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LETTER TO THE EDITOR

Quintessence and scalar dark matter in the Universe

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Abstract. Continuing on from previous works, we present a cosmological model in which dark matter and dark energy are modelled by scalar fields Φ and Ψ, respectively, endowed with the scalar potentials $V(\Phi) = V_o\left[\cosh(\lambda\sqrt{\kappa_o}\Phi) - 1\right]$ and $\tilde{V}(\Psi) = \tilde{V}_o\left[\sinh(\alpha\sqrt{\kappa_o}\Psi)\right]^{\beta}$. This model contains a 95% scalar field. We obtain that the scalar dark matter mass is $m_\Phi \sim 10^{-26}$ eV. The solution obtained allows us to recover the success of standard cold dark matter. The implications on the formation of structure are reviewed. We obtain that the minimal cut-off radio for this model is $r_c \sim 1.2$ kpc.

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Over many years there has been much evidence found about the missing matter in the Universe. It is known that the components of the Universe are radiation, baryons, neutrinos, etc, but observations show that their contribution is less than 5% of the total mass of the Cosmos, in agreement with big-bang nucleosynthesis predictions. This suggests that there must exist a non-baryonic type of matter in galaxies and clusters of galaxies [1, 2]. Recently, observations of type-Ia supernovae [3, 4] have shown that there must exist another component that accelerates the expansion of the Universe. This new component must have a negative equation of state $\omega < -\frac{1}{3}$, where $p = \omega\rho$ [5]. The observations point to a flat Universe filled with radiation, plus baryons, neutrinos, etc contributing $\sim 5\%$, a dark matter component $\sim 25\%$ and so-called dark energy contributing $\sim 70\%$ to the total mass of the Cosmos [6]. One of the most successful models up to now has been the Λ cold dark matter (ΛCDM) model, where the dark energy is a cosmological constant [7]. However, some of the problems with this model have not yet been solved. First of all, if a cosmological constant exists, why is its contribution to the total matter of the same order of magnitude as baryons and cold dark matter? This is the cosmic coincidence problem. Also, the suggested value for the cosmological constant appears to be well below the values predicted by particle physics. On the other hand, the existence of a cosmological constant leads to a strong fine tuning problem over the initial conditions of the Universe.

These latter facts open the possibility of scalar fields as strong candidates for the missing matter of the Universe [9–12]. A reliable model for dark energy is a fluctuating, inhomogeneous scalar field, rolling down a scalar potential, called quintessence (Q) [13]. For this case, great efforts have been made to determine the appropriate scalar potential that could explain current cosmological observations [10, 11, 14]. One example is the pure exponential potential [10, 14]. It has the advantages that it mimics the dominant density background and it appears naturally

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as a solution for a completely scalar-dominated Universe [15]. However, nucleosynthesis constraints require that the scalar field contribution be $\Omega_\Phi \leqslant 0.2$, which indicates that the scalar field would never dominate the Universe [10]. However, a special group of scalar potentials has been proposed in order to avoid the fine tuning and coincidence problem, namely tracker solutions [11], where the cosmology at late times is extremely insensitive to initial conditions. A typical potential is the pure inverse power-law one, $V(\Phi) \sim \Phi^{-\alpha}$ $(\alpha > 0)$ [11, 16]. Although it reduces the fine tuning and the cosmic coincidence problem, the predicted value for the current equation of state for the quintessence is not in good agreement with supernovae results [11]. The same problem arises with the inverse power-law-like potentials. Another example are the potentials proposed in [17]. They efficiently avoid the troubles stated above, but it is not possible to determine their free parameters unambiguously.

In this letter we use a cosh potential, in order to mimic a standard cold dark matter with a quintessential dark energy. Then we will investigate the scalar field fluctuations and the implications for structure formation in directions suggested by some authors. We find that the scalar field is an ultra-light particle which behaves just like cold dark matter. Using previous works [9, 18, 19], it is then possible that a scalar field fluctuation could explain the formation of the galaxy halos.

In a recent paper [12], we showed that the potential

$$\tilde{V}(\Psi) = \tilde{V}_o\left[\sinh\left(\alpha\sqrt{\kappa_o}\Psi\right)\right]^\beta = \begin{cases} \tilde{V}_o\left(\alpha\sqrt{\kappa_o}\Psi\right)^\beta & |\alpha\sqrt{\kappa_o}\Psi| \ll 1 \\ (\tilde{V}_o/2^\beta)\exp\left(\alpha\beta\sqrt{\kappa_o}\Psi\right) & |\alpha\sqrt{\kappa_o}\Psi| \gg 1; \end{cases} \tag{1}$$

is a good candidate for the dark energy. Its asymptotic behaviour at early (late) times is the attractive inverse power-law (exponential) one. Its parameters are given by

$$\begin{aligned} \alpha &= \frac{-3\omega_\Psi}{2\sqrt{3(1+\omega_\Psi)}}, \\ \beta &= \frac{2\,(1+\omega_\Psi)}{\omega_\Psi}, \\ \rho_{o\Psi} &= \left(\frac{2\,\tilde{V}_o}{1-\omega_\Psi}\,\rho_{oCDM}^{-\beta/2}\right)^{\frac{1}{1-\beta/2}}, \end{aligned} \tag{2}$$

where ρ_{oCDM} and $\rho_{o\Psi}$ are the current energy densities of cold dark matter and dark energy, respectively, and ω_Ψ is the current equation of state for the dark energy. It eliminates the fine tuning problem and dominates only at late times, driving the Universe to a power-law inflationary stage (for which the scale factor $a \sim t^p$, with $p > 1$). Thus, again, we will take it as our model for the dark energy.

At the same time, there exists strong evidence for the scalar fields to be the dark matter at a galactic level. If the dark matter component is the scalar field, then it was demonstrated in [9] that a scalar field fluctuation could behave in exactly the same way as the halo of a galaxy. The halos of galaxies (the scalar field fluctuations) could be axially symmetric [18] or spherically symmetric [19], in both cases the geodesics of exact solutions of the Einstein equations with an exponential potential fit the rotation curves of galaxies quite well. In addition, the ΛCDM model over-predicts subgalactic structure and singular cores of the halos of galaxies [20]. In order to solve these problems, some authors have proposed power-law and power-law-like scalar potentials [17, 21–23] to be the dark matter in the Universe, and it is worth mentioning that some of them could be tracker solutions themselves [23]. Much attention has been put on the quadratic potential Φ^2, because of the well known fact that it behaves as pressureless matter due to its oscillations [24], implying that $\omega_\Phi \simeq 0$, for $\langle p_\Phi \rangle = \omega_\Phi \langle \rho_\Phi \rangle$. A reliable model for the dark matter can then be the potential (see [8, 17] and references therein)

$$V(\Phi) = V_o \left[\cosh\left(\lambda \sqrt{\kappa_o}\Phi\right) - 1\right] = \begin{cases} (V_o/2)\left(\lambda\sqrt{\kappa_o}\Phi\right)^2 & |\lambda \sqrt{\kappa_o}\Phi| \ll 1 \\ (V_o/2)\exp\left(\lambda\sqrt{\kappa_o}\Phi\right) & |\lambda \sqrt{\kappa_o}\Phi| \gg 1; \end{cases} \tag{3}$$

because this potential joins together the attractive properties of an exponential potential and the already mentioned quadratic potential, as can be seen from its asymptotic behaviour.

We consider a flat, homogenous and isotropic Universe. Thus we use the flat Friedmann–Robertson–Walker (FRW) metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2(\phi)\,\mathrm{d}\phi^2\right)\right]. \tag{4}$$

The components of the Universe are baryons, radiation, three species of light neutrinos, etc, and two minimally coupled and homogenous scalar fields Φ and Ψ, which represent the dark matter and the dark energy, respectively. Thus, the evolution equations for this Universe are

$$\begin{aligned} &H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \tfrac{1}{3}\kappa_o\,(\rho + \rho_\Phi + \rho_\Psi), \\ &\dot\rho + 3H\,(\rho + p) = 0, \\ &\ddot\Phi + 3\dot H\dot\Phi + \frac{\mathrm{d}V(\Phi)}{\mathrm{d}\Phi} = 0, \\ &\ddot\Psi + 3\dot H\dot\Psi + \frac{\mathrm{d}\tilde V(\Psi)}{\mathrm{d}\Psi} = 0, \end{aligned} \tag{5}$$

where $\kappa_o \equiv 8\pi G$ and ρ are the energy density of radiation, plus baryons, plus neutrinos, etc, $\rho_\Phi = \frac{1}{2}\dot\Phi^2 + V(\Phi)$ and $\rho_\Psi = \frac{1}{2}\dot\Psi^2 + \tilde V(\Psi)$.

We start the evolution of the Universe in the radiation-dominated era (RD), with large (small) and negative (positive) values for the scalar field Φ (Ψ). Taking the initial condition $\rho_{i\Psi} < \rho_{i\gamma}$, the energy density ρ_Ψ is subdominant and behaves as a cosmological constant. The tracker solution (2) will be reached only until the matter-dominated era (MD), that is, until the background equation of state becomes $\omega_b = 0$. After that, it will evolve with a constant equation of state ω_Ψ and will dominate the current evolution of the Universe as an effective exponential potential. The Universe would then be in a power-law inflationary stage. More details can be found in [12]. Now, we will focus our attention on the potential (3).

During RD, the scalar field energy density ρ_Φ tracks the radiation energy density. The ratio of ρ_Φ to the total energy density is constant and is equal to (see [10, 25] and references therein)

$$\frac{\rho_\Phi}{\rho_\gamma + \rho_\Phi} = \frac{4}{\lambda^2}, \tag{6}$$

with ρ_γ being the contribution due to radiation. In order to recover the success of the CDM model, we will make the scalar energy follow standard cold dark matter at the epoch of its oscillations. Thus, we investigate the behaviour of the scalar field Φ near to the transition point $|\sqrt{\kappa_o}\lambda\Phi| = 1$ (see equations (3)), i.e. the point when the scalar field is leaving the radiation solution (exponential-like potential) and entering the dust solution (quadratic-like potential). Taking a_* as the value for the scale factor when this transition occurred (the scale factor has been normalized to $a = 1$ today), we find that it can be approximately given by

$$a_* \approx \frac{4}{\lambda^2 - 4}\left(\frac{\Omega_{o\gamma}}{\Omega_{oCDM}}\right). \tag{7}$$

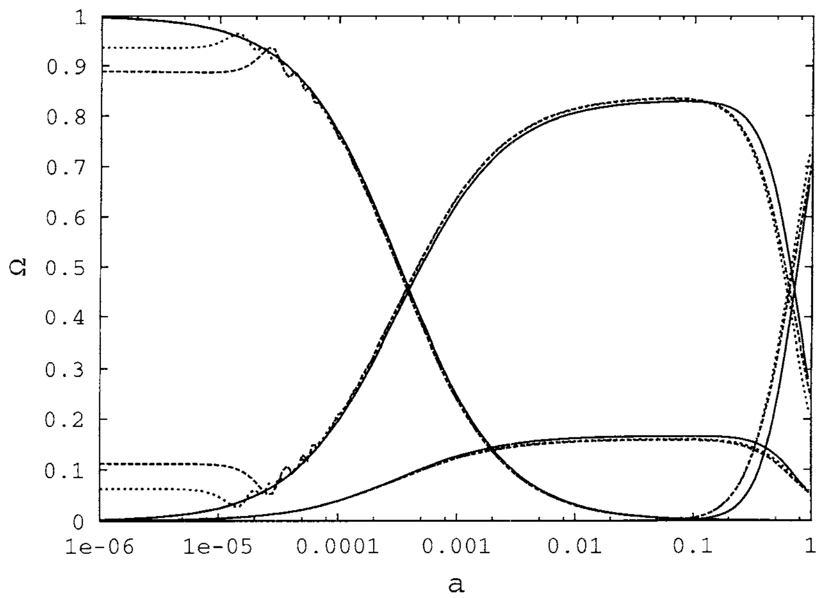

Figure 1. Evolution of the dimensionless density parameters Ω versus the scale factor a with $\Omega_{oM} = 0.30$: ΛCDM (solid curves) and $\Psi\Phi$ DM for two values of $\lambda = 6$ (broken curves), $\lambda = 8$ (dotted curves). The equation of state for the dark energy is $\omega_\Psi = -0.8$.

From this it can be shown that for potential (3) it follows that

$$\frac{\kappa_o V_o}{\left(\lambda^2 - 4\right)^3} \simeq \frac{1.7}{3}\left[\left(\frac{\Omega_{oCDM}}{\Omega_{o\gamma}}\right)^3 \Omega_{oCDM}\right] H_o^2, \tag{8}$$

where Ω_{oCDM} and $\Omega_{o\gamma}$ are the current measured values for the densities of dark matter and radiation, respectively, and H_o is the current Hubble parameter. The restriction from nucleosynthesis for the early exponential behaviour of the potential requires

$$\frac{\rho_\Phi}{\rho_\gamma} = \frac{4}{\lambda^2 - 4} < 0.2 \tag{9}$$

at the radiation-dominated era [10]. Then we have that $\lambda > 2\sqrt{6}$. A numerical solution for the density parameters $\Omega_X = (\kappa_o \rho_X)/(3H^2)$ is shown in figure 1. The time when oscillations start is given by equation (7), and with the values from equation (8) the solution mimics quite well the standard CDM model until today (see, for example, [12]). Note that the change of ρ_Φ to a dust solution occurred before the radiation–matter equality for the values given by equation (8). This allows the scalar field Φ to dominate the evolution of the Universe later, and to provoke an MD era [26].

Now we will investigate the fluctuations in the scalar dark matter component. Using an amended version of CMBFAST [27] and taking adiabatic initial conditions [10], we observe that the scalar fluctuations of Φ make the scalar density contrast $\delta_\Phi = (\delta\rho_\Phi/\rho_\Phi)$ follow the standard dark matter density contrast (see figure 2) [28]. We have then a kind of tracker solution for the fluctuations of the scalar dark matter, too. This last fact makes the potential (3) a reliable dark matter one.

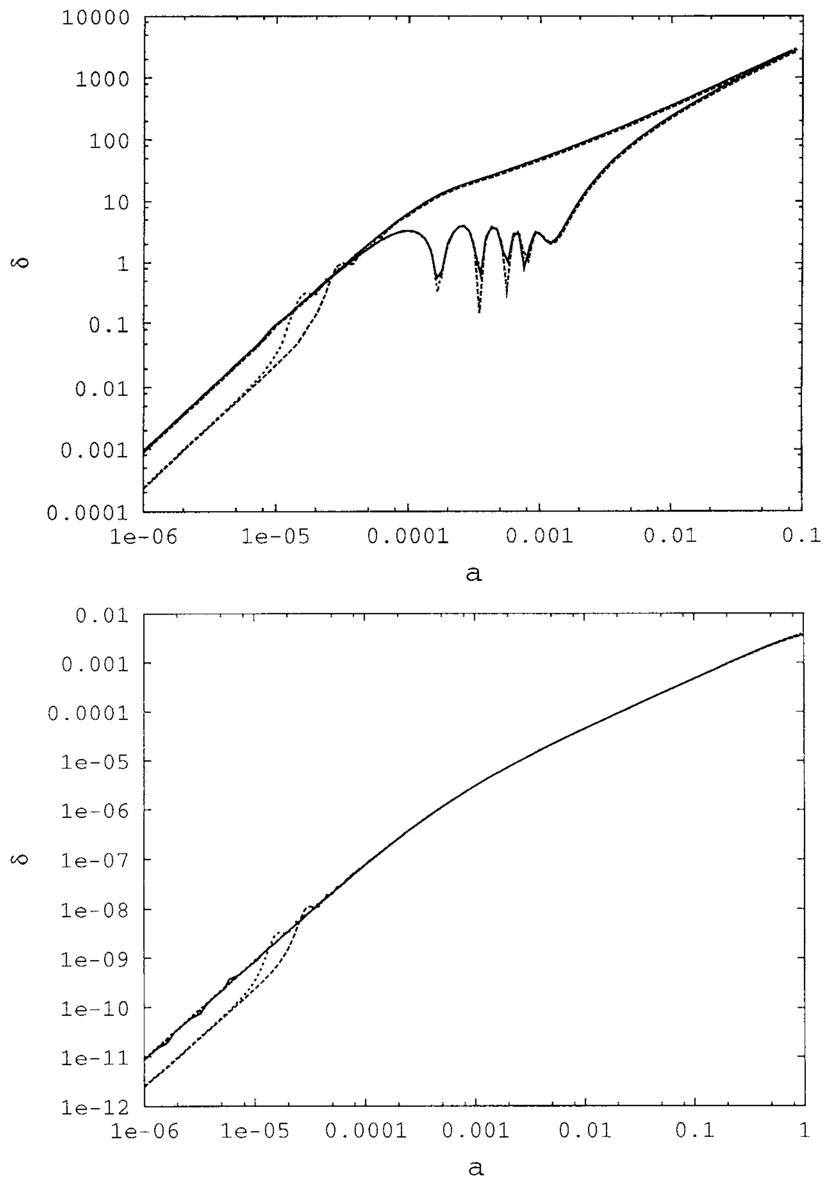

Figure 2. Evolution of the density contrasts δ_b (baryons), δ_{CDM} (standard cold dark matter) and δ_Φ (scalar dark matter) versus the scale factor a taking $\Omega_{oM} = 0.30$ for the models given in figure 1. The modes shown are $k = 0.1$ Mpc^{-1} (top) and $k = 1.0 \times 10^{-5}$ Mpc^{-1} (bottom).

The mass for the scalar field Φ is $m_\Phi^2 = V''(0) = \kappa_o V_o \lambda^2$. Observe that from equation (9) we have a minimal value for the mass of the field. Using equation (8) we obtain

$$m^2_{\Phi,min} \simeq 1.08 \times 10^5 \left[\left(\frac{\Omega_{oCDM}}{\Omega_{o\gamma}} \right)^3 \Omega_{oCDM} \right] H_o^2, \tag{10}$$

implying that $m_\Phi > 3 \times 10^{-26}$ eV; thus, we are dealing with an ultra-light particle as dark matter. Since the Compton length is related to the mass by $\lambda_C = m_\Phi^{-1}$, there will be a maximum value for λ_C given by

$$\lambda_{C,max} \simeq 3.0 \times 10^{-3} \left[\left(\frac{\Omega_{oCDM}}{\Omega_{o\gamma}} \right)^3 \Omega_{oCDM} \right]^{-1/2} H_o^{-1}, \tag{11}$$

and then $\lambda_C < 200$ pc.

From equation (8), we can see that there is a degeneracy because of the infinite pairs (V_o, λ) that are available for the same values of Ω_{oCDM} and $\Omega_{o\gamma}$, and that we recover the standard dark matter model if $\lambda \to \infty$. In fact, observe that we have a one-parameter theory where we can chose V_o, λ or m_Φ as a free parameter. Then, we need another observational constraint to fix completely the parameters of the potential. In [17] it was suggested that the Compton length could be a cut-off for structure formation, but its value is not big enough to be useful. In [22] a similar model is studied, but here the scalar particles behave like a relativistic gas before the time of radiation–matter equality, where the gas is non-relativistic at the current epoch. This last fact ensures that the minimal scale for dark matter halos is of the order of kpc. The potential used in [22] is

$$V(\Phi) = \tfrac{1}{2} m_\Phi^2 \Phi^2 + \kappa \Phi^4, \tag{12}$$

where κ is the dimensionless free parameter of the model. For the potential (3), κ is no longer a free parameter, but $\kappa = \kappa_o \lambda^2 m_\Phi^2 / 4!$. Then, in our case the minimal radius for compact equilibrium now reads [22]

$$r_c = \tfrac{3}{2} (\kappa_o V_o)^{-1/2} . \tag{13}$$

Taking the minimal value for λ allowed by nucleosynthesis and using equation (8), the available values for r_c are

$$r_c \leqslant 2 \times 10^{-2} \left[\left(\frac{\Omega_{oCDM}}{\Omega_{o\gamma}} \right)^3 \Omega_{oCDM} \right]^{-1/2} H_o^{-1}, \tag{14}$$

thus $r_c \leqslant 6\lambda_C \simeq 1.2$ kpc. This value can be useful in order to explain the suppression of galactic substructure and could give us the new constraint we need to fix all the parameters of the model.

Summarizing, a model for the Universe where 95% of the energy density is of scalar nature can be possible. This would have strong consequences in structure formation, like the suppression of subgalactic objects due to the dark matter composed of a ultra-light particle.

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References

[1] Schramm D N 1998 *Nuclear and Particle Astrophysics (Cambridge Contemporary Astrophysics)* ed J G Hirsch and D Page (Cambridge: Cambridge University Press)
Shi X, Schramm D N and Dearborn D 1995 *Phys. Rev.* D **50** 2414–20
[2] Ostriker J P and Steinhardt P J 1995 *Nature* **377** 600
[3] Perlmutter S *et al* 1999 *Astrophys. J.* **517** 565
[4] Riess A G *et al* 1998 *Astron. J.* **116** 1009–38
[5] Steinhardt P J 1996 *Nature* **382** 768
[6] Turner M S 1999 *Astron. Soc. Pac. Conf. Ser.* vol 666
(Turner M S 1999 *Preprint* astro-ph/9811454)

[7] Bahcall N A, Ostriker J P, Perlmutter S and Steinhardt P J 1999 *Science* **284** 1481–8
(Bahcall N A, Ostriker J P, Perlmutter S and Steinhardt P J 1999 *Preprint* astro-ph 9906463)
[8] Sahni V and Starobinsky A 2000 *Int. J. Mod. Phys* D to appear
(Sahni V and Starobinsky A 1999 *Preprint* astro-ph/9904398)
[9] Guzmán F S and Matos T 2000 *Class. Quantum Grav.* **17** L9–16
(Guzmán F S and Matos T 2000 *Preprint* 1998 gr-qc/9810028)
[10] Ferreira P G and Joyce M 1998 *Phys. Rev.* D **58** 023503
[11] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582–5
Zlatev I, Wang L and Steinhardt P J 1999 *Phys. Rev. Lett.* **82** 896–9
[12] Ureña-López L A and Matos T 2000 *Preprint* astro-ph/0003364
[13] Wang L, Caldwell R R, Ostriker J P and Steinhardt P J 2000 *Astrophys. J.* **530** 17–35
(Wang L, Caldwell R R, Ostriker J P and Steinhardt P J 1999 *Preprint* astro-ph/9901388)
[14] Barreiro T, Copeland E J and Nunes N J 1999 Report no SUSX-TH-016
(Barreiro T, Copeland E J and Nunes N J 1999 *Preprint* astro-ph/9910214)
[15] Matos T, Guzmán F S and Ureña-López L A 2000 *Class. Quantum Grav.* **17** 1707–12
(Matos T, Guzmán F S and Ureña-López L A 1999 *Preprint* astro-ph/9908152)
[16] Peebles P J E and Ratra B 1988 *Astrophys. J. Lett.* **325** L17
Ratra B and Peebles P J E 1988 *Phys. Rev.* D **37** 3406
[17] Sahni V and Wang L 1999 *Preprint* astro-ph/9910097
[18] Matos T and Guzmán F S 2000 *Ann. Phys., Lpz.* **9** SI133–6
(Matos T and Guzmán F S 2000 *Preprint* astro-ph/0002126)
[19] Matos T, Guzmán F S and Nuñez D 2000 *Preprint* astro-ph/0003398 (*Phys. Rev.* D, at press)
[20] Navarro J F and Steinmetz M 2000 *Astrophys. J.* **528** 607
[21] Peebles P J E and Vilenkin A 1999 *Phys. Rev.* D **60** 103506
Peebles P J E 2000 Fluid dark matter *Preprint* astro-ph/0002495
[22] Goodman J 2000 Repulsive dark matter *Preprint* astro-ph/0003018
[23] Zlatev I and Steinhardt P J 1999 *Phys. Lett.* B **459** 570–4
(Zlatev I and Steinhardt P J 1999 *Preprint* astro-ph/9906481)
[24] Turner M S 1983 *Phys. Rev.* D **28** 1253
Ford L H 1987 *Phys. Rev.* D **35** 2955
[25] Chimento L P and Jakubi A S 1996 *Int. J. Mod. Phys.* D **5** 71
[26] de la Macorra A and Piccinelli G 2000 *Phys. Rev.* D **61** 123503
(de la Macorra A and Piccinelli G 1999 *Preprint* astro-ph/9909459)
[27] Seljak U and Zaldarriaga M 1996 *Astrophys. J.* **469** 437
[28] Ma C P and Berthshinger E 1995 *Astrophys. J.* **455** 7