Scalar Field Dark Matter from Two Concentric Spherical Branes Universe

2011 J. Phys.: Conf. Ser. 315 012025

(http://iopscience.iop.org/1742-6596/315/1/012025)

Journal of Physics: Conference Series **315** (2011) 012025 doi:10.1088/1742-6596/315/1/012025

Scalar Field Dark Matter from Two Concentric Spherical Branes Universe

Tonatiuh Matos[1]

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN,
A.P. 14-740, 07000 México D.F., México.

Miguel A. García Aspeitia

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN,
A.P. 14-740, 07000 México D.F., México.

Abstract. The Scalar Field Dark Matter (SFDM) model has provided to be a very successful one to explain the large scale structure of the Universe. Nevertheless, one of the main problems of the SFDM model is the ultralight mass of the scalar field $m_\Phi \sim 10^{-22}$eV. In this work we explore a five-dimensional Universe with a special topology with the aim to explain this mass using the Randal-Sumdrum prescription for the hierarchy problem using two branes. In this model we show two limits where the branes have mutual interaction and have no interaction. We analyze the Friedmann equations and the linear corrections introduced by this model.

1. INTRODUCTION

The Scalar Field Dark Matter (SFDM) model has provided to be very successful model for explaining the large scale structure of the Universe [1]. The hypothesis of this model is that the dark matter of the Universe is a scalar field with mass $m_\Phi \sim 10^{-22}$eV. Of course this mass is too small and provokes a strong hierarchy problem. Nevertheless, this model has very interesting features which makes the model worth to keep studying. For example, with this mass, that means, with only one free parameter, the scalar field contains the following important features:
The ultra-light scalar field mass ($m_\Phi \sim 10^{-22}$eV) fits:

(i) The evolution of the cosmological densities [2].
(ii) The rotation curves of galaxies [5] and the central density profile of LSB galaxies [4],
(iii) With this mass, the critical mass of collapse for a real scalar field is just $10^{12}\,M_\odot$, i.e., the one obeserved in galaxies halos [3].
(iv) The central density profile of the dark matter is flat [4].
(v) The scalar field has a natural cut off, thus the substructure in clusters of galaxies is avoided naturally. With a scalar field mass of $m_\Phi \sim 10^{-22}$eV the amount of substructure is compatible with the observed one [6].

[1] Part of the Instituto Avanzado de Cosmología (IAC) collaboration http://www.iac.edu.mx/

(vi) SFDM forms galaxies earlier than the cold dark matter model, because they form Bose-Einstein Condensates at a critical temperature $T_c >>$TeV. So, if SFDM is right, we have to observe big galaxies at big redshifts.

In this work we present a model using two branes, where the mass of the scalar field is not too small, but the brane where we are living reads an ultralight scalar field mass, using the Randal Sundrum prescription. Recent works [8]-[15] assume that the Universe have more than the four dimensions and in fact we live in a membrane that lives in a five dimensional space time obtaining a new degree of freedom (the radion). The imposition of extra dimensions generate different behavior in the Newtonian law at long distances with the correction in the potential and solving the problem of the acceleration as a consecuence of the fifth dimension.

Particulary we porpose a model with two concentric spherical branes embedded in a five dimensional AdS-S space-time. Each brane is filled with different field with the aim of solve the problem of the dark matter. The shape of the action to model this physical structure is given by

$$S = \int dX^5 \sqrt{-g_{(5)}}\, m^3_{(5)} \left(R_{(5)} + \Lambda\right) - \sum_{\pm} \int_{\pm} dx^4 \sqrt{-g^{\pm}} \left(2m^3_{(5)} K^{\pm} + \pounds^{\pm}\right),$$

being $\pm$ the exterior or interior region of the brane respectively, $g_{(5)}$ is the determinant of the five-dimensional (5D) metric and g the determinant of the four-dimensional (4D) one, $m_{(5)}$ is the 5D Planck mass, $R_{(5)}$ is the 5D scalar curvature, K is the extrinsic curvature and Λ is the 5D cosmological constant, $\pounds^+$ is the Lagrangian of the fields content in the exterior brane (spin zero) and $\pounds^-$ is the Lagrangian of the fields content in the interior brane (spin one).
Following the next ideas it is possible to enumerate the next characteristics of the branes and the bulk

(i) The bulk is a five dimensional AdS-S with two branes embedded and with no energy momentum tensor in the bulk $T_{AB} = 0$ $i.e.$ no fields in the bulk.
(ii) The inner brane is our Universe and content the fields predicted by the standard model of particles
(iii) The outer brane content a particular scalar field that behave like a dark matter [15] and imprint the gravitational well potential in the evolution of our Universe (inner brane).

With the next features it is possible to analyze when the branes have no dynamical interaction between them and with interaction. The next section analyze the behavior of this two cases in the Friedmann equations and the fields that lives in the branes.

2. Non interaction between the branes.

In this case we propose two concentric branes embedded in a five dimensional bulk (FIG. 1). For simplicity the problem of each brane can be treaty in separate way in concordance with the superposition method due to the theory is linear in the same way as the electromagnetism. Generalizing the metric associated with the bulk it is possible to write

$$ds^2_{(5)} = -A(a)_{\pm} dt^2_{\pm} + \frac{1}{A(a)_{\pm}} da^2 + a^2 \left[d\chi^2 + \sin^2(\chi)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \tag{1}$$

with the functions $A(a)_{\pm}$ directly solved by the five dimensional Einstein equations. From here, we use natural units ($\hbar = c = 1$).

Following the idea, it is possible to find the Friedmann equation in each brane with the junction conditions in the next way [15]

$$\left(\frac{\dot{a}_1}{a_1}\right)^2 + \frac{1}{a_1^2} = \frac{k^4_{(5)}\rho_1^2}{36} + \frac{\Lambda_1 + \Lambda_2}{12} + \frac{M_1}{m^3_{(5)} a_1^4} + \frac{(\Lambda_2 - \Lambda_1)^2}{16\rho_1^2 k^4_{(5)}} + \frac{9M_1^2}{m^6_{(5)} k^4_{(5)} \rho_1^2 a_1^8} + \frac{3M_1(\Lambda_2 - \Lambda_1)}{2m^3_{(5)} a_1^4 k^4_{(5)} \rho_1^2}, \tag{2}$$

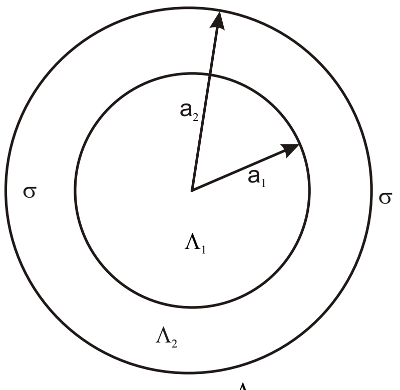

Figure 1. Schematic representation of the interior and exterior branes and the different values of the 5D cosmological constant Λ_i $i = 1, 2, 3$ and the brane tensions σ.

and

$$\left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a_2^2} = \frac{k_{(5)}^4 \rho_2^2}{36} + \frac{\Lambda_3 + \Lambda_2}{12} + \frac{2M_1 + M_2}{m_{(5)}^3 a_2^4} + \frac{(\Lambda_3 - \Lambda_2)^2}{16\rho_2^2 k_{(5)}^4} + \frac{9M_2^2}{m_{(5)}^6 k_{(5)}^4 \rho_2^2 a_2^8} + \frac{3M_2(\Lambda_3 - \Lambda_2)}{2m_{(5)}^3 a_2^4 k_{(5)}^4 \rho_2^2}. \tag{3}$$

with ρ_1, ρ_2 the density matter, M_1, M_2 is the mass of each branes respectively. The constants Λ_1, Λ_2 and Λ_3 is associated with the vacumm energy in the five dimensional space time and $k_{(5)}$ is the five dimensional gravitational constant. $k_{(5)}$ can be associated with the five dimensional Planck mass in the next way

$$\kappa_{(5)}^2 = 8\pi G_{(5)} = \frac{8\pi}{M_{(5)}^3}, \tag{4}$$

where $G_{(5)}$ is the 5-dimensional Newtonian constant and $M_{(5)}$ is the 5-dimensional Planck mass.

It is possible to observe both regimes in the Friedmann equations (2), (3) with an apropiate limit between the brane tension and the energy density of the brane.

2.1. *High Energy Limit*

In this scenario the Universe is very young and the corrections in the Friedmann equations (2), (3) appear when the energy density is much bigger than the brane tension $\rho >> \sigma$ in the next way

$$\left(\frac{\dot{a}_1}{a_1}\right)^2 + \frac{1}{a_1^2} = \frac{\kappa_{(4)}^2}{3}\frac{\rho^2}{2\lambda} + \frac{\Lambda_1}{3} + \frac{M}{m_{(5)}^3 a_1^4}, \tag{5}$$

and

$$\left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a_2^2} = \frac{\kappa_{(4)}^2}{3}\frac{\rho^2}{2\lambda} + \frac{\Lambda_2}{3} - \frac{M}{m_{(5)}^3 a_2^4}, \tag{6}$$

with

$$\Lambda_1 = -\frac{\lambda_{(5)}}{2}, \;\; \Lambda_2 = -\frac{\lambda_{(5)}}{2}, \tag{7}$$

where $m_{(5)}$ is the five dimensional Planck mass and λ is associated with the tension in the brane. This is because in the high energy limit it follows that

$$\rho^2 \sim 2\sigma\rho\left(1 + \frac{\rho}{2\sigma}\right) + \sigma^2,$$

for simplicity we assume the ansatz $M_1 = -M_2 = -M$ with non physical interpretation, where we impose $\kappa_{(4)}^2 \sigma \approx \lambda_{(5)}$.

2.2. Low Energy Limit
In this limit we choose that the energy density is much less than the brane tension on the brane $\rho << \sigma$, obtaining the classical Friedmann equation with corrections provided by the brane theory.

$$\left(\frac{\dot{a}_1}{a_1}\right)^2 + \frac{1}{a_1^2} = \frac{\kappa_{(4)}^2}{3}\rho + \frac{\Lambda_1}{3} + \frac{3M^2}{2m_{(5)}^5 \lambda a_1^8}, \tag{8}$$

and

$$\left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a_2^2} = \frac{\kappa_{(4)}^2}{3}\rho + \frac{\Lambda_2}{3} + \frac{3M^2}{2m_{(5)}^5 \lambda a_2^8}, \tag{9}$$

with

$$\Lambda_{1,2} = \Lambda_{(5)}, \tag{10}$$

where $m_{(5)}$ is the five dimensional Planck mass and λ is associated with the tension in the brane. In the same way as before, this is because $\rho^2 = 2\sigma\rho\left(1+\frac{\rho}{2\sigma}\right)+\sigma^2,$. where we impose the condition $\kappa_{(4)}^2\sigma \approx \lambda_{(5)}$.

3. Interaction between the branes.

In this scenario, the branes have topological interactions between them, obtaining restrictions in the matter contents of each brane. This scenario assume flat five dimensional bulk embedded in the next way

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\delta_{ij}dx^i dx^j + b^2(t,y)dy^2. \tag{11}$$

with two energy momentum tensor for each brane fixed in the orbifold at $y = 0$ and $y = 1/2$ as

$$T_{AB} = \frac{\delta(y)}{b} diag(-\rho, \vec{p}, 0), \tag{12}$$

$$T_{AB*} = \frac{\delta(y-1/2)}{b} diag(-\rho_*, \vec{p}_*, 0), \tag{13}$$

If we assume that the five dimensional Einstein equations can be written as $G_{AB} = \kappa_{(5)}^2 (T_{AB}|_{bulk} + \widetilde{T}_{AB}|_{branes})$, where $A, B = 0, 1, 2, 3, 4$ and the energy-momentum tensor $T_{AB}|_{bulk}$ is for the bulk and $\widetilde{T}_{AB}|_{brane} = T_{AB} + T_{AB*}$ is for the visible brane and the hidden brane respectively.
Then we obtain the following constraint equations in the next way [11]

$$a(t,y)_0\rho = -a(t,y)_{1/2}\rho_*. \tag{14}$$

$$(3p + 2\rho)n(t,y)_0 = -(3p_* + 2\rho_*)n(t,y)_{1/2}. \tag{15}$$

where ρ, p and ρ_*, p_* it is the energy density and pressure of our Universe and Hidden brane respectively. Assuming that $p = \omega\rho$ and $p_* = \omega_*\rho_*$. If we assume linear solution to the functions $a(t,y)$, $n(t,y)$ and $b(t,y)$ in the next way [11]

$$a(t,y) = a_0(t)\left(1 + \lambda|y|\right), \tag{16}$$

$$n(t,y) = n_0(t)\left(1 + \mu|y|\right), \tag{17}$$

$$b(t, y) = b_0, \tag{18}$$

the constraint equations can be reduced as

$$\omega_* = \frac{1}{3}\left((2+3\omega)\frac{2-b_0H}{2+b_0H(2+3\omega)} - 2\right), \tag{19}$$

$$\rho_* = -\rho(1-b_0H)^{-1}, \tag{20}$$

where H is the Hubble parameter of our Universe. If $H = 0$ we demonstrate that $\omega_* = \omega$ and $\rho_* = -\rho$, where the sign is caused by the imposition of Z_2-symmetry. In similar way exist corrections in the Friedmann equations caused by the constraints.

3.1. The Cosmology

For our Universe, the Friedmann equation can be written in the next way [12]

$$\left(\frac{\dot{a}_1}{a_1}\right)^2 = \frac{\kappa_{(5)}^2}{36}\sum_i \rho_i^2, \tag{21}$$

where i is for baryons, neutrinos, radiation, cosmological constant etc.. The equation of energy conservation in the visible brane can be written as

$$\dot{\rho} + 3\frac{\dot{a}_1^2}{a_1^2}(1+\omega)\rho = 0. \tag{22}$$

For the hidden brane, the Friedmann equation can be written as [10]

$$\left(\frac{\dot{a}_2}{a_2}\right)^2 = \frac{\kappa_{(5)}^2}{36}\sum_i H_i^2\frac{(2+b_0H_i(2+3\omega_i))^2}{(2-b_0H_i)^2}, \tag{23}$$

the equivalent energy conservation equation for the hidden brane

$$\dot{\rho}_* + 3\frac{\dot{a}_2^2}{a_2^2}(1+\omega_*)\rho_* = 0. \tag{24}$$

3.2. The Poisson Equation For The Hidden Brane.

It is possible to study the Poisson equation for the hidden branen in the case when the Newtonian potential it is principally influenced by the scalar field as dark matter. In a region around of a cluster of matter, is better work the Klein-Gordon equation in a spherical coordinates in the next way

$$\Phi'' + \frac{2}{r}\Phi' - m_\Phi^2\Phi = \ddot{\Phi}, \tag{25}$$

where $'$ indicates differentiation of r. It is well known that the solution of the above equation (25) can be written as

$$\Phi(t, r) = \frac{e^{\pm ikr}}{r}e^{\pm i\omega t}. \tag{26}$$

and the dispersion relation $k^2 = \omega^2 - m_\Phi^2$ [7]. To study this case, we choose the particular solution of the equation (26) used by Bernal *et al* [7] in the next way

$$\Phi(t,x) = \Phi_0 \frac{sin(x)}{x} cos(\omega t), \tag{27}$$

where Φ_0 is an a constant and $x = kr$ and ρ_Φ can be written in the next way

$$\rho_\Phi = \frac{\Phi_0^2}{2}\left(\omega^2 - k^2 cos^2(\omega t)\right)\left(\frac{sin(x)}{x}\right)^2. \tag{28}$$

Then the Poisson equation can be written as

$$\nabla^2 \Psi_* = -2\pi G \Phi_0^2 \left(\omega^2 - k^2 cos^2(\omega t)\right)\left(\frac{sin(x)}{x}\right)^2 (1 - b_0 H)^{-1}, \tag{29}$$

in spherical coordinates

$$\begin{aligned}\frac{k^2}{x^2}\frac{\partial}{\partial x}\left(x^2 \frac{\partial \Psi_*}{\partial x}\right) + \frac{k^2}{x^2 sin(\theta)}\frac{\partial}{\partial \theta}\left(sin(\theta)\frac{\partial \Psi_*}{\partial \theta}\right) + \frac{k^2}{x^2 sin^2(\theta)}\frac{\partial^2 \Psi_*}{\partial \varphi^2} &= -2\pi G \Phi_0^2 \left(\omega^2 - k^2 cos^2(\omega t)\right)\left(\frac{sin(x)}{x}\right)^2 \\ &\quad \times (1 - b_0 H)^{-1}, \end{aligned} \tag{30}$$

where we use the next change of variable $x = rk$.

Now, it is possible to solve the last differential equation (30) when the Hubble parameter depend of $\vec{r}$ and t and H only depends of t.

(i) *Non dependence of the spatial variables in H.* In this case, we assume that H only depends of t for simplicity, in general H depends of t and the spatial variables $\vec{r}$.
To solve the last equation we assume the next separation of variables $\Psi(x,\theta,\varphi)_* = R(x)Y(\theta,\varphi)$. Then we obtain the next two differential equations

$$\frac{1}{sin(\theta)}\frac{d}{d\theta}\left(sin(\theta)\frac{dY(\theta,\varphi)}{d\theta}\right) + \frac{1}{sin^2(\theta)}\frac{d^2 Y(\theta,\varphi)}{d\varphi^2} = -l(l+1)Y(\theta,\varphi), \tag{31}$$

and

$$\frac{d}{dx}\left(x^2 \frac{dR(x)}{dx}\right) - \left(l(l+1) - \frac{\gamma(t)}{k^2} sin^2 x\right) R(x) = 0, \tag{32}$$

where $\gamma(t) = 2\pi G \Phi_0^2 \left(\omega^2 - k^2 cos^2(\omega t)\right)(1 - b_0 H(t))^{-1}$ only depends of t. From the equation (32) it is evident that the solution is the spherical harmonics

$$Y_l^m(\theta,\varphi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(cos\theta) e^{im\varphi}, \tag{33}$$

where $P_l^m(cos\theta)$ are the Legendre Polynomial.
For the equation 31 it is neccesary obtain a numerical solution.

(ii) *Dependence of the spatial variables in H.* In this general case, H depends of the spatial variables and the time, this implies that ρ has the same characteristics because $H \sim \rho$. Then we obtain the same differential equation for the angular variables (33), but the equation (31) can be generalized in the next way

$$\frac{d}{dx}\left(x^2 \frac{dR(x)}{dx}\right) - \left(l(l+1) - \frac{\beta(t) x^2 sin^2 x}{x^2 - \alpha(t) sin^2 x}\right) R(x) = 0, \tag{34}$$

where $\beta(t) = 2\pi G \Phi_0^2 \left(\omega^2 - k^2 cos^2(\omega t)\right)$ and $\alpha(t) = b_0 \frac{\kappa_{(5)}^2 \Phi_0^2}{12}\left(\omega^2 - k^2 cos^2(\omega t)\right)$ in similar way the last equation (34) must be integrated numerically.

4. CONCLUSIONS

Braneworld theory is an exciting possibility to understand the real topology of the Universe and the fields inside of the universe we observe. As it is possible to show, the dynamic equations change with high energy corrections in the energy density, predicting new observations in the very early Universe and posible evidence in the CMB imprints in the next few year with the Planck mission. Similarly, the topological constraints caused by the imposition of higher dimensions, cause a different behavior in the fields contents in the branes. As we observe in the section III, the dynamic of the fields in the hidden brane is correlated with the evolution parameter of our Universe and the fields contents giving new dynamics. In similar way, the equations in the hidden brane change drastically when we impose interaction between them and it is related with the evolution of our universe and the field contents.

Recently, new experiments (LHC, Planck satellite, etc...) could obtain evidence of the existence of higher dimensions and giving us the real behavior of the gravitation, the real topology and the dynamics of the fields that lives in the Universe.

ACKNOWLEDGEMENT

This work was partially supported by CONACyT México, under grant 49865-F, 216536/219673. Instituto Avanzado de Cosmologia (IAC) collaboration.

References

[1] Matos Tonatiuh and Siddhartha Guzmán F. *et al.*, Class. Quant. Grav. **17**, L9-l16 (2000), arXiv: gr-qc /9810028
[2] Matos Tonatiuh, Vazquez Alberto and Magaña J.A. *et al* MNRAS., **389**, 13957 (2009) arXiv:0806.0683
[3] Alcubierre Miguel, Siddhartha Guzmán F., Matos Tonatiuh, Núñez Darío, Ureña Luis A. and Wiederhold Petra. Class. Quant. Grav. **19** 5017-5024 (2002), gr-qc /0110102.
[4] Bernal Argelia, Matos Tonatiuh and Núñez Darío., Rev. Mex. A.A. **44**, 149-160 (2008), arXiv:astro-ph/0303455
[5] C. G. Boehmer and T. Harko., *et al.*, JCAP **0706** 025 (2007), arXiv:astro-ph/0303455.
[6] Matos Tonatiuh and Ureña Luis A., *et al.*, Phys Rev. D **63**, 063506 (2001), arXiv:astro-ph/ 0006024
[7] Bernal A., Matos T., Nuñez D., *et al* Rev. Mex. A. A., **44** 149-160 (2008), arXiv:astro-ph/0303455v3 Flat Central Density Profiles from Scalar Field Dark Matter.
[8] Randall Lisa and Sundrum Raman., *et al.*, Phys. Rev. Lett. **83** 3370-3373 (1999), arXiv:hep-ph/9905221v1 A Large Mass Hierarchy from a Small Extra Dimension
[9] Dvali G., Gabadadze G., Porrati M. *et al.*, Phys. Lett. B **485** 208-214, (2000), arXiv:hep-th/0005016v2 4D Gravity on a Brane in 5D Minkowski Space
[10] Khoury, Ovrut, Steinhardt, Turok., *et al.*, Phys. Rev. D **64**:123522 (2001)
[11] Binétruy P., Deffayet C., Lanoglois D., *et al.*, Nucl. Phys. **B** 565: 269-287 (2000), arXiv:hep-th/9905012v2. Non-conventional cosmology from a brane-universe
[12] García Aspeitia M. A. and Matos T. To be Published. Φ^2 Scalar Field Dark Matter in a Braneworld
[13] Räsänen S., Ph.D. thesis, (2002). arXiv:astro-ph/0208282v2 A primer on the ekpyrotic scenario
[14] Steinhardt, Turok., arXiv:hep-th/0111098
[15] García Aspeitia M. A. and Matos T., arXiv:gr-qc/0906.3278 (2009). To be Published. The Universe Dynamics from Topological Considerations