mass of the dark matter halo, based on the observations of the rotation curves of test particles or of the gravitational lensing. As I will show you, it is not uncommon that some studies on the nature of dark matter include extra assumption, some even on the very nature of the dark matter, what we want to determine!, that bais the studies and the results obtained from the observation and, in some cases, imply an inconsistent system altogether.

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INTRODUCTION

Einstein's equations (including the cosmological constant) have proven to accurately describe the Universe at the Solar System scale (Precession, GPS, and gravitational lensing, the general relativistic experiment per excellence), as well as at large scales, 100 Mpc, where the Friedmann-Robertson-Walker spacetime determines the standard cosmological model, which is observationally corrobotated by the Cosmic Microwave Background Radiation; by the redshits in galaxies, and by the relative abundance of Hidrogen to Helio, which is 1 : 4.

Based on these facts, our position regarding the new set of observations, is to consider the Einstein's equations to be valid at all the intermediate scales, without any modification, and try to understand its consequences.

These new set of cosmological observations are the following four main groups: the acoustic peaks in the cosmic microwave background radiation, Supernovae type Ia data, rotation curves of spirals and dynamics of galactic clusters, and their gravitational lensing [1, 2, 3]. The last two of these observations are related most significantly with the presence of dark matter (DM), where a consistent model considers a Dark Matter halo surrounding the galaxies and galactic clusters. All of these cosmological observations are consistently described by the ΛCDM model. We mean by this that, as we consider that our hypothesis is that the Einstein's equations do describe the dynamics of the bodies moving on a curved background, which in turn is curved due to the presence of matter, then, when a given observation, say the rotational curve profile, is not explained by the

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amount and disctribution of matter that we see, there must be extra matter that we are not seeing, but that affects the motion as the Einstein's equation dictates.

There has been many attempts to model DM halos, some of them have shown that general relativistic effects can be important [4, 5, 6, 7, 8, 9, 10, 11], but in general, it is fair to say that the make use of approximations or limits that in some cases represents assumptions about DM properties. Even lensing images or distortions of background galaxies due to the space-time curvature [12, 13] are usually described within the Newtonian framework, which is remarkable as long as lensing is a purely relativistic effect. The point is that usually assumptions are made precisely on the nature of dark matter, which is what one is trying to determine, and those extra assumptions can bais the conclusions of produce an altogther inconsisten description.

The nature of the Dark matter is unknown. The scalar field DM model is an alternative proposed in the past [14, 15] to fit the observed amount of substructure [16], the critical mass of galaxies [17], the rotation curves of galaxies [18], the central density profile of LSB galaxies [19], the evolution of the cosmological densities [20], among other topics.

In the present work we focus on a given simple model for the DM halo and explicitly show what exactly is determined by the observations and what comes as extra assumptions. This is particularly important as the unknown nature of the DM is one of the most relevant questions that one would like to solve. It is clear that to make assumptions specifically on the nature of DM, in order to obtain information on the nature of the DM, is skewing the problem.

THE MODEL

In our study we consider a static and spherically symmetric space-time in General Relativity, described by the line element:

$$ds^2 = -e^{2\Phi/c^2}\,c^2\,dt^2 + \frac{dr^2}{1-\frac{2Gm}{c^2\,r}} + r^2\,d\Omega^2, \tag{1}$$

where $d\Omega = d\theta^2 + \sin^2\theta d\varphi^2$. The gravitational potential $\Phi(r)$ and the mass function $m(r)$ are functions of the radial coordinate only. In fact, due to the symmetries of this space-time, all physical quantities depend only on r. The stress energy tensor has the general form: $T^{\mu}{}_{\nu} = \mathrm{diag}\left(T^t_t, T^r_r, T^{\theta}_{\theta}, T^{\theta}_{\theta}\right)$, and the system of equations is, so the Einstein's equations are the known set [21]:

$$m' = -\tfrac{4\pi r^2}{c^2} T^t{}_t, \tag{2}$$

$$\left(1 - 2\,\tfrac{mG}{c^2 r}\right)\tfrac{\Phi'}{c^2} - \tfrac{mG}{c^2 r^2} = \tfrac{4\pi G\,r}{c^4} T^r{}_r, \tag{3}$$

$$T^r{}_r{}' + (T^r_r - T^t_t)\left(\tfrac{\Phi'}{c^2} + \tfrac{2\left(T^r_r - T^{\theta}_{\theta}\right)}{r}\right) = 0, \tag{4}$$

where prime $' \equiv \partial/\partial r$. And we used the conservation equation, $T^{\mu\nu}{}_{;\mu} = 0$, instead of the angular Einstein's equation. The potentials of such space-time are determined by the DM halo.

In this way, the system of equations which must be solved are, the Einstein's equations, Eqs. (2,3), and the field equation (4). In either case, there are five unknown functions, namely: m, Φ and three componentes of the energy momentum tensor, $T_t^t, T_r^r, T_\theta^\theta$. Thus, we have three equations for five unknown functions. We need extra data. It is important to underline this fact.

Following the line of work presented in [6], we will use observational results to close the system of equations. In the case of galactic halos, two main observations can serve to obtain the usefull information: measurements of rotation curves in spirals and light deflection by lensing. Once the extra data is given, there will be no more room left for any other assumption, the rest of the functions are determined by the system of equations. These point will become clear further when we consider an specific energy momentum tensor.

Rotation curves

The motion of test particles in such spacetime is determined by the geodesic equations and, for test particles in circular motion, there is a relationship between the gravitational potential, Φ, and the tangential velocity of those particles, v_c:

$$\frac{\Phi'}{c^2} = \frac{\beta^2}{r}, \tag{5}$$

where we have defined $\beta^2 = \frac{{v_c}^2}{c^2}$. This tangential velocity is the one measured by observations of rotation curves in galaxies. Thus, v_c is an observable function, and by means of Eq. (5), the gravitational potential can be determined.

Moreover, as long as the magnitude of the observed velocities are small with respect to the speed of light, this justifies the validity of one of the weak field approximations $\Phi/c^2 << 1$ that one usually assumes by taking the weak field limit.

Lensing

The other observation concerns the gravitational lensing, that for the line element given by Eq. (1), the deflection of the light ray, $\Delta\varphi$, at the radius of maximal approach, r_m, is given by [22],

$$\Delta\varphi = -\int_\infty^{r_m} \frac{r_m\, dr}{r^2\sqrt{\left(1-\frac{2Gm}{rc^2}\right)\left[e^{-2\frac{\Phi}{c^2}}e^{2\frac{\Phi(r_m)}{c^2}} - \frac{r_m^2}{r^2}\right]}}. \tag{6}$$

We remark that from the expression of the deflection angle, it is a large step to infer the mass of the DM halo based solely on the observation of the deflection angle. A supposition has to be made on the relation between the gravitational potential, Φ, and the mass function, m [22]. Such supposition, as we will show, not only strongly depends on the type of matter, but also on the specific characteristics of the type of matter considered.

Now the system of equations is closed, five equations and five unknowns. In what follows, we will present two specific cases for stress-energy tensors describing the Dark Matter: Perfect fluid and Scalar field.

Perfect fluid

In the case of the perfect fluid, the stress-energy tensor is given by $T_{\mu\nu} = (\rho c^2 + p)u_\mu u_\nu + p g_{\mu\nu}$, where the density is $\rho = (1+\varepsilon)\rho_0$, where ρ_0 is the rest mass energy density and ε the internal energy per unit mass, u^μ is the co-movil four velocity, normalized as $u_\mu u^\mu = -c^2$, and p is the pressure. The conservation equation, $T^\mu{}_{\nu;\mu} = 0$, implies the field equation:

$$\left(\rho c^2 + p\right)\frac{\Phi'}{c^2} + p' = 0, \tag{7}$$

which can be rewritten as

$$T^r{}_r{}' + \left(T^r_r - T^t_t\right)\frac{\Phi'}{c^2} = 0. \tag{8}$$

Scalar field

Now, for the scalar field the stress energy tensor is given by

$$T_{\mu\nu} = \phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\left(g^{\alpha\beta}\phi_{,\alpha}\,\phi_{,\beta} + 2V(\phi)\right), \tag{9}$$

where $\phi_{,\alpha} = \partial\phi/\partial x^\alpha$; and $V(\phi)$ is the scalar potential. The components of the stress-energy tensor are

$$\begin{aligned} T^t{}_t &= -\frac{1}{2}\left(1 - \frac{2Gm}{c^2 r}\right)\phi'^2 - V(\phi), \\ T^r{}_r &= \frac{1}{2}\left(1 - \frac{2Gm}{c^2 r}\right)\phi'^2 - V(\phi), \\ T^\theta{}_\theta &= T^\varphi{}_\varphi = T^t{}_t. \end{aligned} \tag{10}$$

>From the conservation equation for the scalar field, $T^\mu{}_{\nu;\mu} = 0$, one obtains a field equation, the Klein-Gordon equation,

$$\phi'' + \left(\frac{\frac{m'G}{c^2 r} + \frac{\frac{3mG}{c^2} - 2r}{r^2}}{1 - \frac{2mG}{c^2 r}} - \frac{\Phi'}{c^2}\right)\phi' + \frac{\frac{\partial V}{\partial\phi}}{1 - \frac{2mG}{c^2 r}} = 0, \tag{11}$$

that can be written as

$$T^r{}_r{}' + \left(T^r_r - T^t_t\right)\left(\frac{\Phi'}{c^2} + \frac{2}{r}\right) = 0, \tag{12}$$

which is remarkable similar to the field equation for the perfect fluid, Eq.(8). Given this similarity, it is convenient for our mathematical description to consider the single field equation for both types of matter

$$T^r{}_r{}' + \left(T^r_r - T^t_t\right)\left(\frac{\Phi'}{c^2} + \frac{2a}{r}\right) = 0. \tag{13}$$

in which $a = 0$ for the perfect fluid, and $a = 1$ for the scalar field. Notice that if one considers a sort of perfect fluid given by $T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p_i, p_i)$, p_i (some times called "tangential" pressure) denotes a term representing the ignorance we have on the features of the fluid. This presure p_i is related to the other fluid variables as $p_i = (1-a)\,p - a\,\rho$, (see Eq. (13)), where "a" takes, in principle, any value. There are works which have discussed this field equation considering a as a free parameter [11, 9, 10]. For the purpose of the present work we will consider only the two extremal cases, $a = 0$ and $a = 1$, but the discussion can be directly applied for these cases as well.

Here we want to emphasize that the approximations $2\,G m / c^2 r << 1$ and especially $p << \rho$ are, in general, extra hypothesis which strongly depend upon the nature of the DM type. It is clear that if all these conditions are satisfied, then the above system of equations, Eqs. (2, 3, 7), reduces to the hydrodynamic set of equations for the case of the perfect fluid model. But, for example in the case of the scalar field, there is no Newtonian limit, and one has to be careful with these approximations.

Since the gravitational potential is already determined by the rotation curves of spirals and these types of matter has particular relations between components of the energy momentum tensor, these reduces the number of unknown functions to three, and so the deflection angle measurements are not longer needed to close the systmem of equations but it can serve to discriminate between models.

Another posibility is, for instance, if we give an equation of state for the perfect fluid, $p = p(\rho)$ or, in the case of the scalar field, an explicit form for the potential, $V(\phi)$, there is no freedom left to choose the form of the rest of the functions,they will be determined by the system of equations.

THE HALO MASS

After substituting Eq. (5) into the gravity equations, Eqs. (2, 3) and in the field equation Eq. 4, we obtain an equation (with no approximations) for the mass function as the only unknown function

$$m' + P(r)\,m = Q(r) \tag{14}$$

with

$$\begin{aligned} P(r) &= \frac{2\,r\,\beta^{2\prime} - \left(1 + 2\,\beta^2\right)\left(3 - 2\,a - \beta^2\right)}{\left(1 - 2a + \beta^2\right)\,r}, \\ Q(r) &= \frac{c^2}{G}\,\frac{r\,\beta^{2\prime} - \beta^2\left(2 - 2\,a - \beta^2\right)}{\left(1 - 2a + \beta^2\right)}. \end{aligned} \tag{15}$$

The functions $P(r)$ and $Q(r)$ depend on the type of fluid we are dealing with (a) and on the rotation curves profile.

For the case of the perfect fluid, the density and pressure are directly computed from Eqs. (2) and (3), respectively.

For the scalar field, using the expressions Eqs. (10), we obtain that

$$\phi'^2 = \frac{c^4}{4\pi G r^2}\left(\frac{\frac{Gm'}{c^2}-\frac{Gm}{c^2 r}}{1-\frac{2Gm}{c^2 r}}+\beta^2\right), \tag{16}$$

$$V(\phi(r)) = \frac{c^2}{8\pi r^3}\left[m\left(1+2\beta^2\right)+m' r\right]-\frac{c^4}{8\pi G}\left(\frac{\beta^2}{r^2}\right). \tag{17}$$

Once the function $\beta(r)$ is given, the scalar field and its potential are straightforwardly determined in terms of the radial coordinate. In order to obtain the form $V(\phi)$, one needs to invert the solution for the scalar field ($r = r(\phi)$), and to substitute it into Eq. (17). As shown below, this procedure works at least for simple β functions.

In this way, we have shown that the mass function m, associated to a galactic halo by means of the rotation velocity strongly depends on the DM model that is being considered. The single observation of the rotation curve is not sufficient to determine the mass associated with the halo, since it depends on the nature of it. Moreover, we have shown that the relationship between the pressure and the density, or between the scalar field and the scalar potential is fixed, up to integration constants, once the rotation velocity is employed.

EXAMPLE

The idea in this section is to stress the conclusions that we are presenting by means of considering a typical observation of rotation velocities in spirals and to directly determine the gravitational mass in each case, when the DM is a perfect fluid (dust) and when it is a scalar field.

In practice we can consider a velocity distribution, as a phenomenological model, for instance the velocity profile coming from N-body simulations given by NFW [23, 24] or a Burkert profile [25] given by the phenomenological of rotation curves [26], to determine the mass of each type of fluid. We will show that the gravitational mass inferred by the same velocity profile is strikingly different for the perfect fluid and scalar field cases.

Navarro-Frenk-White (NFW) velocity profile

Independently from its origin, the NFW profile [23, 24] is considered *per se* as a viable fitting model to describe galactic kinematics. This profile has been subject to geometrical studies elsewhere [27, 28, 27]. In this example, we assume this profile as a valid phenomenological galactic profile for the galactic data. We obtain the usual expression for the mass derived within this description, and compare it with the same form of the rotation velocity, but considering that it is due to a DM halo composed of a scalar field.

The velocity profile in the NFW model [23, 24] is given by

$$v_T^2 = \frac{\sigma_0{}^2 r_0}{r}\left(-\frac{r/r_0}{1+r/r_0} + \ln\left[1+r/r_0\right]\right). \quad (18)$$

where $\sigma_0 = 4\pi G \rho_0 r_0{}^2$ is a characteristic velocity of stars in the halo, given in terms of a characteristic density, and r_0 is a scale radius. Given this velocity profile, we have to solve Eq. (14) with $a = 0$ for the perfect fluid and with $a = 1$ for the scalar field. In neither case there is an analytical solution, thus we have integrated the equations numerically. We do not want to treat here specific galaxies but to emphasize the differences between the galaxy models. Therefore, we set σ_0 and r_0 to some typical values. In our plots we assume geometric units ($G = c = 1$), and therefore the characteristic velocity takes values, $0 < \sigma_0 < 1$, and the mass is less than the unity. For definiteness, we assume $\sigma_0 = 0.001$ and $r_0 = 1$. In figure (1) we plot both halo masses (perfect fluid and scalar field). Disregarding the behavior near the origin, as long as we are considering the outside region, as mentioned above, we see that the mass associated to the halo in each case are different. We now consider lensing. By integrating Eq. (5) for the given rotation

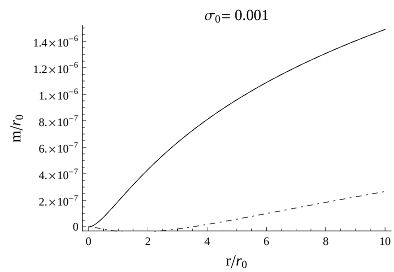

FIGURE 1. Comparison of the masses using the rotation velocity profile from the NFW model with a perfect fluid (upper curve) and scalar field (lower curve).

velocity, Eq. (18), we obtain the following expression for the gravitational potential:

$$\Phi = -\sigma_0 r_0 \frac{\ln\left(1+\frac{r}{r_0}\right)}{r}. \quad (19)$$

We substitute this expression, together with the corresponding numerical solution one for the mass in each case, in the equation for the deflection angle, Eq. (6), and perform the integration varying the value of the radius of maximal approach, r_m. The results are plotted in figure 2. As we see, the observation of the deflection angle can determine which type of matter is actually composing the DM halo.

In this example, the mass associated to the scalar field model is essentially positive and a well–behaved function, that is, it does not follow the no-go result mentioned in the previous example. Though there is a small region where the mass becomes negative. This is due to the fact that we are demanding the velocity profile to grow in that region. In a real setting however the stellar disc adds to the velocity profile, thus we expect that its contribution avoids negative mass regions for the scalar field.

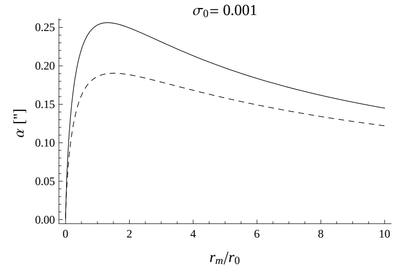

FIGURE 2. Deflection angle generated by the gravitational lensing of a NFW rotation profile with a perfect fluid (upper curve) and scalar field (lower curve).

With these examples it is clear how two different types of matter (perfect fluid and scalar field) can be consistent with the observation of rotation curves of DM halos, though they lead to different conclusions to the mass function inferred. The deflection of light can then be used to discriminate between the two models. Even though the mass function for some model has not an intuitively expected behavior, it is necessary to use the observation in order to discard the model, being aware of the assumptions made during the derivation of such conclusions.

See [29] for details and further analysis.

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